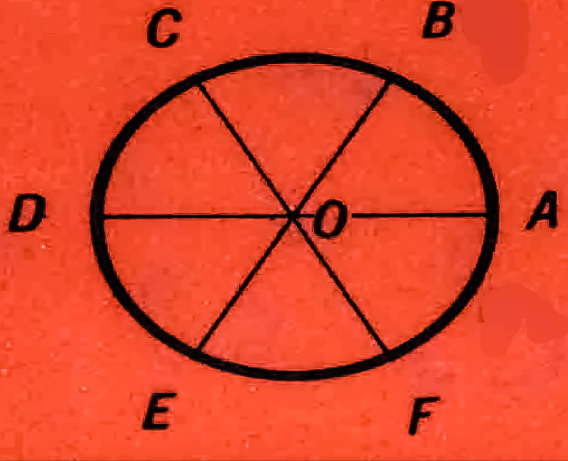
PROBLEM BOOK:

Algebra and Elementary Functions

A. KUTEPOV
and
A. RUBANOV

MIR PUBLISHERS
MOSCOW



А. К. КУТЕПОВ, А. Т. РУБАНОВ

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Translated from the Russian
by LEONID LEVANT

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FOREWORD TO THE FIRST RUSSIAN EDITION

This problem book is intended for secondary school students. In this problem book, the authors have selected and methodologically arranged problems and excercises on algebra and elementary functions which encompass all the standard secondary school mathematics program.

Moreover, this problem book contains two review chapters which help the instructor to organize work for the students with the goal in mind of preparing them for examinations. In each paragraph, the problems and exercises are arranged in an ascending order of complexity.

The most difficult problems are noted with an asterisk. These problems can be used for group work or for homework.

In this work, the authors accepted much valuable advice from their review editors R. A. Kalnin and I. M. Smychkovich, and the authors would like to express their deep thanks to them.

The Authors

FOREWORD TO THE SECOND RUSSIAN EDITION

Because of requests from mathematics teachers, the number of problems and exercises in almost all of the chapters have been increased. In addition, there are three new chapters in the problem book: *Limits*, *The Derivative and its Application*, and *Review Problems Covering the Entire Course*. This problem book is very handy for students and teachers in those technical high schools where the elements of higher mathematics are not taught in a special course.

For the second edition, the authors studied all of the remarks and suggestions which were sent to them. Moreover, the authors would like to thank those individuals who took the time to send in their remarks and ideas. The authors, however, would like to express their special gratitude to their review editor A. T. Rogov who with great attention read the proofs and gave us a number of very valuable suggestions.

The Authors

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CHAPTER I

Review Exercises

1. SHORT-CUT MULTIPLICATION AND DIVISION WITH THE AID OF FORMULAS

Perform the indicated operations using the formulas for short-cut multiplication:

1. (Orally). (1) $(2b^3-1)^2$; (2) $(0.5+2a^5)^2$; (3) 52^2; (4) 1001^2; (5) 198×202; (6) 84^2-16^2; (7) $\left(2\frac{3}{4}\right)^2-\left(1\frac{1}{4}\right)^2$; (8) $\sqrt{313^2-312^2}$.

2. (1) $(0.5x^2-0.2y)^3$; (2) $(-2a-3b)^2$; (3) $(x^n+4)^3$; (4) $(x-2)(x^2+2x+4)$.

3. (1) $(1+2u)(1-2u)(1-4u^2)$;
(2) $(2mn+1)(4m^2n^2-2mn+1)(8m^3n^3-1)$;
(3) $(4^{2m}+4^{3n})(4^{2m}-4^{3n})(4^{4m}-4^{6n})$;
(4) $(1+x-y)(1+x+y)(1+2x+x^2+y^2)$.

4. (1) $(16-25z^4):(4+5z^2)$; (2) $\left(\frac{1}{8}x^3+\frac{1}{27}y^3\right):\left(\frac{1}{2}x+\frac{1}{3}y\right)$;
(3) $(a^{2x}-b^{2x}):(a^x+b^x)$;
(4) $(3^{3x}+3^{3a}):(3^x+3^a)$.

5. (1) $[(2a+3b)^2-1]:(2a+3b+1)$;

(2) $(2^{4m}-2^{4n}):(2^{2m}-2^{2n})$; (3) $(1-x^{6k}):(1-x^{2k})$;

(4) $[(m-n)^3-p^3]:(m-n-p)$.

6. Prove the validity of the following identities:

(1) $(a-b)^2=(b-a)^2$; (2) $(-a-b)^2=(a+b)^2$;

(3) $(a-b)^3=-(b-a)^3$; (4) $(-a-b)^3=-(a+b)^3$.

7. (1) Find the sum of the squared difference between two numbers and their quadruple product.

(2) Find the difference between the squared sum of two numbers and their quadruple product.

8. (1) Prove the identity $(10a+5)^2=100a(a+1)+25$ and (a) check its validity for $a=8$; (b) formulate the rule for an oral squaring of numbers whose last digit is 5.

(2) Calculate orally: 35^2; 95^2; 105^2; 145^2; 195^2.

9. (1) Prove the identity $\left(a+\frac{1}{2}\right)^2=a(a+1)+\frac{1}{4}$ and (a) check its validity for $a=4$, $a=5$; (b) formulate the rule for an oral squaring of mixed numbers whose fractional part equals $\frac{1}{2}$.

(2) Calculate orally: $\left(7\frac{1}{2}\right)^2$; $\left(8\frac{1}{2}\right)^2$; $\left(60\frac{1}{2}\right)^2$; $\left(100\frac{1}{2}\right)^2$.

10. Complete the following expressions to perfect squares:

(1) $x^2+4x+?$; (2) $36x^2-?+49y^2$; (3) $m^4-3m^2+?$;

(4) $4a^2+5a+?$; (5) $?-2a+\frac{1}{4}$; (6) $p^2-0.5p+?$.

11. Single the perfect squares of the sum or difference of two numbers out of the following expressions:

(1) x^2+2x+4; (2) $a^2-6a+13$; (3) $4a^4+5a^2-2$;

(4) $9x^2-8x+2$; (5) $m^2-0.2m+1$; (6) x^2+px+q.

12. Carry out the following operations:

(1) $(2x+y^2)^2+(2x-y^2)^2$;

(2) $\dfrac{\left(\frac{1}{x}+\frac{1}{y}\right)^2-\left(\frac{1}{x}-\frac{1}{y}\right)^2-xy}{2-xy}$;

(3) $\dfrac{(2a+3)^3-(2a-3)^3}{(3a+4)^2+3a^2-24a-7}$;

(4) $\dfrac{(2x+3)^2+2(4x^2-9)+(2x-3)^2}{(2x+3)^2-2(4x^2-9)+(2x-3)^2}$.

13. Prove the following identities:

(1) $(4x^2+4ax+a^2):(2x+a)-(2x+a)^3:(4x^2+4ax+a^2)=0$;

(2) $\left(\dfrac{x-3}{2}\right)^3-\left(\dfrac{x+3}{2}\right)^3=-\dfrac{9(x^2+3)}{4}$;

(3) $\left(\dfrac{x-3}{3}\right)^3+\left(\dfrac{x+3}{3}\right)^3=\dfrac{2x(x^2+27)}{27}$.

14. Simplify the following expressions and find their numerical values:

(1) $9m^2+24mn+16n^2$ for $m=4,\ n=-2$;

(2) $27a^3-54a^2b+36ab^2-8b^3$ for $a=3,\ b=-\dfrac{1}{2}$;

(3) $\dfrac{a^6+8}{a^4-2a^2+4}$ for $a=-4$;

(4) $\dfrac{4x^2+12xy+9y^2-4}{2x+3y+2}$ for $x=4,\ y=-3$.

2. FACTORIZATION

Factorize the following:

15. (1) $2ab^3+6a^2b^2+8a^4b^4$; (2) $3x(x-3)-x+3$;

(3) $m(m-1)+(1-m)^2$; (4) $4a^2(b^2-1)+4b^2(1-b^2)$.

16. (1) $(2p+3q)(p-q)+(2p-3q)(q-p)$;

(2) $(a^2-b^2)(a^2+ab+b^2)-a^3+b^3$;

(3) $3x^3(x^2-x+1)+2x^2(x-x^2-1)+x(-x^2+1-1)$.

17. (1) 2^x+2^{2x}; (2) $a^{3m}+2a^{2m}+a^m$;

(3) 3^x+3^{x+1}; (4) $p^{2x+1}-2p^{2x}+p^{2x-1}$.

18. (1) In the polynomial ax^2+bx+c factor out a;
(2) in the polynomial $3a^2b-5ab^2-b^3$ factor out $3a^2b$.

Factorize the following:

19. (1) $a^3-5a^2-4a+20$; (2) x^3+x^2+x+1;
(3) a^3+3+3a^2+a.

20. (1) $2ax+ay-az-2bx-by+bz$;
(2) $4ax+3xy^2-3ay^2-4x^2$;
(3) $4a^2x-6x-24a^2y+36y$.

21. Factorize the following polynomials replacing the middle term by the sum or the difference of two terms:

(1) $2a^2+10a+12$; (2) a^4+a^2-20;
(3) $x^2-7xy+12y^2$; (4) x^3-3x+2.

Factorize:

22. (1) $64a^2-25b^4$; (2) $a^2+4ab+4b^2-1$;
(3) $4(a+b)^2-9(a-b)^2$.

23. (1) $a^4-a^2(b^2+1)+b^2$; (2) $a^5-4a^3+8a^2-32$;
(3) $a^2+ax-2x^2+a+2x$; (4*) a^4+4;
(5*) x^4-x^2+16; (6*) x^4+6x^2+25.

24. Perform the following operations (orally):

(1) $394\times28+606\times28$; (2) $7.35\times2.5-1.35\times2.5$;
(3) $14\frac{3}{8}\times1\frac{1}{4}-4\frac{3}{8}\times1\frac{1}{4}$;
(4) $24\times2.78+35\times2.78+41\times2.78$.

25. Reduce the given fractions:

(1) $\frac{x^2+2x-8}{(x-1)^2-1}$; (2) $\frac{(x-1)^3-8}{x-3}$;
(3) $\frac{9-12x+4x^2}{2x-3}$.

26. (1) Prove that the polynomial $x^5-x^4-2x^3+2x^2+x-1$ is divisible by (a) x^2-1; (b) $(x+1)^2$; (c) $(x-1)^3$.

(2) Prove that the fraction $\frac{(2x^2+2xy)(x-1)}{x^2-x+xy-y}$ takes on even values for any integral x.

27. Will the value of the following fractions change if x and y are doubled:

(1) $\frac{2x+3y}{3y-x}$; (2) $\frac{2x^2-3y^2}{x+y}$; (3) $\frac{2x^2-3y^2}{x^2+y^2}$.

28. Solve the given equations:

(1) $y+40+(y+3)^2 = (2y+7)^2$;

(2) $(a+2)(a-2)(a^2+4) = 25a^2-16$.

29. (1) Prove that if a is an integer, then a^3-a is exactly divisible by 6.

(2) Knowing that a^3-a, where a is an integer, is divisible by 6, prove that the expressions a^3-7a, a^3+11a, and $5a^3+13a-30$ are also divisible by 6.

3. SIMPLIFYING AND EVALUATING ALGEBRAIC EXPRESSIONS

30. For what values of x do the following fractions have no sense?

(1) $\frac{2x+1}{x-1}$; (2) $\frac{3x^2-2}{x^2-1}$; (3) $\frac{x}{x^3-1}$;

(4) $\frac{x^2}{x^3+1}$; (5) $\frac{2x-1}{3x+1}$; (6) $\frac{3}{3x^2+5}$.

Simplify the following expressions:

31. (1) $\left(\frac{3}{9-m^2}+\frac{2m-1}{m-3}-\frac{m^2-4}{m^2+6m+9}\times\frac{m+3}{m-2}\right):\frac{m}{m-3}$;

(2) $\left(\frac{6a^2-2b^2-ab}{b^2-4a^2}+\frac{3a+1}{5a-1}\times\frac{5a^2-a}{6a^2-3ab-b+2a}+1\right)\frac{a-b}{2b}$.

32. $$\frac{\frac{5z^2}{10z-8}-9(z+2)\left(\frac{1}{14z+28}+\frac{1}{35z-28}\right)+\frac{2}{5z-4}}{\frac{18z-6}{1-7z}\left(\frac{z+3}{3-9z}-\frac{4z-12}{54z^2-6}+\frac{3z+1}{6z+2}\right)}.$$

33. (1) $\left(\frac{3x+2}{2x+3}-\frac{4x-1}{2x+3}-\frac{2x^2+3x}{4x^2+12x+9}\right):\frac{3-2x}{2x+3}$;

(2) $\left(\frac{1}{a^2-9}:\frac{x-a}{3a^2+9a}-\frac{3a}{9-3x-3a+ax}\right)\times\frac{x^3-27}{3a}$.

34. Prove the following identities:

(1) $\left(\frac{y}{x}-\frac{x}{y}\right):\left(2-\frac{x}{y}-\frac{y}{x}\right):\left(\frac{y}{x}+1\right)=\frac{x}{x-y}$;

(2) $\left[\left(\frac{3a}{a^3-b^3}\times\frac{a^2+ab+b^2}{a+b}-\frac{3}{b-a}\right):\frac{2a+b}{a^2+2ab+b^2}\right]\frac{3}{a+b}=\frac{9}{a-b}$.

35. Compute orally:

(1) $\frac{a}{a+1}$ if $a=-\frac{3}{4}$, $a=3$;

(2) $\frac{7y}{(x-y)^2}$ if $x=-5$, $y=2$.

36. Simplify the following expressions and find their numerical values:

(1) $\left[\left(\frac{1}{x}+\frac{1}{y}\right):\frac{x+y}{2}+\frac{1}{x^2}+\frac{1}{y^2}\right]\frac{xy}{(x+y)^2}$ for $x=-\frac{1}{2}$, $y=-2$;

(2) $\left(\frac{a+1}{a-1}-\frac{a-1}{a+1}\right)\left(\frac{1}{2}-\frac{a}{4}-\frac{1}{4a}\right)$ for $a=-3\frac{3}{4}$.

37. Carry out the operation of division and check on the results by evaluating the dividend, the divisor and the quotient for the given values of the letters:

$(x^4+x^3y+7xy^3-3y^4):(3y^2+x^2-xy)$ for $x=4$, $y=-4$.

38. How many times is the number 3^{3^3} greater than 3^{3^2}?

39. A batch of a kg of berries has been stored at a supply depot. The first analysis showed that the berries contain 99% of water. With time the water content reduced and has become equal to 98%. What is now the weight of the berries?

40. Find x if

$$\frac{(17.125+19.38:x)0.2+3\frac{1}{12}:2\frac{1}{18}}{\left(5\frac{17}{32}-4\frac{11}{27}:2+2\frac{1}{4}\times1\frac{3}{8}\right):27.74+\frac{7}{9}}=6.48.$$

4. CARRYING OUT COMPUTATIONS ON A SLIDE RULE

41. Read the following numbers considering them as a number series and rounding them off to three significant digits:

60.1, 0.00601, 17.54, 632.562, 74.033, 540, 483.52.

42. Set the following numbers on the main number scale with the aid of the sighting line of the slide:

(1) 1, 0.5, 2000, 16.250, 85, 0.0064, 10, 0.111;

(2) 46.2, 85.23, 280, 10.2, 0.8426, 318.5, 513;

(3) 1043, 46 452, 51 300, 2.07, 102.4, 0.00487.

43. Find the number of significant digits in the following numbers:

(1) 3, 48, 735, 5007, 27 000, 0.125, 2.13, 156.7;

(2) 14.8, 1002.3, 83745.7, 0.137, 0.0150, 0.00034.

44. Determine the number if its digits are 1, 3, and 8; the number of its significant digits being $+3$, $+1$, $+6$, 0, -2, -5.

Multiply the given numbers using the basic scale of the slide rule:

45. (1) 2×4, 30×800, 0.6×0.7, 14.5×4.9, 5.25×0.8;

(2) 29.6×75, 0.015×46, 1.54×326, 2.08×465;

(3) 0.0064×0.00056, 6430×0.00017, 9820×0.000542.

46. (1) 44×3.5, 23.8×3.1, 202×4.95, 0.874×119;

(2) 0.0312×0.183, 0.202×30.3, 0.022×51.3, 419×0.0368.

47. (1) $0.285\times31.2\times42$; (2) $89.2\times0.63\times1.5$; (3) $42.4\times0.78\times$ $\times0.0075$; (4) $435\times0.538\times0.207\times323$; (5) $18.4\times0.0248\times0.926\times$ $\times41\,600\times0.0576$.

48. Find 10%, 15%, 25.3%, 44%, 56%, 68.5%, 92%, 112% of the numbers 480, 248, and 1120.

49. (1) Compute the weight of a pine board 5.75 m long, 6.5 cm thick, 34 cm wide. The density of pine is 580 kg/m^3.

(2) Compute the volume of a room, whose dimensions are $8.4\times$ $\times5.8\times3.2$ m.

50. (1) Compile a table of wages if the former rates 62.3, 74.8, 86.5 roubles are increased by 18%.

(2) Compile a table for converting inches into millimetres (1 inch = 25.4 mm).

(3) Convert the following from degrees Centigrade to degrees Kelvin, 10°, 18.5°, 19.8°, 180°, 39.5°.

51. Divide the given numbers using the basic scale of the slide rule:

(1) 6 : 3, 6.4 : 3, 5.25 : 32, 12.7 : 1.6, 0.18 : 0.45;

(2) 7.12 : 2.37, 8.98 : 0.136, 19.8 : 1.06, 42.6 : 2.68;

(3) 849 : 82, 0.642 : 0.0254, 0.196 : 127, 86.7 : 0.621;

(4) 0.003 : 0.0347, 0.008 : 0.00543, 0.00056 : 0.00392.

52. Solve the following equations:

(1) $0.0392x = -8.44$; (2) $6.749x = 5.37$;

(3) $18.7x = -249.2$.

53. (1) Find the altitude of a parallelogram if its area is equal to 254.7 cm^2, and the base to 27.3 cm.

(2) The piecework payment for the whole work fulfilled amounts to 4.35 roubles. The time required for its fulfilment is 8.5 hours. What is the time-work payment?

CHAPTER II

Approximate Computations

5. APPROXIMATE COMPUTATIONS

54. In 1976 the Soviet Union produced 520 million tons of oil, 712 million tons of coal, 862,000 tractors, 2,025,000 automobiles, 7.1 million TV-sets. Which of the mentioned numbers should be considered as approximate, and which as exact?

55. Round the numbers 7.365 and 8.455 to the second and then to the first decimal place.

56. Round the given numbers successively to the third, second and first decimal places, then to units, tens, hundreds and thousands:

4375.5494, 5497.1857, 8040.5048, 9485.3755, 2243.5245.

57. Carry out the successive rounding of the following fractions to the third, second and first decimal places:

$$\frac{3}{7},\ \frac{7}{99},\ \frac{5}{6},\ \frac{2}{13},\ \frac{4}{11},\ \frac{7}{8},\ \frac{2}{17},\ \frac{5}{18},\ \frac{7}{19}.$$

58. Find the absolute error of the following approximate numbers if all their significant digits are correct:

124, 347, 13.7, 18.1, 1.75, 2.43, 0.034,

9.051, 2.0537, 8.2354, 4.20448, 7.11831.

59. Find the lower and the upper bounds of the following approximate quantities:

$17.8\ (\pm 0.1)$, $36.5\ (\pm 0.05)$, $24.3\ (\pm 0.1)$, 42.4 g $(\pm 0.1$ g),
10.5 m $(\pm 0.1$ m), 36.7 s $(\pm 0.1$ s), 25.7 km $(\pm 0.1$ km),
$14°13'\ (\pm 1')$, $12°15'18''\ (\pm 2'')$.

60. Find the upper and the lower bounds of the approximate numbers obtained as a result of replacing the common fractions $\frac{3}{7}$, $\frac{2}{3}$, $\frac{4}{9}$, $\frac{7}{13}$ by respective decimal fractions accurate to the second decimal place.

61. As is known, the radius of the Earth is approximately equal to 6371 km. Find the absolute error and the upper and lower bounds of this approximate number.

62. Determine the relative error of the following approximate numbers if all their significant digits are correct:

42.3, 3.75, 4.837, 142.484, 15.7574, 18.340.

63. Which measurement is more exact?

(1) 750 m $(\pm 0.5$ m), 12.3 m $(\pm 0.05$ m) or 1.25 m $(\pm 0.01$ m);

(2) 10.6 s $(\pm 0.2$ s) or 12.5 s $(\pm 0.1$ s).

64. When measuring the length of a beam the number 4.46 m is obtained. Determine the relative error and the bounds within which the true length of the beam lies.

65. Given the relative error of a number, find its absolute error and the bounds within which the true number is contained if:

(1) $x = 34.6$, $\delta = 0.4\%$; (2) $x = 75.8$, $\delta = 0.3\%$;

(3) $x = 100$, $\delta = 0.5\%$; (4) $x = 12.7$, $\delta = 1.2\%$.

66. How many correct significant digits are there in the following approximate numbers:

1895, 230, 1.303, 20.04, 0.345, 0.04, 2.5020?

67. Express 12,500 m, 1340 m, 5800 m in kilometres considering the zeros as (1) correct significant digits, (2) non-significant digits.

68. Is there any difference between the following entries: "The water has been heated to the temperature of 45 °C", and "The water has been heated to the temperature of 45.0 °C"; "The car runs with the speed 80 km/h", and "The car runs with the speed 80.0 km/h"?

69. How many correct digits have the following approximate numbers:

1542 (± 6), 1542 (± 4), 8.44 (± 0.05), 8.44 (± 0.02), 8.44 (± 0.07), 34.546 (± 0.003), 8.447 (± 0.005)?

70. Rewrite the following approximate numbers using only correct digits:

1344 (± 3), 1344 (± 7), 9.47 (± 0.04), 30.03 (± 0.02).

71. Carry out the addition of the approximate numbers:

(1) $2.3+0.47+3.144+0.1568$;

(2) $4.42+3.747+0.0347+2.45467$.

72. Carry out the subtraction of the approximate numbers:

(1) $34.71-21.4234$; (2) $87.3-4.456$;

(3) $25.48-9.752$.

73. Calculate to within 0.01:

(1) $\frac{1}{2}+\frac{1}{3}+\frac{1}{5}+\frac{1}{7}$;

(2) $1+\frac{1}{4}+\frac{3}{7}+\frac{2}{9}+\frac{2}{11}$.

74. Multiply the following pairs of approximate numbers:

2.003×3.14, 3.76×0.04, 1.05×2.301, 75.3×2.102, 4.05×0.0037, 2.0503×15.1, 0.003×25.7.

75. Carry out the division of the given approximate numbers:

$7.354 : 2.123$, $4.58 : 22.158$, $4.34 : 3.1424$, $23.4 : 0.4536$, $48.5 : 1.3759$, $7.5 : 15.358$.

76. Perform the indicated operations with the approximate numbers:

(1) $\frac{45.372\times 2.17+34.5\times 2.1}{7.8432}$;

(2) $\frac{153.413\times 0.873-34.873\times 1.543}{435.17}$;

(3) $\left(3\frac{2}{3}-1\frac{7}{8}\right)\times 14\frac{5}{6} : 3\frac{11}{12}$ to within 0.01.

77. Compute the weight of a metal rod having the form of a circular cylinder whose diameter $d \approx 3.64$ cm and length $l \approx 20.7$ cm if its density $\varrho \approx 7800$ kg/m^3.

78. Find the weight of copper wire ($d \approx 0.4$ cm and $l \approx 1.56$ m) if its density $\varrho = 8800$ kg/m^3.

79. Compute (with the aid of the Tables):

(1) 3.23^2, 7.337^2, 1.584^2, 0.3468^2, 0.05437^2, 25.74^2, 4.347^2, 1.75^3, 2.37^3, 45.74^3, 53.12^3, 67.32^3, 0.345^3, 0.6487^3, 0.0373^3, 0.04586^3;

(2) $\sqrt{3745}$, $\sqrt{8432}$, $\sqrt{954.3}$, $\sqrt{747.67}$, $\sqrt{82.44}$, $\sqrt{13.72}$, $\sqrt{1.458}$, $\sqrt{3.744}$, $\sqrt{0.4537}$, $\sqrt{0.9247}$, $\sqrt{0.08347}$, $\sqrt{0.04721}$, $\sqrt{0.00573}$, $\sqrt{0.007483}$, $\sqrt{0.0000562}$;

(3) the numbers reciprocal to the given ones:

1.3754, 2.1741, 8.3454, 0.3893, 0.00456,
0.0007537, 12.73, 15.748, 164.76, 183.91;

(4) the length of the circumference and the area of the circle by the given diameter:

0.3754, 0.4037, 2.8475, 4.537, 85.37, 41.03, 12.74,
9.05, 1.0034, 6.0087.

6. CARRYING OUT COMPUTATIONS WITH THE AID OF A SLIDE RULE

Check on the following results:

80. (1) $\dfrac{75.8 \times 4.65}{22.3} \approx 15.8$;

(2) $\dfrac{27.5 \times 36.9}{4.35} \approx 233$;

(3) $\dfrac{6.42 \times 42.5 \times 8.5}{9.75 \times 36} \approx 6.61$;

(4) $\dfrac{3.25 \times 5.2 \times 6.86}{8.6 \times 143} \approx 0.0943$.

81. (1) $\dfrac{23.8 \times 0.94 \times 112}{21.3 \times 1.71} \approx 68.8$;

(2) $\dfrac{5.69 \times 2.17 \times 0.0074}{3.26 \times 18.85} \approx 0.00149$;

(3) $\dfrac{40.5\times26.6\times0.134\times0.96}{66.3\times12.45\times5.85}\approx 0.0287$;

(4) $\dfrac{7.45\times62.5\times0.52\times12.2}{14.05\times37.6\times4.82}\approx 1.16$.

82. (1) $\dfrac{0.456\times1.025\times0.843\times75.4}{75.8\times.0.905\times17.25}\approx 0.0251$;

(2) $\dfrac{3.14\times192\times27.5\times72.5}{74\times21.8\times14.4}\approx 51.7$;

(3) $\dfrac{5.8\times7.5\times86}{2.6\times28.8}\approx 50$;

(4) $\dfrac{0.74\times42.8\times0.094}{2.13\times0.033\times17.1}\approx 2.48$.

83. (1) $\dfrac{20.4\times0.079\times23.4}{52.6\times0.146\times86}\approx 0.057$;

(2) $\dfrac{0.081\times428.6\times100}{0.52\times83.8\times57.3}\approx 1.39$;

(3) $\dfrac{27.6\times0.00615\times1.07}{85.6\times3.46\times0.0216}\approx 0.0284$;

(4) $\dfrac{64.2\times0.42}{14.5\times0.0191\times7.87}\approx 12.4$.

84. Find x from the proportions:

(1) $\dfrac{755}{620}=\dfrac{570}{x}$; (2) $\dfrac{2.5}{1.4}=\dfrac{0.75}{x}$;

(3) $\dfrac{2.5}{6.8}=\dfrac{x}{1.5}$; (4) $\dfrac{x}{3.2}=\dfrac{14}{57.3}$;

(5) $\dfrac{62.3}{x}=\dfrac{7.48}{0.865}$; (6) $\dfrac{x}{37}=\dfrac{7}{10}$.

85. (1) Find the numerators of the following equal fractions:

$$\frac{x_1}{4.3}=\frac{x_2}{6}=\frac{x_3}{8}=\frac{x_4}{2.5}=\frac{7}{9}.$$

(2) Find the denominators of the following equal fractions:

$$\frac{1.4}{0.23}=\frac{18}{x_1}=\frac{28}{x_2}=\frac{0.48}{x_3}=\frac{61}{x_4}.$$

86. Find x, y, and z from the equations

$$\frac{16.9}{x}=\frac{4.46}{y}=\frac{6.25}{z}=7.7.$$

87. (1) Find the following numbers on the left end of the scale of squares:

8, 167, 284, 3.56, 456, 895, 0.04, 0.0745.

(2) Find the following numbers on the right end of the scale of squares:

16, 0.2, 19.2, 3650, 87.5, 0.475, 1000.

88. Carry out multiplication on the scale of squares:

25×1.2, 3.4×5.7, 0.25×3.7, 0.039×5.6, 24.7×0.0348.

89. Carry out the operation of division using the scale of squares:

34 : 23, 1.5 : 2.3, 5.4 : 0.75, 8.7 : 0.37, 480 : 2.36, 54.7 : 0.036.

90. Find:

(1) 6.8^2, 2.36^2, 0.295^2, 69.3^2, 1.165^2, 0.205^2;

(2) 0.00592^2, 368^2, 14.5^2, 4.25^2, 0.202^2, 427^2.

91. Set:

(1) the numbers 8, 4, 4.58, 0.009, 6580, 1000, 2.89 in the left part of the scale of cubes.

(2) the numbers 40, 45, 45.8, 0.09, 75.5, 10 000, 38.5 in the middle part of the scale of cubes.

(3) the numbers 800, 840, 844, 0.9, 573, 100, 385.8 in the right-hand part of the scale of cubes.

92. Find:

(1) 2.8^3, 13.5^3, 0.57^3, 8.6^3, 0.34^3, 2.81^3;

(2) 1.725^3, 1.062^3, 0.084^3, 0.746^3, 4.22^3, 5.345^3, 1.005^3.

93. Check on the following results:

(1) $(5.3)^2\times270\approx7590$; (2) $\dfrac{(3.2)^2\times0.65}{0.0215}\approx310$;

(3) $\left(\dfrac{107\times0.206}{30.04}\right)^2\approx0.526$; (4) $\dfrac{31.4\times(2.56)^2\times3.47}{25.1\times(0.2274)^2}\approx38$.

94. What distance will a freely falling body cover during 13.6 s ($g = 9.8$ m/s^2) from the start?

95. Find the area of a circle, knowing its diameter, for $d =$ 20.5 m; $d = 0.184$ m; $d = 102$ m.

96. Compute the volume of a cylinder if (1) $d = 30.2$ dm; $H = 5.4$ dm; (2) $d = 4.6$ m; $H = 3$ m.

97. Compute the surface area of a sphere given its diameter: $d = 13.5$ cm; $d = 2.4$ m; $d = 0.83$ m.

98. Compute the volume of a sphere given its diameter: $d = 35.1$ dm; $d = 4.2$ dm; $d = 0.58$ m.

99. Compute the area of a circus arena if its diameter $d = 13$ m.

100. What is the weight of a cylindrical iron pig if $d = 300$ mm and $H = 800$ mm? Specific gravity of iron is equal to 7.2.

CHAPTER III

First-Degree Equations

7. EQUATIONS IN ONE UNKNOWN

101. Which of the following equalities are identities and which are equations:

(1) $3x+1 = 3(x-1)+4$; (2) $(x-5)^2 = x^2-10x+25$;

(3) $\frac{1}{2}(5-x) = 4x$; (4) $\frac{1}{5}x+1 = \frac{x+5}{5}$?

102. Are the following equations equivalent?

(1) $10x-2 = 4$ and $5x-1 = 2$;

(2) $(x-2)(x+3) = 0$ and $x-2 = 0$;

(3) $(x+4)(x-2) = 4(x+4)$ and $x-2 = 4$;

(4) $\frac{1}{6}(5x-3) = 3$ and $5x-3 = 18$;

(5) $(x-2)^2 = 3(x-2)$ and $x-2 = 3$;

(6) $2x-3+\frac{x+1}{x-4} = 5+\frac{x+1}{x-4}$ and $2x-3 = 5$;

(7) $2x-3+\frac{x+1}{x+4} = 5+\frac{x+1}{x+4}$ and $2x-3 = 5$;

(8) $\frac{x+3}{x+1} = \frac{1-x}{x+1}$ and $x+3 = 1-x$;

(9) $\frac{x+3}{x+1} = \frac{2x-1}{x+1}$ and $x+3 = 2x-1$;

(10) $x^2-5x+8 = 2$ and $x^2-5x+6 = 0$;

(11) $3x-1 = 5$ and $|3x-1| = 5$.

103. 1. For what values of m will the given equations be equivalent?

(a) $2x+3 = 12$ and $2x+3 = 12m$;

(b) $2x+3 = 12$ and $(2x+3)\,m = 12m$.

2. Set up an equation which has no roots.

3. Set up an identity with respect to x.

4. Set up an equation equivalent to the equation $2x+3 = 9$.

Solve the following equations:

104. (1) $x-7-9x = 4x-3-8x$;

(2) $2\frac{1}{3}x-3\frac{1}{2}x = 3\frac{1}{5}x-4\frac{2}{3}x-9$.

105. (1) $(y+5)(y+2)-3(4y-3) = (5-y)^2$;

(2) $(x+2)^3-(x-2)^3 = 12(x^2-x)-8$;

(3) $(3y-1)^2-5(2y+1)^2+(6y-3)(2y+1) = (y-1)^2$;

(4) $6x+(x-2)^2 = (x+3)(x-3)+2x+13$.

106. (1) $7-2x-\frac{1-3x}{7} = 2-\frac{2x-1}{3}$;

(2) $\frac{9x-5}{3x+1}+\frac{12x+1}{2-6x} = \frac{9-108x+36x^2}{4(9x^2-1)}$;

(3) $\frac{4-x}{x-3} = 5+\frac{1}{x-3}$;

(4) $\frac{3x-2}{3}-\frac{4-x}{2}+2 = \frac{3x-5}{6}+2x$.

107. (1) $\frac{5y-6}{25y^2-4}+\frac{5y-18}{12y-30y^2}+\frac{1}{6y} = \frac{4y-5}{20y^2+8y}$;

(2) $\frac{8y-5}{2y+5} = 5-\frac{3y+7}{3y+2}$;

(3) $\frac{6}{x+2}+\frac{x+2}{2-x}-\frac{x^2}{4-x^2} = 0$.

108. (1) $2x^2 = 5x$; (2) $(x+2)(x-4) = x+2$;

(3) $3a^2x^2 = 5abx$; (4) $(x-b)^2 = 2a(x-b)$.

109. (1) $3(a-4x)+7(2x-a)-5(3x+2a)+a=0$;

(2) $\dfrac{y}{y-1}-\dfrac{4m-3y^2}{my-m-3y+3}-\dfrac{3y}{m-3}=0$;

(3) $\dfrac{ax+b}{x}:\dfrac{a}{d}=\dfrac{cx+d}{x}:\dfrac{c}{b}$.

110. (1) $\dfrac{a+x}{b}-\dfrac{x-b}{a}=2$;

(2) $\dfrac{a^3-1}{a^3+1}=\dfrac{a(x-1)+a^2-x}{a(x-1)-a^2+x}$;

(3) $\dfrac{x}{a}(3ab+1)=\dfrac{3ab}{a+1}+\dfrac{(2a+1)x}{a(a+1)^2}+\dfrac{a^2}{(a+1)^3}$.

111. Solve the equation $\dfrac{a}{a-c}=\dfrac{b}{b-c}$ with respect to a, b, c.

112. The aerial velocity of sound depends on the air temperature and is expressed by the formula $v=(331+0.6t)$ m/s. Find the time t and determine at what temperature the sound velocity is equal to 355 m/s.

113. (1) $S=\dfrac{a+b}{2}\times h$. Find h, a;

(2) $R=\dfrac{r_1r_2}{r_1+r_2}$. Find r_1; (3) $t_0=\dfrac{m_1t_1+m_2t_2}{m_0}$.

Find m_0, t_1; (4) $Q=mc(T-t)$. Find c, T.

114. Solve the following equations:

(1) $|x|-1=5$; (2) $3|x|+1=|x|+7$;

(3) $|x+4|=4$; (4) $|2x-3|=5$;

(5) $|5-|x||=3$; (6) $||2x-3|-4|=6$;

(7) $|x+3|+|x-1|=6$; (8) $|3x-2|-|3-x|=3$;

(9)* $|x-1|-2|x+3|+x+7=0$.

115. (1) Given the expressions $y_1=4x+5$ and $y_2=-3x-2$. For what values of x do we have the equality $y_1=y_2$? Verify the results by plotting the graphs of the functions.

(2) For what values of x does each of the expressions $(x-3)(x+4)$ and $(2x+1)(x+7)\left(x-\frac{1}{3}\right)$ vanish?

8. DETERMINANTS

116. Compute the following determinants:

(1) $\begin{vmatrix} 2 & 3 \\ 3 & 4 \end{vmatrix}$; (2) $\begin{vmatrix} -2 & -2 \\ 4 & 0 \end{vmatrix}$; (3) $\begin{vmatrix} 2 & 3 \\ 3 & 4.5 \end{vmatrix}$;

(4) $\begin{vmatrix} -\frac{3}{4} & \frac{1}{3} \\ -1 & -\frac{8}{9} \end{vmatrix}$; (5) $\begin{vmatrix} a & b \\ b & a \end{vmatrix}$; (6) $\begin{vmatrix} \sin\alpha & -\cos\alpha \\ \cos\alpha & \sin\alpha \end{vmatrix}$.

117. Write in the form of determinants the given expressions:
(1) x^2+2x+1; (2) $4a^2-9a-9$; (3) $x^3+y^3+z^3+3xyz$.

118. Compute the determinant $\begin{vmatrix} a & b \\ a & b \end{vmatrix}$ and formulate the property of such determinants. Give examples.

119. Compute the determinant $\begin{vmatrix} a & ka \\ a & ka \end{vmatrix}$ and formulate the property of such determinants. Give examples.

120. Prove that $\begin{vmatrix} a_1 & kb_1 \\ a_2 & kb_2 \end{vmatrix} = k \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}$ and formulate the property of such determinants. Give examples.

121. Perform the following operations:

(1) $\dfrac{\begin{vmatrix} 2a & b \\ b & 2a \end{vmatrix} + \begin{vmatrix} b & 2b \\ -2a & 2b \end{vmatrix}}{\begin{vmatrix} 2 & b \\ -1 & a \end{vmatrix}}$; (2) $\dfrac{\begin{vmatrix} a & -2 \\ b & 3 \end{vmatrix}^2 - \begin{vmatrix} 3a+b & 2 \\ -2 & 4b \end{vmatrix}}{3a-2}$.

122. Solve the following equations:

(1) $\begin{vmatrix} 2x-3x \\ 5x-3 \end{vmatrix} = \begin{vmatrix} 2x+1 & -4 \\ 4 & x \end{vmatrix}$;

(2) $\begin{vmatrix} 3y-1 & \frac{1}{2}(3+3y) \\ 4y & y \end{vmatrix} - \begin{vmatrix} -y & -1 \\ 1 & 3y \end{vmatrix} = 0.$

123. (1) For what x is the determinant $\begin{vmatrix} 2x & 1 \\ -3 & 2 \end{vmatrix}$ equal to the doubled determinant $\begin{vmatrix} 2 & 1 \\ -1 & 3 \end{vmatrix}$?

(2) For what value of m does the equation $\begin{vmatrix} 2x-1 & 2 \\ 3m & -1 \end{vmatrix} = 0$ have the root $-\frac{5}{2}$?

124. (1) For what value of x does the fraction $\frac{D_1}{D_2}$, if $D_1 = \begin{vmatrix} 1 & -2 \\ x & 3 \end{vmatrix}$, $D_2 = \begin{vmatrix} -1 & x \\ 2 & -3 \end{vmatrix}$, (a) equal $\frac{2}{5}$, (b) have no sense?

(2) Find x from the proportion $\frac{D_1}{D_2} = \frac{D_3}{D_4}$ where $D_1 = \begin{vmatrix} 2x & -5 \\ 2 & 3 \end{vmatrix}$, $D_2 = \begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix}$, $D_3 = \begin{vmatrix} 2 & 1 \\ -3 & 2x \end{vmatrix}$, $D_4 = \begin{vmatrix} -3 & 1 \\ 2 & -1 \end{vmatrix}$.

9. SYSTEMS OF FIRST-DEGREE EQUATIONS

125. The difference between two numbers equals 10. Find these numbers. How many solutions has this problem? May we assert that any pair of numbers will be a solution for this problem?

126. Find (by trial) several solutions for each of the following equations: (a) $x-y = 5$, (b) $y = 3x$, (c) $3x-2y = 16$. Construct the corresponding straight lines and show that each of the equations is indeterminate.

127. For the equation $x+2y = 4$ compile a table of values of x and y which satisfy this equation and construct the straight line $x+2y = 4$. Using the graph, find x for the following values of y: 3.5, -2, -1.5, 2, 6.

128. Find several solutions for each of the following equations:

$$5y-8x = 4y+3-9x \quad \text{and} \quad \frac{x+y}{5} - \frac{x-1}{10} = \frac{x-2}{4}.$$

129. Solve the following systems of equations and check on the solutions graphically:

(1) $\begin{cases} x-y = 2 \\ 3x-2y = 9; \end{cases}$ (2) $\begin{cases} 2x+y = 11 \\ 3x-y = 9; \end{cases}$ (3) $\begin{cases} 3x+y = 2 \\ 6x+2y = 3. \end{cases}$

130. (1) For what a does the system $\begin{cases} x+ay = 3 \\ x-y = 3 \end{cases}$ have (a) an infinite number of solutions; (b) the solution $x = 3, y = 0$?

(2) For what m does the system $\begin{cases} x+my = 4 \\ x-2y = 3 \end{cases}$ have no solution?

131. (1) Write a system of equations which has no solution; give a graphical explanation.

(2) Write a system of equations which has an infinite number of solutions. Give a graphical explanation.

Solve the following systems of equations:

132. (1) $\begin{cases} 4x+3y+4 = 0 \\ 6x+5y+7 = 0; \end{cases}$ (2) $\begin{cases} 15x+23y = -10 \\ 3x+4y = -2. \end{cases}$

133. (1) $\begin{cases} (2x+y-1) : (3x+2y+11) = 0.5 \\ (5x-3y+4) : (6x-3y+3) = 0.75; \end{cases}$

(2) $\begin{cases} (x+2y-3) \times (3x-2y-7) = 0 \\ (2x-y-4) \times (x-2y+5) = 0; \end{cases}$

(3) $2-\frac{1}{3}(x+5y) = 4x+2y+5 = \frac{1}{4}[4-3(x-7)+10y-2].$

134. (1) $\begin{cases} \frac{4}{x}+\frac{6}{y} = 0 \\ \frac{3}{x}-\frac{4}{y} = -\frac{17}{6}; \end{cases}$

(2) $\begin{cases} \frac{1}{3-x}+\frac{1}{y-2} = 2 \\ \frac{2}{3-x}-\frac{5}{y-2} = -3. \end{cases}$

135. Solve the following systems with respect to x and y. For what values of the remaining letters will the systems have a unique solution?

(1) $\begin{cases} ax+2y=3 \\ 3x-ay=4; \end{cases}$ (2) $\begin{cases} x+ay=m \\ x+a^2y=n; \end{cases}$ (3) $\begin{cases} x:a+y:b=2 \\ x:a-y:b=0. \end{cases}$

136. (1) $\begin{cases} x+5y-2z=5 \\ 7x-3y-4z=-9 \\ 5x-y+2z=31; \end{cases}$ (2) $\begin{cases} 2x-3y-6z=10 \\ x+y+2z=0 \\ 3x+2y-4z=-2; \end{cases}$

(3) $\begin{cases} 2x-3y+4z+8=0 \\ x+2y-3z-3=0 \\ 3x+y-2z+1=0; \end{cases}$ (4) $\begin{cases} 4x+3y-8z+1=0 \\ x-6y+2z-3=0 \\ 2x-3y+4z-3=0. \end{cases}$

137. (1) $\begin{cases} ax+by+cz=k \\ x:y=m:n \\ y:z=p:q; \end{cases}$ (2) $\begin{cases} x=\dfrac{y-3}{4}=\dfrac{z+5}{-2} \\ x+3y-4z=-13. \end{cases}$

138. Solve the following systems. Give the geometrical interpretation for (1) and (2):

(1) $\begin{cases} x-2y=11 \\ 3x+2y=9 \\ 4x+7y=-1; \end{cases}$ (2) $\begin{cases} 2x+3y=8 \\ x-2y=-10 \\ 3x-y=5; \end{cases}$

(3) $\begin{cases} 2x+8y-5z=0 \\ 3x-2y+3z=0. \end{cases}$

10. SETTING UP EQUATIONS AND SYSTEMS OF FIRST-DEGREE EQUATIONS

139. The sum of the digits of a two-digit number is equal to 12. When the digits are interchanged, the number increases by 75%. Find this number.

140. The numerator of a fraction is by 5 less than its denominator. If one adds 17 to the numerator of this fraction, and 2 to the denominator, one will obtain a fraction which is the reverse of the given one. Find this fraction.

141. By how much need the number 100 be multiplied or divided so that, whether the number obtained is divided by 5 or by 7, there will be a remainder of 1, and so that at the same time the first quotient will be greater by 4 than the second?

142. A two-digit number divided by the sum of its digits has a quotient of 4 and a remainder of 3. If one interchanges the digits,

one will obtain a number which is by 5 greater than the sum of the digits multiplied by 6. Find the number.

143. The total mass of the two first Soviet artificial Earth satellites was 591.9 kg. The first satellite was 1243.4 kg lighter than the third, and the second was 818.7 kg lighter than the third. How much did each of the first three Soviet Earth satellites weigh?

144. In 1950 the socialist countries' share in world industrial production increased by 10% in comparison with 1937, and was $\frac{10}{19}$ of their share in 1964. How great a percentage of world industrial production did the socialist countries produce in 1937, 1950 and 1964, if their share in production in 1964 rose by 18% in comparison with 1950?

145. A ship reached Cherkassy from Kiev in an appointed time, moving at a fixed average speed. If it had increased this speed by 8 km/h, it would have reached its destination 48 minutes earlier than the time alloted. But if its speed had been 2 km/h less than it was in reality, then it would have been 15 minutes late. Determine the distance between Kiev and Cherkassy, the average speed of the ship and its travelling time.

146. Two hours after a plane took off in a westerly direction, another plane IL18, flying at a constant speed of 600 km/h, took off from the same airport and in the same direction. How long did it take for the second plane to overtake the first, if the latter was flying at a speed of 350 km/h?

147. A bus set off from city *A* for city *B*, 288 km away. An hour later a passenger car, the ratio of whose speed to the bus' was 3 : 2, took off after it. Find the speed of the bus and of the car, knowing that the car arrived in city *B* an hour earlier than the bus.

148. The diameter of the sheave of an electric motor rotating at 1200 revolutions per minute is equal to 180 mm. How should the sheave's diameter be altered, so that if the number of revolutions of the electric motor is diminished to 1080 per minute, the speed of the driving shaft will stay the same?

149. To complete a planned task, a team of workers had to manufacture a certain amount of parts in 13 days. By increasing the productivity of their labour, the team manufactured 50 parts more each day than was projected in the plan. Consequently, after only 12 days, the plan had not only been fulfilled, but 100 parts had been made in excess of the target plan. How many parts did the team manufacture?

150. A team of workers had to carry out a job in 12 days. By overfulfilling the norm by 25% each day, the team not only carried out its assignment after 10 days of work, but also manufactured 42 parts above and beyond it. How many parts did the team manufacture per day?

151. Three pumps were installed to pump out water from a mine shaft. The first pump, working alone, could pump out the water in 12 hours; the second, in 15 hours; the third, in 20 hours. For the first three hours the first and third pumps were working, and then the second pump was also turned on. How much time was required altogether to pump out the water from the shaft?

152. Two workers working together could complete a task in 8 hours. It so happened that the first worker worked 5 hours and the second, 8 hours, and as a result they carried out 11/14 of the total task. In how many hours could each worker separately complete this task?

153. 25- and 12.5-m rails were delivered for the construction of a 10-km section of a double-track railway. If all the long rails available were laid, then to complete the construction of the section, 41 2/3 per cent of the number of short rails should also be used. But if all the short rails were laid, then 84 4/9 per cent of the number of long rails would have to be used. How many long and short rails were delivered?

154. Two shops in a factory had to release 180 machine tools a year according to the plan. The first shop fulfilled the plan by 112 per cent, and the second fulfilled it by 110 per cent, and therefore both shops released 200 tools. How many machine tools did each shop release in excess of the plan?

155. Two teams had to complete a certain job in 30 days according to the plan. Six days after they started the job one team was transferred to another zone, while the other continued working alone and completed the remaining part of the job in 40 days. In how many days could each team, working separately, complete the whole job?

156. A tank of 10,000 l. capacity is being filled with gasoline by two pumps, the second of which supplies 10 l. less a minute than the first. In 10 minutes the tank is half full. How many litres of gasoline does each pump pour in?

157. How much water need one add to 30 l. of 90 per cent alcohol to obtain 80 per cent alcohol?

158. 4 kg of hot water and 3 kg of 10 °C water were mixed together. The temperature of the mixture turned out to be equal to 40 °C. Find the temperature of the hot water used for the mixture.

159. Two pieces of equal weight were cut off from two hunks of alloys containing different percentages of lead and weighing 6 kg and 12 kg. Each of the cut-off pieces was fused with the remainder of the other hunk, after which the percentage of lead content in both alloys became the same. How much did each of the cut-off pieces weigh?

160. (A problem of Leo Tolstoy.) An artel of mowers went out into the field. It was supposed to mow two meadows, one of which was two times bigger than the other. For half of the day the artel mowed the larger meadow; but in the second half of the day the artel split into two, and one half stayed to finish off the large meadow, while the other began to mow the small meadow. By the evening the large meadow was mowed, but there remained a section of the small one, which was mowed the next day by one mower working the whole day. How many mowers were there in the artel?

161. Think of your date and month of birth. Multiply your date of birth by 10, add to the product the number of your month of birth, multiply the sum obtained by two, subtract the number of the month from the result. Name the final number. How can you find out the date and month of birth of an acquaintance of yours, if at your command he performs all these operations on his data?

162. a tons of grain were poured into two elevators. Every day b tons of grain were taken from the first elevator, and c tons from the second. t days later an equal amount of grain was left in both elevators. How many tons of grain had been poured into each elevator separately? Substitute plausible values for the letters in the formula obtained, and evaluate the unknowns.

163. Two airplanes took off from an airfield for city A at the same time and in the same direction. The first plane flew at a speed of v km/h, while the second plane's speed was d km/h greater. Determine the distance between the airfield and city A, it being known that the second plane arrived there t hours earlier than the first.

CHAPTER IV

Inequalities

11. INEQUALITIES AND SYSTEMS OF FIRST-DEGREE INEQUALITIES

164. (Orally). Put the sign of equality or inequality between the following pairs of numbers:

$\frac{2}{3}$ and $\frac{5}{8}$, 0.125 and $\frac{1}{8}$, 0.1 and 0.0986, 0.(9) and 1, $a+3$ and $a+4$,

$$b+1 \quad \text{and} \quad b-1, \quad (a-2)^3 \quad \text{and} \quad (a-1)^3.$$

165. Combine the following inequalities into double ones:

$x < 3$ and $x > -2$, $x > -1$ and $x < 0$, $x+y < -5$ and $x+y > -10$, $m-n > 0$ and $m-n < 5$.

166. (1) Multiply both members of the following inequalities by the given numbers:

$4 > 1$ by 3; $4 > 1$ by -1; $-3 < -2$ by -2; $a > -2$ by b.

(2) Divide both members of the following inequalities by the given numbers:

$8 > -2$ by 2; $0 > -4$ by 4; $-2 > -8$ by -2;

$a^6+1 > a^4-1$ by a^2+1; $a^4+x^3 > 2x^2$ by x^2;

$x^2+y^2 > 0$ by $-a$.

167. (1) Add the following inequalities:

(a) $2 > -5$ and $3 > 2$; (b) $-15 > -20$ and $10 > -10$;

(c) $m+n > m-n$ and $2m > m-n$; (d) $12 < 15$ and $3 > -2$.

(2) Subtract the second inequality from the first one: (a) $3 > -2$ and $5 < 6$; (b) $12 > -1$ and $-10 < -2$; (c) $2x+3 > 4$ and $5 < 10-x$; (d) $5a+1 > 2a-7$ and $a-4 > 3-a$.

(3) Multiply the following inequalities termwise:

$5 > 4$ and $4 > 2$; $4 > -5$ and $3 > 2$; $2 > 0$ and $-2 < 1$.

(4) Square and cube the following inequalities termwise:

$$3 < 5; \quad -3 < 0; \quad 5 > -7; \quad -4 > -7; \quad 2a^2 > 5.$$

168. What can be said about the numbers a and b if it is known that

$$ab > 0; \quad ab < 0; \quad \frac{a}{b} > 0; \quad \frac{a}{b} < 0; \quad a^2b > 0; \quad \frac{a}{b^2} < 0.$$

169. By what sign of inequality can the following letters be connected without changing their places?

(a) m and n if $m-n = 2$;

(b) x and y if $x-y = -3$;

(c) p and q if $p+q = 5$;

(d) a and b if $a^2-b^2 = -c$, $c > 0$.

170. (1) For what m and n is the number $-(m+n)$ positive? For what values is it negative?

(2) For what m and n is the inequality $m+n < m-n$ true?

(3) For what values of x and y are the inequalities $x^2 > y^2$; $x^2 < y^2$ satisfied?

171. Are the following inequalities equivalent?

(1) $x^2+3x-2 > 2$ and $x^2+3x-4 > 0$;

(2) $4x+\frac{5}{x+3} > 8+\frac{5}{x+3}$ and $4x > 8$;

(3) $4x+\frac{5}{x-3} > 8+\frac{5}{x-3}$ and $4x > 8$;

(4) $3x^2 < 6x$ and $x < 2$;

(5) $\frac{x+3}{2-x} > 0$ and $(x+3)(2-x) > 0$.

172. Solve the following inequalities:

(1) $x-(5-x) > 3$; (2) $6(2x+7) < 15(x+2)$;

(3) $7(4x-3)-3(8x-7) > 1$; (4) $(x+5)^2-(x-9)^2 < 140$.

173. (1) For what values of p do the following equations have positive solutions; negative solutions:

(a) $2x+4 = p$; (b) $3x-p+1 = 2x-1$; (c) $4x+2p-3 = 1-3x$?

(2) For what values of m does the equation

(a) $mx+3x = 5$ have the root greater than 3,

(b) $5(x-m) = -2$ have the root less than -2?

Mark on the number scale the solutions of the following inequalities:

174. (1) $\frac{2x+1}{5}-\frac{2-x}{3} > 2$;

(2) $-\frac{x}{2} < 3x-\frac{3+2x}{4}$;

(3) $\frac{1}{2}(3x-1)+\frac{x}{5} < 7x+10.1$;

(4) $4-\frac{3}{2}x > \frac{13}{8}-\frac{1}{6}(4x-3)$.

175. (1) $\frac{x}{3}-3\left[5x-\frac{2-3(x-5)}{4}\right] > x+3\frac{19}{24}$;

(2) $\frac{x-\frac{1}{3}(4x-9)}{6}-\frac{3x}{2}+\frac{5(2-3x)}{4}-1 < x+27\frac{2}{9}$;

(3) $\frac{x-3}{2}-\frac{3(3-x)}{10}+\frac{7x-6}{4} < \frac{x+10}{3}-\frac{3-16x}{20}$;

(4) $\frac{2x^2-5x+3}{6}-\frac{4-x}{12}+\frac{5-x^2}{3} \geqslant \frac{2x-1}{9}$.

176. (1) $\frac{1}{2}+[(x^2-1)x+2]\times\frac{1}{4} > 3\frac{1}{2}x+\frac{x^3-1}{4}$;

(2) $\frac{1.3-3x}{2}+\frac{5x-0.4}{0.3}<\frac{1.8-8x}{1.2}$;

(3) $2(3y-2)y-3\left[1-(2y+3)(2-y)+\frac{3-y}{2}\right]\leqslant 13$;

(4) $3\left\{t+\frac{1-3t}{4}-\left[1-2\left(t-\frac{t+3}{5}\right)\right]\right\}>5t-2$.

177. (1) $|x|<3$; (2) $|x|>3$; (3) $\left|\frac{1}{2}x\right|<2$;

(4) $|x-3|>5$; (5) $|x+3|<7$; (6) $|4-3x|>7$;

(7) $|2x-4|\leqslant 0.5$; (8) $2\leqslant|x|\leqslant 3$;

(9) $|x+3|>|2x-1|$.

Solve the following inequalities:

178. (1) $a(x+2)>x-1$; (2) $4m^2x-4n^2x>m^4-n^4$;

(3) $\frac{2y+k}{a}>b$; (4) $\frac{3z}{m}+n<p$;

(5) $(m-x)^2-(m+x)^2+2x^2<(2m-x)^2+(2m+x)^2$.

179. (1) $\frac{x}{a}-\frac{3x-1}{2}>\frac{x+2}{4a}$, $a\neq 0$;

(2) $\frac{px}{q}-\frac{qx}{p}<a$, $p\neq 0, q\neq 0$;

(3) $\frac{x}{m}+\frac{n}{m^2}<\frac{4x}{m}-\frac{n}{m^2}$, $m\neq 0$;

(4) $x+\frac{x-1}{a+1}>\frac{x+1}{a+1}-ax$, $a\neq -1$;

(5) $\frac{bx}{2-b}+\frac{2x+3}{4}>\frac{1-x}{3}$; $b\neq 2$.

180. (1) Find the least integer satisfying the inequality

$$49.4-\frac{27-x}{10}<47.4-\frac{27-9x}{10}.$$

(2) Find the greatest integer satisfying the inequality

$$\frac{1}{4}(2x+1)-0.2(3x+1)>-\frac{1}{3}.$$

(3) For what values of x do the fractions $-\frac{1}{3-x}$, $\frac{x^2+1}{x+1}$, $\frac{2}{x^4+3}$ attain (a) positive values, (b) negative values?

181. (1)* Find all positive integral roots of the equation $ax = a+10x$.

(2) For what values of a does the equation $\frac{2a+5}{3} = \frac{5ax+1}{4}$ have a negative solution?

182. Solve the following inequalities using the method of intervals:

(1) $(x-2)(x-3)(x-5)(x-8) > 0$;

(2) $(x+3)(x+1)(x-4)^2(x-5) < 0$;

(3) $(2x-5)(3-x)^3(x-5)^2(5+2x) \geqslant 0$;

(4) $\frac{2x+1}{3-x} < 0$; (5) $\frac{(x+3)^2(x+1)(x-5)}{(x-4)^2(x-2)} \leqslant 0$;

(6) $\frac{3x-1}{x+5} < 2$.

183. Graph the following functions:

(1) $y = x-4$; (2) $y = 2x-5$, (3) $y = -2x-4$.

Using the graphs, determine for what values of the argument the functions (a) take on positive values, (b) take on negative values, (c) vanish.

184. (1) What will happen to a totality of inequalities if an inequality having no solution is added to it?

(2) What will happen to a system of inequalities if an inequality having no solution is added to it?

Solve the given systems of inequalities and mark the obtained solutions on the number scale:

185. (1) $\begin{cases} 2x-4 > 0 \\ 4x-20 < 0; \end{cases}$ (2) $\begin{cases} 5x+3 > 0 \\ 2x-1 > 0; \end{cases}$

(3) $\begin{cases} 3x-5 < 0 \\ 7x+21 < 0; \end{cases}$ (4) $\begin{cases} 12x-6 < 0 \\ 12-3x < 0; \end{cases}$

(5) $\begin{cases} 5(x+1)-9x-3 > -6(x+2) \\ 3(3+2x) < 7x-2(x-8); \end{cases}$

(6) $\begin{cases} (x-0.4)^2-(x+0.4)(x-0.4) > 0.1 \\ (x+0.1)(x+0.01)-(x-0.1)(x-0.01) > 0.22. \end{cases}$

186. (1) $\begin{cases} \frac{x-3}{4} - x < \frac{x-1}{2} - \frac{x-2}{3} \\ 2-x > 2x-8; \end{cases}$

(2) $\begin{cases} \frac{3}{4}x + \frac{7}{8} < \frac{1}{4}x + \frac{5}{2} \\ \frac{x+1}{4} - \frac{2x-1}{3} < 2; \end{cases}$

(3) $\begin{cases} \frac{x}{8} - \frac{5x-4}{12} < \frac{x-2}{6} - \frac{x+1}{3} - \frac{3x}{4} + 6 \\ x > \frac{5x-14}{25} - \frac{3x-5}{20} - 9\frac{3}{4}; \end{cases}$

(4) $\begin{cases} \frac{x^2-1}{5} + \frac{x+1}{2} < \frac{2x^2+3}{10} + \frac{x}{2} + 3 \\ 1-x > \frac{0.5(x-3)}{2} - \frac{2x+3.5}{3}. \end{cases}$

187. (1) $\begin{cases} x < -2, \quad 1 \leqslant x \leqslant 2 \\ x < -3, \quad 1.5 \leqslant x \leqslant 3 \\ -2.5 < x \leqslant 2, \quad x > 2.7; \end{cases}$ (2) $\begin{cases} -2 < x < 1 \\ x < -3, \quad x > 0 \\ x > -1; \end{cases}$

(3) $\begin{cases} 2x-3 < 3x-4 \\ x+8 > 3x+1 \\ 3x-2 > x+2 \\ 5x-1 < 4x+6; \end{cases}$

(4) $-1 < \frac{3a+10}{a+7} < 2;$

(5) $\begin{cases} |x| > 2 \\ (x+3)\,x(x-1)\,(x-4) < 0. \end{cases}$

188. Find the integral solutions of the following systems of inequalities:

(1) $\begin{cases} 1 - \frac{3x-88}{7} > 5x \\ 4x+5 < 5x+4.5; \end{cases}$ (2) $\begin{cases} x+2 > \frac{2x-8}{6} - \frac{18-4x}{3} \\ \frac{2-x}{4} + 9\frac{2}{3} > x. \end{cases}$

189. (1) $\begin{cases} \frac{5}{x} > 9 \\ \frac{12}{x} + 5 < 8; \end{cases}$ (2) $\begin{cases} \frac{9}{x} - \frac{4}{9} > \frac{10}{x} - \frac{1}{2} \\ \frac{7}{x} + \frac{1}{3} > \frac{7}{12} - \frac{1}{4x}; \end{cases}$

(3) $\begin{cases} \dfrac{1}{x-1} < \dfrac{2}{x+1} \\ \dfrac{2}{x-3} > \dfrac{3}{x-2}; \end{cases}$ (4) $\begin{cases} \dfrac{2}{3x-7} > \dfrac{5}{17-4x} \\ \dfrac{12}{5x-8} < \dfrac{11}{2x+3}; \end{cases}$

(5) $\begin{cases} \dfrac{x}{2x+1} > \dfrac{1}{4} \\ \dfrac{6x}{4x-1} < \dfrac{1}{2}; \end{cases}$ (6) $\begin{cases} \dfrac{1}{x+1} - \dfrac{1}{7(x+1)} < \dfrac{2}{x+9} \\ \dfrac{1}{2x+5} < \dfrac{1}{8x+3}. \end{cases}$

190. For what values of m do the systems

(1) $\begin{cases} 3x-6y = 1 \\ 5x+my = 2; \end{cases}$ (2) $\begin{cases} 3x+my = m \\ 2x-5y = 20; \end{cases}$

(a) have solutions satisfying the conditions: $x > 0$, $y > 0$; $x < 0$, $y < 0$; $x = 0$, $y = 0$; (b) have no solutions; (c) have an infinite set of solutions?

Solve graphically the following inequalities and systems of inequalities:

191. (1) $2x+7 > 0$; (2) $10-3x < 0$; (3) $\begin{cases} 3x-4 < 0 \\ x+2 < 0; \end{cases}$

(4) $\begin{cases} 2x-5 > 0 \\ x+1 > 0; \end{cases}$ (5) $\begin{cases} \frac{1}{3}x-1 > 0 \\ 7-x > 0; \end{cases}$ (6) $\begin{cases} \frac{1}{2}x-3 > 0 \\ 2x+1 > 0. \end{cases}$

192. (1) $2y-4x-5 > 0$; (2) $3y+6x-5 < 0$.

193*. (1) $\begin{cases} y-x-1 < 0 \\ 3x+y-6 > 0; \end{cases}$ (2) $\begin{cases} 2x+y-2 > 0 \\ x-2y+2 < 0; \end{cases}$

(3) $\begin{cases} x-y = -3 \\ x+2y > 6; \end{cases}$ (4) $\begin{cases} x+y = 4 \\ 2x-y < -1. \end{cases}$

194. Find the two-digit number, whose tens digit exceeds its units digit by 3, if this number is greater than 70 and less than 80.

195. A rifleman hits a target a metres away. An observer hidden in a shelter between the rifleman and the target at a distance of b metres from the rifleman and c metres from the target hears simultaneously the report of the shot and the sound produced by the bullet hitting against the target. The sound velocity equals v m/s. Find the velocity of the bullet, and determine which distance is longer, b or c.

196. If 29 is added to $\frac{1}{4}$ and $\frac{1}{3}$ of the required number, then the first sum will be less than $\frac{3}{2}$ of the required number, and the second one will exceed the required number by more than 1.5 times. Find this number if it is known to be an integer.

197. If the tourist daily travelled 20 km more than the distance he actually covers, then he would have travelled more than 900 km during 8 days. On the other hand, if he covered daily 12 km less than his actual average, then he would not be able to travel the distance of 900 km in 10 days. How many kilometres did the tourist make per day?

198. One side of a triangle is equal to 6 cm, and the difference between the remaining two to 2 cm. Find the sides of this triangle if they are whole numbers not exceeding 8 cm.

199*. It is necessary to buy 20 metres of cloth of two different grades. One metre of the first-grade cloth costs 15 roubles, and that of the second-grade cloth, 12 roubles. How many metres of cloth of both grades should be bought so that the cost of the whole purchase does not exceed 276 roubles?

12. PROVING INEQUALITIES

Prove the following inequalities:

200. (1) $a^2+b^2 \geqslant 2ab$; (2) $a^2-ab+b^2 \geqslant 0$; (3) $a^2+ab+b^2 \geqslant 0$; (4) $a^3+b^3 \geqslant a^2b+ab^2$ if $a > 0$, $b > 0$; (5) $ab+ac+bc \geqslant 3abc$, where a, b, c are real non-negative numbers which are less than unity; (6) $ad > bc$ if $\frac{a}{b} > \frac{c}{d}$, and $bd > 0$.

201. (1) $\frac{a}{b}+\frac{b}{a} \geqslant 2$, $a>0$, $b > 0$; (2) $\frac{x^2}{x^4+1} \leqslant \frac{1}{2}$; (3) $(x+y)\times$ $\times\left(\frac{1}{x}+\frac{1}{y}\right) \geqslant 4$, $x > 0$, $y > 0$; (4) $a^2+b^2+c^2+3 \geqslant 2(a+b+c)$; (5) $x^5+y^5 \geqslant x^4y+xy^4$ if $x \geqslant 0$, $y \geqslant 0$; (6) $\frac{bc}{a}+\frac{ac}{b}+\frac{ab}{c} \geqslant a+$ $+b+c$, $a > 0$, $b > 0$, $c > 0$; (7) $a^4+b^4 \geqslant 2$ if $a+b = 2$.

202. (1) $|x|-\frac{1}{x} > 2$ for $x < -1$;

(2) $\frac{1}{x^2}+\frac{1}{x} > 2$ for $0 < x < 1$;

(3) $2+\frac{2}{x}<3$ for $x>2$; (4) $\frac{8}{x^2}-\frac{x}{4}<1\frac{1}{2}$ for $x>2$;

(5) $4x-\frac{8}{x^3}>7$ for $x>2$.

203. (1) $\frac{x^2+4}{\sqrt{2x^2+4}}\geqslant 2$; (2) $\frac{x^2}{1+x^2}\leqslant\frac{1}{2}$ for $|x|\leqslant 1$;

(3) $x^4y^2+x^4+y^2+1\geqslant 4x^2y$;

(4) $(a+b)(ab+1)\geqslant 4ab$ if $a>0$, $b>0$. Find the condition for which the equality will hold.

204. (1) $\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\geqslant\frac{9}{a+b+c}$, where a, b, and c are positive numbers.

(2) $\frac{a^3+b^3}{2}\geqslant\left(\frac{a+b}{2}\right)^3$, where $a>0$, $b>0$;

(3) $(a+b)(b+c)(c+d)(d+a)\geqslant 16\,abcd$, where $a\geqslant 0$, $b\geqslant 0$, $c\geqslant 0$, $d\geqslant 0$;

(4) $a^2+b^2+c^2<2(ab+ac+bc)$, where a, b, c are the sides of a triangle.

205. Prove that a positive proper fraction with positive numerator and denominator increases with an increase both in the numerator and the denominator by one and the same positive number, and an improper fraction decreases.

206. Prove that in any triangle the half-perimeter is greater than each of its sides.

207. Prove that the sum of the hypotenuse and the altitude of a right triangle dropped on the hypotenuse exceeds the half-perimeter of the triangle.

208. Prove that the sum of the cubed legs in a right triangle is less than the cubed hypotenuse.

209. Prove that of all rectangles having a given perimeter the square has the greatest area.

210. A ship covered the distance from Volgograd to Astrakhan and returned back. Its speed with the stream is v_1 and against the stream v_2. Prove that the average speed of the ship throughout the route Volgograd-Astrakhan-Volgograd $v_{av}<\frac{v_1+v_2}{2}$.

CHAPTER V

Real Numbers

13. REAL NUMBERS AND THEIR GEOMETRICAL REPRESENTATION

211. Which of the given numbers are rational, and which are irrational: $-3.56999\ldots$, $2.17345345\ldots$, $-3.1753434217\ldots$, $1.18345483454\ldots$, $0.37823782\ldots$, $3.1415926\ldots$, $\sqrt{5}$, $\sqrt[3]{\frac{8}{27}}$?

212. 1. Can the sum $a+b$ be rational if (a) one of the addends is rational and the other irrational; (b) both addends are irrational numbers?

2. Can the difference $a-b$ be a rational number if (a) one of the numbers is rational and the other irrational; (b) a and b are irrational numbers?

3. Can the product ab be a rational number if (a) one of the factors is rational, and the other irrational; (b) both factors are irrational?

4. Can the fraction $\frac{a}{b}$ be a rational number if (a) a is rational; b, irrational; (b) a is irrational; b, rational; (c) a and b are irrational numbers?

213. 1. The line segment a is incommensurable with unity. Can the segment $a+2$, $a-4$, $2a$ be commensurable with unity?

2. Let a and b be incommensurable segments. Will the segments: a and $\frac{b}{2}$; $\frac{a}{3}$ and b; $\frac{a}{5}$ and $\frac{b}{6}$ be commensurable?

214. Lay from zero on the number scale a line segment equal to the diagonal of a square whose side is equal to the scale unit. The right-hand end-point of this segment represents an irrational number. Prove that there is an infinite number of points on the number scale representing irrational numbers.

215. Prove that in an isosceles triangle, whose vertex angle is equal to 108°, the base is incommensurable with the side.

216. 1. Is the assertion that the square root of a rational number is always an irrational number true?

2. Give examples illustrating that the square root of a rational number can be: (a) a whole number, (b) a finite decimal fraction, (c) an infinite periodical decimal fraction.

217. 1. For one and the same unit of measurement the lengths of line segments are expressed by the following numbers: 2, 3.1, $\sqrt{5}$, 0.8, $3\frac{8}{13}$, $\sqrt{7}$, 0.02 (3). Which of those segments are commensurable with the unit length, and which are not?

2. Find out whether the two line segments are commensurable if for the same unit of measurement their lengths are expressed by the following numbers: (a) 3 and $\frac{3}{5}$; (b) 3 and $\sqrt{3}$; (c) 0.3 $\sqrt{3}$ and $\frac{2}{7}\sqrt{3}$; (d) 0.2 (3) and $3\frac{2}{3}$; (e) $3\sqrt{2}$ and 0.2 $\sqrt{0.125}$.

218. Construct the following intervals on the number scale [2, 4], [−3, 1], [−5, 2], [−4, −1], (0, 3), [−6, 2], $[x_1, x_2]$.

219. What are the solutions of the following inequalities—a totality or a system? Indicate the solutions on the number axis:

(1) $|x| < 3$; (2) $|x| \geqslant 5$; (3) $|2x| < 7$;

(4) $|x+2| < 3$; (5) $|x-2| > 3$; (6) $|2x-3| \geqslant 5$.

220. Using the sign of absolute value, write the following number sets with the aid of inequalities:

(1) $-2 < x < 2$; (2) $x \leqslant -3$ and $x \geqslant 3$;

(3) $2 < x < 6$; (4) $-4 \leqslant x \leqslant 2$; (5) $0 < x < 10$;

(6) $x < -4$ and $x > 5$.

221. On the number axis indicate the points corresponding to the numbers 0, 1, 2, $\sqrt{2}$, $\sqrt{3}$, $-\sqrt{5}$, −2.6. To this end, using the Pythagorean theorem, construct the line segments whose lengths are expressed by these numbers.

222. Find the distance between the points on the number axis corresponding to the following numbers: (1) 2 and 4; (2) -3 and -1; (3) 0 and 5; (4) -2.5 and 0; (5) $-\sqrt{2}$ and 2; (6) $-\sqrt{3}$ and $2\sqrt{3}$; (7) -3 and 3; (8) x_1 and x_2. Write the formula for determining the distance between two points on the number axis.

223. 1. Write rational approximations, to within 0.1; 0.01; 0.001 (with deficit and with excess), to the following numbers: 2.3883..., 0.73054..., 0.8343703..., 8.00374..., 3.70503..., 7.030573..., $-0.9899777...$, $-2.09093...$, 4.999998.

2. Compare (by magnitude) the following numbers: 7.5 and 7.4893; 2.3754 and 2.3755; 0.734894 and 0.734889; 0.000001 and -0.5; 0.37485 and 0.37543; -2.178354 and -2.178837.

224. 1. Write the following rational numbers: $\frac{2}{3}, \frac{3}{7}, 1.4, 3, 0, -\frac{5}{6}$ in the form of infinite decimal fractions.

2. Convert the following decimal fractions: 0.15, 2.45999..., $-3.1212...$, 0.(36), 3.12(3), 0.1(6) into common ones.

225. Carry out the indicated operations involving real numbers (to within 0.01):

(1) $\frac{1}{3}+2.17374...$; $3(83)+2.03453...$;

(2) $8.1543...-3.14$; $10.7373...-11.0354$; $\frac{2}{7}-\frac{1}{5}$;

(3) $3.112...\times 2.115...$; $0.3437...\times 2.073...$; $0.1838...\times\frac{1}{3}$.

226. Find the opposite and the reciprocal number for each of the given numbers:

$$5;\ -3.5;\ 0.5;\ \sqrt{2};\ -4;\ \frac{2}{\sqrt{3}};\ 2.29999....$$

227. For what values of x do the following expressions have no sense:

(1) $\frac{2}{x-3}$; (2) $\frac{1}{x^2+4}$; (3) $\frac{2}{x^2-2}$;

(4) $\frac{5}{x^3+4}$; (5) $\frac{-4}{(x+1)^3}$; (6) $\frac{1}{\sqrt{x^2-16}}$?

228. Find the domains of permissible values for the unknowns in the following equations:

(a) $\sqrt{2x-3} = \sqrt{5x-4}$; (b) $\sqrt{x-7} - \sqrt{5-x} = 3$;

(c) $\sqrt{3x-1} + \frac{2}{\sqrt{3x-1}} = \sqrt{5x+3}$.

229. 1. It is known that $|a| > |b|$. Is it possible that $a < b$? Give examples.

2. Can two opposite numbers be reciprocal by magnitude? Give examples.

3. May we assert that the number $-a$ is negative?

230. 1. It is known that $a > 0$ and $b < 0$. May we assert that $ab = 0$? $a+b = 0$?

2. May we say that $(x-y)^4$ is a positive number for any values of x and y?

3. May we assert that for certain values of a and b the equality $(a+b)^2 = a^2+b^2$ is correct?

231. 1. May one assert that any two real numbers can undergo any of the four arithmetical operations?

2. Which operations involving two rational numbers are performable within the set of rational numbers?

232. 1. Which is broader—the set of rational numbers or that of real numbers?

2. How the following assertion should be understood: "A one-to-one correspondence is established between the set of all real numbers and the set of points on the number axis"?

233. 1. Give examples of elements of the set of (a) natural numbers, (b) even numbers, (c) odd numbers, (d) simple numbers, (e) numbers multiple of 9, (f) numbers which, when divided by 7, yield 5 as the remainder, (g) whole numbers, (h) rational numbers, (i) irrational numbers, (j) real numbers.

234. Give examples of common elements of:

(a) a set of simple numbers and a set of even numbers,

(b) a set of simple numbers and a set of odd numbers,

(c) a set of negative numbers and a set of rational numbers,

(d) a set of real numbers and a set of rational numbers.

235. 1. Compile a table of approximate values of the sum $\sqrt{2}+\sqrt{5}$ calculated to within 1, 0.1, 0.01 and so on both with deficit and with excess. Proceeding from the definition of the sum of irra-

tional numbers, find three decimal places of the infinite fraction representing the sum $\sqrt{2}+\sqrt{5}$.

2. Same for the product $\sqrt{2}\times\sqrt{5}$

236. Find the first two decimal digits of each of the infinite fractions representing the results of the following operations:

(1) $3+\sqrt{2}$; (2) $\sqrt{2}+\sqrt{3}+\sqrt{5}$; (3) $\sqrt{5}-\sqrt{3}$;

(4) $0.810810081\ldots -\sqrt{6}$; (5) $4\sqrt{2}$; (6) $\sqrt{2}\times\sqrt{5}$.

CHAPTER VI

Power with a Rational Exponent

14. OPERATIONS INVOLVING POWERS WITH NATURAL EXPONENTS. THE SQUARE OF A POLYNOMIAL

237. Calculate orally:

$$(-2)^3,\ (-2)^6,\ (-0.12)^2,\ -3^2,\ (-1)^{2n},\ (-1)^{2n-1},\ \left(\frac{2}{3}\right)^4.$$

238. Evaluate:

(1) $a^3,\ -a^3,\ (-a)^3$ for $a = 2$ and $a = -2$;

(2) $10x^4-4x^3-2x^2-4x-3$ for $x = -1\frac{1}{2}$.

239. Compile and learn by heart the table of squares of the natural numbers from 10 to 32.

240. 1. Compute 21^2, 210^2, 2100^2. Deduce a general rule for squaring the numbers ending in zeros.

2. Compute 12^3, 120^3, 1200^3. Deduce a general rule for cubing the numbers ending in zeros.

241. 1. Compile a table of powers of the number 2, the 10th power inclusive.

2. Compile a table of powers of the number 3, the 7th power inclusive.

242. Write the numbers obtained by substituting $x = 10$ in the following expressions:

(1) $5x^3+2x^2+3x+9$; (2) $6x^4+8x^3+4x^2+x$; (3) x^9.

243. Write in the form of powers of 10 the following numbers:
100,000; 1,000,000; 10,000,000;
1,000,000,000,000,000,000,000,000,000,000;
12,000; 135,000; 273,000,000.

244. 1. Write the general expression of a (a) two-, (b) three-, (c) four-, (d) n-digit number to the base 10.

2. Write the general expression of a two-, three-, and four-digit number to the base: (a) 2, (b) 5, (c) 12.

245. Construct the parabola $y = \frac{1}{2}x^2$. What peculiarity has the location of its points corresponding to the following values of x:

3 and -3? 2 and -2? $\frac{1}{3}$ and $-\frac{1}{3}$? $\frac{1}{2}$ and $-\frac{1}{2}$?

246. How will the volume of a cube change if its edge is (a) increased 2 times, 3 times, by 400%, (b) reduced to $\frac{1}{5}$, by 50%?

247. What is the nth term of the sequence

$$\frac{1}{2^2}, \frac{2}{3^2}, \frac{3}{4^2}, \frac{4}{5^2}, \frac{5}{6^2}, \frac{6}{7^2}, \frac{7}{8^2}, \frac{8}{9^2}, \frac{9}{10^2}, \frac{10}{11^2}, \ldots$$

equal to? Compute the 15th term of this sequence.

248. 1. Simplify: $a^3-(-a^3)$, $(-a)^9-a^9$.

2. Compute: $5^4+(-5)^4$, $125^9+(-125)^9$.

249. 1. Compute:

$$\left(-\frac{2}{5}\right)^2, \left(-\frac{2}{3}\right)^3, \left(2\frac{1}{3}\right)^2, (-0.02)^2, \left(\frac{5}{2}\right)^3, \left(\frac{5}{3}\right)^3.$$

2. Compute the following using two methods:

$(2\times10)^2$, $(1\times3\times5)^3$, $4^2\times25^2\times2^2$, $(3^3)^2$, $(-2^2)^3$, $[(-2)^3]^2$.

3. Knowing that $2^5 = 32$, compute in the shortest way 2^{15}.

250. 1. For what values of x are the given inequalities correct:

$$x^3 > 1000, \quad x^2 > 900, \quad x^6 < 15\,625?$$

2. For what values of x and y is the equality $\frac{x}{y} = \left(\frac{x}{y}\right)^2$ valid?

251. 1. Carry out the indicated operations:

(a) $(3a^2b)^4$; (b) $(-2x)^3(-5y^2)^3$; (c) $(-1)^{2n+3}$;

(d) $\left(-\frac{1}{2}ab^3\right)^{2n}$, (e) $\left(-\frac{1}{2}ab^3\right)^{2n+1}$; (f) $\left(-\frac{1}{2}ab^3\right)^{n}$.

2. Represent in the form of a power:

(a) $3^m \times 3$; (b) $2^n \times 8$; (c) $3^m : 3$; (d) $7^{m+1} : 49$.

Carry out the following operations:

252. (1) $\frac{2cd}{3ab} \times \frac{4a^7b^4}{5c^4d^3} \times \frac{15bc^3}{8a^6d^2}$; (2) $\frac{4a^5x^3y}{5b^3cz^4} : \frac{8a^6xy^4}{3bc^2z^5}$;

(3) $\frac{2a^2b^3c}{3x^2y^3} \times \frac{a^mb^nc^p}{x^my^n} \times \frac{6x^{m-1}y^{n-2}}{a^{m+1}b^{n+2}c^{p+3}}$;

(4) $\frac{5a^nb^{n-1}c^{n-2}}{6x^{n+1}y^{n+2}z^{n+3}} : \frac{3a^{n-1}b \times c^{n+1}}{2xy^nz^{n+1}}$.

253. (1) $\left(\frac{3a^2b^2}{5x^2y^2}\right)^3 \times \left(\frac{2ax^2}{5b^3y}\right)^3 : \left(\frac{4a^4}{5b^2y^4}\right)^2$;

(2) $\left(\frac{m+n}{a+b}\right)^3 \times \left(\frac{n-m}{a-b}\right)^3 \times \left(\frac{a^2-b^2}{m^2-n^2}\right)^2$;

(3) $\left[\left(\frac{mnp}{a^2b}\right)^4 : \left(\frac{m^2n^2}{a^3b^2}\right)^2\right] \times \left[\left(\frac{a^3b^4c}{mp^3}\right)^6 : \left(\frac{a^5b^8c^2}{m^2p^5}\right)^3\right]$.

254. (1) $(ax^{2m}+6x^{2n}) : x^{m+n}$;

(2) $(a^{2n}-b^{2m}) : (a^n-b^m)$;

(3) $(x^{3m}-y^{3n}) : (x^m-y^n)$;

(4) $(a^n+2)(a^{2n}-2a^n+4)$.

255. Simplify:

(1) $\left[\frac{2}{(-a)^3}\right]^2 + \left[-\left(\frac{2}{a}\right)^3\right]^2 + \left(-\frac{2}{a^2}\right)^3 - 2\left(-\frac{2}{a^3}\right)^2 - \left[\frac{1}{2}\left(-\frac{2}{a}\right)^2\right]^3$;

(2) $\left[\left(-\frac{a}{2}\right)^3\right]^2 : \left(-\frac{a}{2}\right)^6 - \left(\frac{1}{a}-\frac{1}{b}\right) \times (ab)^2 : (a^2b-ab^2)$.

256. Solve the equation

$$(-2x)^6-(-8x^3)^2-[-(2x)^2]^3-[2(-x^3)]^2+x-15[(2x^3)^2-x] = 32.$$

257. Prove the following identities:

(1) $\frac{a^5-b^5}{a^5+b^5} + \frac{a^5+b^5}{a^5-b^5} = \frac{2(a^{10}+b^{10})}{a^{10}-b^{10}}$;

(2) $\frac{x^m+y^n}{x^m-y^n} - \frac{x^m-y^n}{x^m+y^n} = \frac{4x^my^n}{x^{2m}-y^{2n}}$.

258. 1. Prove the inequality $(1+m)^{n+1} > (1+m)^n+m$, $m > 0$.

2. Which of the two numbers is greater:

2^{2^2} or $(2^2)^2$? 3^{3^2} or $(3^3)^2$? 3^{2^3} or $(3^2)^3$?

3. Find the error in the proof that any two arbitrary numbers are equal to each other.

Let a and b be different numbers, i.e. $a \neq b$;

$$(a-b)^2 = a^2-2ab+b^2 = b^2-2ab+a^2 = (b-a)^2,$$

$$(a-b)^2 = (b-a)^2,$$

$$a-b = b-a, \quad \text{whence} \quad 2a = 2b \quad \text{and} \quad a = b.$$

259. (1) Prove the identities:

(a) $(a^2+b^2)(x^2+y^2) = (ax+by)^2+(ay-bx)^2$;

(b) $(a^2+b^2)(x^2+y^2) = (ax-by)^2+(ay+bx)^2$.

(2) Formulate the property of the product of two such numbers.

(3) Verify these identities for $a = 4$, $b = 3$, $x = 2$, $y = 1$.

Carry out the following operations:

260. (1) $(a+b+c)^2$; (2) $(2x+3y+z)^2$;

(3) $(3a-4b+c)^2$; (4) $(a+b+c+d)^2$;

(5) $(2a^n-3a^{n-1}+a^{n-2})^2$;

(6) $\left(1-\frac{a}{b}-\frac{b}{a}\right)^2$.

261. (1) $(x+y)^4$; (2) $(x-y)^4$; (3) $(2x+3a)^4$;

(4) $(3m-2n)^4$; (5) $\left(\frac{1}{2}a-2b\right)^4$; (6) $(a^2-2b^3)^4$.

262. (1) $(2a+3b)^3$; (2) $(3a^2-2b)^3$; (3) $(x+y+z)^3$;

(4) $(2x-3y+1)^3$; (5) $(a-b)^6$; (6) $(2x+1)^6$.

263. Compute:

(1) $\frac{2\times 5^{22}-9\times 5^{21}}{25^{10}}$; (2) $\frac{5(3\times 7^{15}-19\times 7^{14})}{7^{16}+3\times 7^{15}}$;

(3) $\frac{(4\times 3^{22}+7\times 3^{21})\times 57}{(19\times 27^4)^2}$; (4) $\frac{10\times 2^n-5\times 2^{n-1}}{6\times 2^{n+1}+3\times 2^n}$.

15. POWERS WITH ZERO AND NEGATIVE INTEGRAL EXPONENTS

264. Calculate orally:

(1) 2^{-3}, $(-1.3)^0$, $(0.2)^{-3}$, $\left(-1\frac{1}{2}\right)^{-3}$, $(-1)^{-4}$;

(2) -5^{-2}, $(-3)^{-2}$, 27×3^{-2}.

265. Compute:

$$(-0.1)^{-4},\ \left(\frac{3}{4}\right)^{-1},\ \left(\frac{2}{5}\right)^{-2},\ 243^{-1},\ (-12)^{-1},\ (-1)^{-3},$$

$$\left(-2\frac{1}{2}\right)^{-4},\ (-0.1)^{-2},\ (-0.1)^{-3},\ -0.25^{-2}.$$

266. 1. Which is greater: 3^2 or 3^0, $\left(\frac{1}{3}\right)^2$ or $\left(\frac{1}{3}\right)^0$, $(-1)^2$ or $(-1)^0$ $(-1)^3$ or $(-1)^0$?

2. How many solutions has the equation $(2x+3)^0 = 1$?

267. 1. In the equations $y = 2^x$ and $y = \left(\frac{1}{2}\right)^x$ put x equal to 0, 1, 2, 3. How will y change with an increase in x? Show it graphically.

2. Graph the following functions:

$$y = 2x^0,\quad y = -4x^{-2},\quad y = (x+1)^{-2},\quad y = 12x^{-1}.$$

268. Transform the following expressions so as to eliminate the zero and negative exponents:

(1) m^2n^{-1}, a^0b^{-2}, $5x^{-1}y^0$, $7^{-1}a^{-1}bc^{-2}$

(2) $(a+b)(a-1)^{-1}$, $(x-y)(x+y)^{-2}$, $(m+n)^0(m-n)^{-2}$;

(3) $\frac{xy^{-2}}{m^0}$, $\frac{a^{-1}b^{-1}}{a^0b^0}$, $\left(\frac{m}{n}+\frac{n}{m}\right)^{-1}(mn)^0$, $\frac{2a(a-b)^{-4}}{b^{-n}(x+y)^{-2}}$.

Represent the following fractions in the form of integral expressions.

269. (Orally).

$$\frac{1}{a^2},\ \frac{x}{x+2},\ -\frac{3}{(a+b)^3},\ \frac{x^m}{y^n},\ \frac{5}{2^{-2}x^{-3}y^{-5}z^{-1}}.$$

270. (1) $\frac{1}{4}$, $\frac{5}{27}$, $-\frac{1}{64}$, $\frac{81}{625}$, $-2\frac{3}{4}$, $\frac{7}{216}$;

(2) 0.1, 0.001, 0.0000001, −0.00023, 0.0238.

271. (1) $\dfrac{m+1}{(m+1)^{-1}\times 2x\times(x-y)^{-2}}$;

(2) $\dfrac{1}{\dfrac{a}{b}-\dfrac{b}{a}}$; (3) $\dfrac{\dfrac{1}{x^3}+\dfrac{1}{y^3}}{\dfrac{1}{x^3}-\dfrac{1}{y^3}}$;

(4) $\left(\dfrac{1+x^{-1}}{1-x^{-1}}\right)^{-1}$; (5) $\dfrac{3^{-1}(a+b)}{2^{-1}a^{-2}(a-b)^{-3}}$;

(6) $\dfrac{\left(\dfrac{2}{a^5}+\dfrac{3}{b^4}\right)^3}{\left(\dfrac{1}{x^2}-\dfrac{1}{y^3}\right)^4}$.

272. Transform the following expressions so as to eliminate negative exponents:

(1) $\dfrac{54x^5y^{-1}z^{-4}}{42x^{-1}y^{-8}z^{-4}}$; (2) $\dfrac{18x^{-4}y^{-5}z}{24x^{-2}y^3z^{-1}}$;

(3) $\dfrac{27x^{-m}y^{-2m}}{18x^{-2n}y^{-m}}$; (4) $\dfrac{17^{-1}a^{-p}b^{-3p}}{31^{-1}a^{-2p}b^{-2p}}$.

273. Compute the following:

(1) $\left[6-4\left(\dfrac{5}{16}\right)^0\right]^{-2}$; (2) $\left[\left(\dfrac{2}{3}\right)^{-1}-\dfrac{3}{4}\right]^{-1}$;

(3) $\dfrac{3\left(\dfrac{2}{3}\right)^{-2}+4^{-1}}{\left(\dfrac{1}{2}\right)^{-1}+5}$; (4) $\dfrac{2^{-3}+\left(\dfrac{3}{4}\right)^{-4}\left(-\dfrac{1}{2}\right)^2}{\left(\dfrac{1}{5}\right)^0-12\times 3^{-3}}$;

(5) $\dfrac{(0.6)^0-(0.1)^{-1}}{\left(\dfrac{3}{2^3}\right)^{-1}\left(\dfrac{3}{2}\right)^3+\left(-\dfrac{1}{3}\right)^{-1}}$;

(6) $\dfrac{\left(\dfrac{1}{2}\right)^{-2}-5(-2)^{-2}+\left(\dfrac{2}{3}\right)^{-2}}{2^{-2}+1^0}$

Carry out the following operations:

274. (1) a^5a^{-5}; (2) $(3^{-2}\times 2^4)(3^3\times 2^{-3})$;

(3) $(6^{-3}\times 49\times 4^{-10})(4^{10}\times 7^2\times 6^4)$; (4) $(5m^3np^{-2})\times$

$\times(25^{-1}m^{-2}n^{-2}p^3)$; (5) $(-7a^{-3}b^{-2})(-4a^2b^{-1})(-a^2b^2x^{-1})$;

(6) $(a-x)^{-2}(x-a)^3$.

275. (1) $x^{-7} : x^4$; (2) $x^7 : x^{-6}$; (3) $x^{-9} : x^{-7}$;

(4) $(2^3a^{-3}b^4) : (2^{-2}a^{-4}b^{-3})$;

(5) $(x-y)^{-2} : (y-x)^{-1}$;

(6) $\dfrac{1}{6a^{n-1}b^{-2}c^{-3}} : \dfrac{5}{3}\, a^{-n}b^{n+2}c^5$;

(7) $\dfrac{7}{3}\, x^{-5}y^{n-1}z : \dfrac{5}{6x^4yz^2}$.

276. (1) $(a^{-2}+b^{-1})^2$; (2) $(x^{-2}+y^{-2})(x^{-2}-y^{-2})$;

(3) $(x^{-1}+y^{-1})(x^{-2}-x^{-1}y^{-1}+y^{-2})$;

(4) $(2x^{-2}-1)(4x^{-4}+2x^{-2}+1)$;

(5) $\dfrac{4x^{-2}-9y^{-2}}{a-b} \times \dfrac{a^{-2}-b^{-2}}{2x^{-1}+3y^{-1}}$;

(6) $\dfrac{a^{-3}-b^{-3}}{x^{-1}+y^{-1}} \times \dfrac{x+y}{a^{-2}+a^{-1}b^{-1}+b^{-2}}$.

277. (1) $(8x^{-3}-27y^{-3}) : (4x^{-2}+6x^{-1}y^{-1}+9y^{-2})$;

(2) $(a^{-3}+a^0 : (a^{-1}+1)$; (3) $(a^2-b^2) : (a^{-1}-b^{-1})$;

(4) $(2x^{-3}+x^{-2}+x^{-1}-4) : (x^{-1}-1)$.

278. (1) $(-a^2)^{-5}$; (2) $(-a^5)^{-2}$; (3) $(-a^{-5})^{-2}$;

(4) $(-x^3)^{-2n}$; (5) $(-x^{2n})^{-3}$; (6) $(-x^{-2n})^{-3}$;

(7) $\left[\left(-\dfrac{1}{5}\right)^{-2}\right]^{-1}$; (8) $\left(\dfrac{a^{-2}b^3}{x^{-1}y^{-4}}\right)^{-2}$; (9) $\left(\dfrac{a^{-3}b}{x^3y^{-2}}\right)^{-3}$.

Simplify:

279. (1) $\left(\dfrac{x+y}{x-y}\right)^{-3}\left(\dfrac{x-y}{x+y}\right)^{-2}\left(\dfrac{x+y}{x-y}\right)^{2}$;

(2) $\left(\dfrac{m+n}{m-n}\right)^{-2} \times \dfrac{k^0}{(m+n)^{-3}} \times (m-n)^{-3}$;

(3) $\left(\dfrac{1}{1+a}\right)^{5}\left(\dfrac{1}{1-a}\right)^{-7}\left(\dfrac{a-1}{a+1}\right)^{-6}$;

(4) $[m-(1-m)^{-1}]^{-1}\left[\dfrac{m(m-2)+m^0}{\dfrac{1}{m^{-2}}-m+1}\right]^{-1}$.

280. (1) $\dfrac{\frac{1}{2}-a^{-1}}{4-\left(\frac{1}{a}\right)^{-2}}:\left[\dfrac{1}{2^{-2}(2+a)}-2a^{-1}-1\right]$,

and evaluate for $a=-\frac{1}{2}$;

(2) $\dfrac{3ab}{c^{-1}}:\left(\dfrac{b}{c^{-1}}+\dfrac{a}{c^{-1}}-\dfrac{a}{b^{-1}}\right)-\dfrac{(a-1)\,a^{-1}+(b-1)\,b^{-1}+(c+1)\,c^{-1}}{a^{-1}+b^{-1}-c^{-1}}$;

(3) $\dfrac{a^{-1}+(b+c)^{-1}}{a^{-1}-(b+c)^{-1}}\times\left[x^0+\left(\dfrac{2bc}{b^2+c^2-a^2}\right)^{-1}\right]$;

(4) $\left\{x+\left[1+\left(\dfrac{3-x}{x+1}\right)^{-1}\right]^{-1}\right\}^{-1}$ and evaluate for $x=-\frac{1}{3}$;

(5) $\dfrac{a^{-2}b^{-1}+a^{-1}b^{-2}}{a^{-2}-b^{-2}}+a^3(a^2-2ab+b^2)^{-2}$.

281. Evaluate the following using the table of values for fractions of the form $\frac{1}{n}$:

(1) 2.5^{-1}, 2.56^{-1}, 2.564^{-1}, -20.43^{-1};

(2) 6.405^{-1}, 1.2536^{-1}, 8.042^{-1}, -1.1109^{-1};

(3) 0.35^{-1}, 0.08075^{-1}, 215.49^{-1}.

16. THE CONCEPT OF THE ROOT. EXTRACTING THE SQUARE ROOT OF A NUMBER. IMAGINARY NUMBERS. EXTRACTING THE ROOT OF A PRODUCT, A FRACTION AND A POWER

282. 1. The area of a square plot equals 64 m^2. How long is the side of the plot?

2. How long is the edge of a cubic tank containing 343 litres?

283. 1. What is the cube of 4 equal to? What is the cube root of 64 equal to?

2. Compute $\sqrt{81}$, $\sqrt{121}$, $\sqrt{169}$, $\sqrt{\frac{1}{25}}$. How can the answers be checked on?

3. Check on the found roots:

$$\sqrt{973.44}=31.2,\quad \sqrt[3]{-1728}=-12,\quad \sqrt[3]{-3375}=-15,$$

$$\sqrt[5]{-1024}=-4.$$

284. 1. For what real values of x will the expressions $\sqrt[2n]{x}$ and $\sqrt[2n+1]{x}$ (n a natural number) be real numbers?

2. Which of the following roots are real numbers:

$$\sqrt{0},\ \sqrt{-1},\ \sqrt[3]{-1},\ \sqrt[5]{4},\ \sqrt[4]{-16},\ \sqrt[3]{-27},\ \sqrt[4]{-1},$$

$$\sqrt{-0.04},\ \sqrt[3]{x},\ \text{where } x<0;\ \sqrt[4]{x},\ \text{where } x<0?$$

285. Find the permissible values of the real argument for the following real functions:

(1) $\sqrt{x-1}$; (2) $\sqrt[6]{1-3x}$; (3) $\sqrt{-7-2x}$;

(4) $\sqrt[3]{5x-6}$; (5) $\sqrt[4]{5x-6}$; (6) $\sqrt[3]{(2-5x)x}$;

(7) $\sqrt{(2-5x)x}$; (8) $\sqrt{(x-1)(x-2)(x-3)}$; (9) $\sqrt{x-3}-\sqrt{x-7}$.

286. 1. For what conditions do the expressions $\sqrt{a}$, $\sqrt{-b}$, $\sqrt[n]{c}$, $\sqrt[n]{-d}$ have sense on the set of real numbers.

2. What condition must be satisfied by m for the relations $\sqrt[n]{m}<m$, $\sqrt[n]{m}=m$, $\sqrt[n]{m}>m$ to be fulfilled?

3. For what values of n is the identity $\sqrt[n]{a^n}=a$ correct?

4. What values does the expression $\frac{\sqrt[n]{a^n}}{a}$ take on if $a>0$, $a<0$, $a=0$? Consider the cases when n is even and when it is odd.

5. Prove the identity $\sqrt[2n+1]{-a}=-\sqrt[2n+1]{a}$, where n is a natural number. Apply the above proved identity to the following radicals:

$$\sqrt[3]{-2},\ \sqrt[7]{-25},\ \sqrt[3]{3-\sqrt{10}},\ \sqrt[5]{\sqrt{2}-\sqrt{3}},\ \sqrt[5]{(a-2)^3}.$$

287. Pick the rational and irrational numbers from the given below:

$$\sqrt[3]{64},\ \sqrt{81},\ \sqrt[3]{-10},\ \sqrt{625},\ \sqrt{101},\ \sqrt{-25},\ \sqrt[4]{9:16},$$

$$\sqrt{0.04},\ \sqrt{0.08}.$$

288. Find the arithmetical value of the root:

(1) (Orally) $\sqrt[4]{81}$, $\sqrt{289}$, $\sqrt{2500}$, $\sqrt{0.81}$, $\sqrt{(-5)^4}$,

$\sqrt{(2-\sqrt{3})^2}$, $\sqrt[4]{(\sqrt{2}-\sqrt[3]{3})^4}$;

(2) $\sqrt{a^2}$, $\sqrt{(3-m)^2}$, $\sqrt{(2x+1)^2}$, $\sqrt{(a-3)^2}$,

$\sqrt{(a+b)^2}$, $\sqrt{(2-x)^2y^4}$, $4\sqrt{x^3}+\sqrt{x(x-5)^2}$.

289. 1. Find the values of x for which the following expressions will be arithmetical roots:

$$\sqrt{2+x},\ \sqrt[3]{2-x},\ \sqrt[4]{|x|-3},\ \sqrt[5]{x^2+x+1}.$$

2. Prove that

$$x+y+\sqrt{(x-y)^2} = \begin{cases} 2x \text{ if } x > y \\ 2y \text{ if } x < y. \end{cases}$$

3. May we assert that for any values of a the expression $\sqrt{(3-a)^2}+$ $+\sqrt{(a-4)^2}$, when simplified equals -1?

4. Evaluate the expression $4x-\sqrt{1-2x+x^2}$ for $x = 5$ using two methods: (a) extracting the root, (b) by direct substitution.

290. Simplify:

(1) $\sqrt{(a-2)^2}$ for $a \leqslant 2$;

(2) $\sqrt{(a-3)^2}$ for $a \geqslant 3$;

(3) $\sqrt{(a-b)^2}$ for $a < b$;

(4) $\sqrt[4]{(x+2)^4}$ for $x \leqslant -2$;

(5) $\sqrt[4]{(5-x)^4}$; (6) $\sqrt[6]{(x-y)^6}$.

291. Graph the functions $y = \sqrt{(x+1)^2}+\sqrt{(x-1)^2}$ and $y =$ $= \sqrt{(x+2)^2}-\sqrt{(x-1)^2}$.

292. How many digits has the square of a one, two, three, ..., n-digit number?

2. How many digits has the integral part of $\sqrt[m]{a}$, if $m = 2, 3$ where a is a positive one-, two-, ..., n-digit number?

293. $\sqrt{7225}$, $\sqrt{44100}$, $\sqrt{166464}$, $\sqrt{\frac{961}{1024}}$, $\sqrt{5\frac{121}{144}}$,

$\sqrt{0.0784}$, $\sqrt{4998.49}$.

294. 1. Accurate to 1: $\sqrt{8317}$, $\sqrt{9310}$, $\sqrt{24334}$.

2. Accurate to 0.1: $\sqrt{979.15}$, $\sqrt{6270}$, $\sqrt{\frac{3}{7}}$.

3. Accurate to 0.01: $\sqrt{535}$, $\sqrt{0.6}$, $\sqrt{1894.25}$, $\sqrt{3\frac{1}{7}}$.

295. Extract the root accurate to 1 and write the answer in the form of a double inequality, for example $2<\sqrt{7}<3$;

$$\sqrt{228},\ \sqrt{9000},\ \sqrt{1340},\ \sqrt{80.9},\ \sqrt[3]{50},\ \sqrt[3]{100},\ \sqrt[4]{60},\ \sqrt[5]{46}.$$

296. Extracting the square root, find the greatest (least) integer whose square is less (greater) than the number 3242; 7437; 8347; 35,975; 483,974.

297. Using the tables of square roots find $\sqrt{x}$ for the following values of x:

2; 4; 5; 0.7; 1.5; 1.96; 110; 2401; 0.22; 892.3;
50.236; 0.841; 0.0841.

Solve problems 298 to 305 with the aid of a slide rule.

298. $\sqrt{4}$, $\sqrt{81}$, $\sqrt{361}$, $\sqrt{1024}$, $\sqrt{52.5}$, $\sqrt{0.535}$, $\sqrt{0.061}$, $\sqrt{0.0005}$, $\sqrt{20.5}$, $\sqrt{25{,}000}$, $\sqrt{0.00141}$, $\sqrt{0.428}$, $\sqrt{6.852}$, $\sqrt{828}$, $\sqrt{1733}$, $\sqrt{0.00484}$.

299. (1) Determine the area of a square by its side equal to 4.5 m, 0.38 m, 109 m, 540 m, 6.33 m;

(2) determine the side of a square by its area, equal to 40.6 m^2, 0.98 m^2, 733 m^2, 64.9 m^2.

300. (1) $\sqrt[3]{8800}$, $\sqrt[3]{0.0000088}$, $\sqrt[3]{0.00000935}$, $\sqrt[3]{3.854}$;

(2) $\sqrt[3]{42.15}$, $\sqrt[3]{0.0836}$, $\sqrt[3]{24.35}$, $\sqrt[3]{0.00375}$, $\sqrt[3]{72.36}$, $\sqrt[3]{0.0954}$;

(3) $\sqrt[3]{95365}$, $\sqrt[3]{0.000093}$, $\sqrt[3]{654348}$, $\sqrt[3]{0.000544}$.

301. (1) Determine the volume of a cube by its edge equal to 1.75 m, 2.04 m, 0.348 m, 0.546 m, 4.025 m;

(2) determine the edge of a cube given its volume equal to 846.3 m^3, 42.56 m^3, 0.00347 m^3, 0.0546 m^3.

302. Check on the following results:

(1) $\dfrac{\sqrt{8.35}\times 0.84}{52.6} \approx 0.0461$; (2) $\sqrt{\dfrac{44.6\times 0.00404}{21.6\times 0.0403}} \approx 0.465$;

(3) $\frac{33.5\times\sqrt{44.8}}{71.8\times0.08}\approx 20.7$; (4) $\left(\frac{73.8\times2400\times\sqrt[3]{0.09}}{81.5\times3.61\times7}\right)^2\approx 1490.$

303. $\sqrt[3]{(4.8)^2}$, $(24.5)^{\frac{2}{3}}$, $(0.505)^{\frac{2}{3}}$, $\sqrt[3]{-734^2}$, $\sqrt{3.6^3}$, $\sqrt{(0.82)^3}$, $(608)^{\frac{3}{2}}$, $(0.23)^{\frac{3}{2}}$.

304. Compute the diameter of a circle given its area equal to 8340 m^2, 0.0495 m^2, 26,400 m^2, 1000 m^2, 100 m^2, 10 m^2, 1 m^2.

305. The capacity of a cylindrical bucket of height 4 dm is 27 litres. Determine the diameter of the bucket.

306. Representing the products and the squares of numbers in the form of the areas of rectangles and squares, respectively, interpret geometrically the formulas

$$(x+y)^2 = x^2+2xy+y^2 \quad \text{and} \quad x^2-y^2 = (x-y)(x+y), \quad x > y.$$

307. 1. Find the leg of the triangle if the other leg is equal to 24 cm, and the hypotenuse to 36 cm.

2. The side of an isosceles triangle equals 16 cm. Find its altitude.

308. A body, freely falling in a vacuum, covers the distance s during the time $t = \sqrt{\frac{2s}{g}}$, where g is the acceleration due to gravity ($g \approx 10$ m/s^2). Find t if $s = 4500$ m.

309. Horizontal range on sea is determined by the formula $d = 4.1\sqrt{h}$, where d is the distance from the horizon in kilometres, and h is the altitude of the observer's eye above the sea level in metres. Compute (to within 1 km) the distance between the observer and the horizon if $h = 10$ m; $h = 300$ m.

310. According to Torricelli's law the velocity of efflux of a fluid from an orifice in a reservoir is determined by the formula $v = 0.6\sqrt{2gH}$, where H is the depth of the orifice. Find v if $H = 30$ cm, $g = 10$ m/s^2.

311. To keep a body orbiting round the Earth as a satellite it is necessary that its centrifugal force counterbalances the attracting force of the Earth. The velocity required to keep a satellite in its orbit is computed by the formula $u = \sqrt{\frac{gr_0^2}{r}}$ where r_0 is the radius of the Earth, r is the distance between the satellite and the centre of the Earth, g is the gravitational acceleration on the Earth's surface. Compute the speed required for the satellite to

retain in orbit 300 km distant from the surface of the Earth; put $r_0 \approx 6370$ km, $g \approx 10$ m/s^2.

312. Solve the following equations:

(1) (Orally) $\sqrt[3]{x} = -2$, $\sqrt[4]{x} = 5$, $3 - \sqrt{x} = 5$, $\sqrt[3]{x} - 1 = 3$, $7 - \sqrt[4]{x} = 4$;

(2) $5x^2 - 45 = 0$, $2x^2 + 32 = 0$, $3x^2 + 7 = 0$, $12x^2 = 0$.

313. The altitude of a rectangle is 75% its base. Find the perimeter of the rectangle knowing that its area equals 48 m^2.

314. Extract the given roots (orally):

$$\sqrt{64 \times 36 \times 100},\ \sqrt[3]{216 \times 64},\ \sqrt[5]{-32 \times 243},\ \sqrt{\frac{36}{121}},$$

$$\sqrt[3]{-2\frac{10}{27}},\ \sqrt[4]{\frac{81}{625}},\ \sqrt{3\frac{1}{16}},\ \sqrt{0.01},\ \sqrt{0.0144},\ \sqrt[3]{0.343},$$

$$\sqrt[4]{6^8},\ \sqrt{\left(\frac{2}{3}\right)^6},\ \sqrt[3]{a^9},\ \sqrt[3]{x^{6n}},\ \sqrt[n]{x^{3n}}.$$

Carry out the following operations:

315. (1) $\sqrt{25a^2}$; (2) $\sqrt[3]{27x^6}$; (3) $\sqrt{\frac{1}{9}a^2b^4}$;

(4) $\sqrt[4]{a^8b^4c^{12}}$; (5) $\sqrt[3]{-125x^3y^6}$; (6) $\sqrt[5]{-32a^5b^{10}}$;

(7) $\sqrt[3]{64a^{-9}}$; (8) $\sqrt[4]{0.0081a^8b^4c^{12}}$.

316. (1) $\sqrt{\frac{4a^2b^4}{25c^2d^6}}$; (2) $\sqrt[3]{-\frac{27a^6b^3}{64x^{12}}}$;

(3) $4xy\sqrt[3]{\frac{27a^3b^6}{64x^3y^6}}$; (4) $\sqrt[n]{\frac{a^nb^{2n}}{x^{3n}}}$;

(5) $\sqrt[n]{\frac{(a-b)^{3n}}{(a+b)^{6n}}}$; (6) $\sqrt[n]{\frac{(x+y)^{2n}}{x^{3n}(x-y)^n}}$.

317. Separate real numbers from the imaginary ones:

$$\sqrt{10},\ \sqrt{-4},\ \sqrt[3]{-27},\ 3.1020030004\ldots,\ \sqrt[4]{-81},\ \sqrt[6]{-3},$$

$$\sqrt[4]{-\frac{81}{16}} - \sqrt{169}.$$

318. Carry out the indicated operations:

(1) $2i+3i-4i$; (2) $\sqrt{-9}+\sqrt{-4}+\sqrt{-16}$;

(3) $\sqrt{-64}-\sqrt{-36}-\sqrt{-1}$; (4) $(\sqrt{-49}-\sqrt{-81})3$.

17. TRANSFORMING RADICALS

17.1. Taking Factors Outside the Radical Sign

319. Take factors outside the radical sign:

(1) $\sqrt{8}$, $\sqrt{48}$, $\sqrt{175}$, $\sqrt[3]{375}$, $\sqrt[4]{243}$, $\sqrt{128}$, $\sqrt{288}$, $\sqrt{\frac{50}{49}}$;

(2) $\sqrt[3]{24}$, $\sqrt[3]{54}$, $\sqrt[3]{135}$, $\sqrt[4]{625}$, $\sqrt[3]{-686}$, $\sqrt[5]{-96}$, $\sqrt[5]{300{,}000}$.

320. Without performing the operations determine which number is greatest:

(1) $3\sqrt{2}$ or $\sqrt{27}$, $3\sqrt[4]{m^3}$ or $\sqrt[4]{162m^3}$, $2a\sqrt{a}$ or $\sqrt{4a^3}$;

(2) $3\sqrt{\frac{3}{8}}$ or $\sqrt{\frac{3}{2}}$, $2\sqrt{\frac{4}{75}}$ or $3\sqrt{\frac{25}{243}}$, $2\sqrt[3]{\frac{1}{54}}$ or $\sqrt[3]{\frac{8}{375}}$;

$-3\sqrt[3]{54}$ or $-2\sqrt[3]{128}$?

Take factors outside the radical sign.

321. (1) $\sqrt{x^3}$, $\sqrt{ax^6}$, $\sqrt[3]{x^7b}$, $\sqrt[3]{16x^2y^6z^8}$;

(2) $\sqrt[3]{x^{3n+1}}$, $\sqrt[3]{a^{3n-1}}$, $\sqrt[n]{a^{n+2}}$, $\sqrt[n]{x^{n+3}}$, $\sqrt[n]{4x^{2n}}$;

(3) $\frac{2}{3}\sqrt[3]{54x^4z^5}$, $\frac{3x}{8}\sqrt[4]{32x^5y^8}$, $\sqrt[n]{a^{3n}b^{2n}}$;

(4) $\sqrt{\frac{a}{9}}$, $\sqrt[3]{\frac{a^6b}{x^9}}$, $\frac{\sqrt[n]{x^{n+1}}}{x}$, $\sqrt{\frac{5a^2}{98}}$, $a\sqrt[m]{\frac{b^{m+2}}{a^{m+1}}}$.

322. (1) $\sqrt{\frac{a^6}{b^7}}$, $\sqrt{\frac{8ab^2}{27x^2y}}$, $\sqrt[3]{\frac{ab^2c^3}{27xy^6z^{10}}}$, $\sqrt{\frac{x}{y^2}-\frac{1}{y}}$;

(2) $\sqrt{\frac{1}{m^2}-\frac{n}{m^3}}$, $\sqrt{\frac{1}{x^2}+\frac{1}{y^2}}$, $\sqrt[3]{\frac{m^3}{n^3}+1}$, $\sqrt[5]{\frac{a}{x^6}+\frac{b}{x^{10}}}$.

323. For what values of a and b do the following equalities hold true?

(1) $\sqrt[2n]{a^{2n}b} = a\sqrt[2n]{b}$; (2) $\sqrt[2n]{a^{2n}b} = -a\sqrt[2n]{b}$;

(3) $\sqrt[2n+1]{a^{2n+1}b} = a\sqrt[2n+1]{b}$.

Give numerical examples for each equality.

324. Take factors outside the radical sign:

(1) $\sqrt{3(2-\sqrt{5})^2}$; (2) $\sqrt{18(\sqrt{3}-2)^2}$; (3) $\sqrt{\frac{(\sqrt{3}-3)^2}{4}}$;

(4) $\sqrt[4]{\frac{80}{(1-\sqrt{2})^4}}$; (5) $\sqrt{8(3-\sqrt{10})^4}$; (6) $\sqrt{24(\sqrt{17}-4)^4}$;

(7) $\sqrt[3]{54(1-\sqrt{5})^3}$; (8) $\sqrt[5]{(5-\sqrt{5})^7}$; (9) $\sqrt{(2-x)^3}$ for $x \leqslant 2$; (10) $\sqrt{x^3(x-3)^5}$ for $x \geqslant 3$;

(11) $\sqrt{(x-4)^3(1-x)^5}$ for $1 < x < 4$.

325. Take the factors outside the radical sign taking into account the permissible values of the letters entering the given radicands:

(1) $\sqrt{(x-2)^3}$; (2) $\sqrt{50(5+a)^5}$; (3) $\sqrt{(5x-3)^4x^5}$;

(4) $\sqrt[4]{(3x+5)^5}$; (5)* $\sqrt{\frac{m^2}{n^2}-\frac{m}{n}}$; (6) $\sqrt[3]{\frac{x}{y^4}+\frac{y}{x^4}}$;

(7)* $\sqrt{\frac{a}{(a-b)^2}-\frac{1}{a-b}}$; (8) $\sqrt{8-12a+6a^2-a^3}$;

(9) $\sqrt[4]{(\sqrt{2}-x)^5x^8}$; (10) $\frac{a}{b}\sqrt[3]{\frac{a^4}{b^3}+\frac{a^4}{b^2}+\frac{a^4}{b}}$;

(11) $\sqrt[4]{2(x-4)^4}$; (12) $\sqrt[4]{2(x-4)^8}$; (13) $\sqrt[4]{2(x-4)^{12}}$;

(14) $\sqrt{\frac{1}{x^2}-\frac{1}{x}}$.

326. 1. A student was asked to take a rational factor outside the radical sign in the expression $\sqrt{8\frac{8}{63}}$. Without turning the mixed number into an improper fraction, he quickly wrote the following equality $\sqrt{8\frac{8}{63}} = 8\sqrt{\frac{8}{63}}$. Is this equality correct?

2. Prove the identity

$$\sqrt{a+\frac{a}{a^2-1}} = a\sqrt{\frac{a}{a^2-1}} \text{ if } a > 1.$$

Take the factors outside the radical sign:

$$\sqrt{7\frac{7}{48}},\ \sqrt{11\frac{11}{120}},\ \sqrt{13\frac{13}{168}}.$$

17.2. Bringing Factors under the Radical Sign

327. For what values of a and b are the following equalities correct?

(1) $a\sqrt[2n]{b} = \sqrt[2n]{a^{2n}b}$; (2) $a\sqrt[2n]{b} = -\sqrt[2n]{a^{2n}b}$;

(3) $a\sqrt[2n+1]{b} = \sqrt[2n+1]{a^{2n+1}b}$.

Give numerical examples for each equality.

328. (Orally). Bring the factors under the radical sign:

$$2\sqrt{5},\ 5\sqrt[3]{2},\ 2\sqrt[4]{6},\ x\sqrt[3]{5},\ \frac{2}{3}\sqrt{6},\ a\sqrt[3]{a},\ x^2\sqrt{a},\ \frac{3}{4}\sqrt{x},\ -4\sqrt{3}.$$

329. (Orally). Without extracting the roots find out which number is greater:

(1) $3\sqrt{2}$ or $2\sqrt{3}$; (2) $3\sqrt{10}$ or $4\sqrt{8}$; (3) $2\sqrt[4]{3}$ or $\sqrt[4]{45}$;

(4) $\frac{1}{2}\sqrt{5}$ or $2\sqrt{\frac{1}{3}}$?

330. 1. Which of the given numbers is greater?

$$6\sqrt[3]{5},\ 8\sqrt[3]{2},\ 2\sqrt[3]{130},\ \sqrt[3]{900}.$$

2. Compute $20\sqrt{5}$ to within 0.1 with the aid of two methods. Why are different results obtained? Which of them is more accurate?

3. Compute accurate to 0.1 making use of two methods:

$$2\sqrt{2},\ 3\sqrt{3},\ 5\sqrt{5},\ 8\sqrt{6},\ 10\sqrt{14},\ 20\sqrt{7}.$$

4. Using the Table of square roots compute to within 0.1:

$$3\sqrt{2},\ 5\sqrt{3},\ -6\sqrt{15},\ 7\sqrt{5}.$$

Bring the factors under the radical sign:

331. (1) $6\sqrt[3]{2\frac{1}{12}}$; (2) $-3\sqrt[4]{1\frac{5}{27}}$; (3) $\frac{2}{3}\sqrt{3}$;

(4) $-\frac{1}{2}\sqrt[3]{-12}$; (5) $1\frac{3}{4}\sqrt{2\frac{2}{7}}$; (6) $0.2\sqrt[3]{25}$.

332. (1) $2a\sqrt{3a}$, $2x\sqrt[3]{3y}$, $3xy^3\sqrt{x}$, $-n^2\sqrt{a}$, $\frac{a}{b}\sqrt[3]{\frac{b^2}{a^2}}$, $2m^2n\sqrt[3]{\frac{1}{2mn}}$;

(2) $\frac{3}{4}\sqrt[3]{2mn}$, $\frac{x}{y}\sqrt[3]{\frac{y}{x}}$, $-\frac{a}{b}\sqrt[3]{-\frac{b^4}{a^5}}$;

(3) $\frac{\sqrt[3]{m^5+1}}{m}$, $(m+n)\sqrt{\frac{1}{m^2-n^2}}$, $\frac{1}{m-n}\sqrt{m^2-n^2}$.

333. (1) $\frac{2a}{3b}\sqrt[5]{-\frac{b^4}{2a^3}}$, $\frac{\sqrt[3]{a^3+a^4}}{a}$, $\frac{m-n}{\sqrt{m^2-n^2}}$, $\frac{v}{\pi\sqrt[3]{\left(\frac{v}{2\pi}\right)^2}}$;

(2) $2x\sqrt[3]{\frac{1}{x^3}+x}$, $mn\sqrt{\frac{1}{m}+\frac{1}{n}}$, $m^2n^2\sqrt{\frac{1}{m^4}-\frac{1}{n^4}}$.

334. (1) $mn\sqrt[n]{\frac{1}{m^{n-1}n^{n-1}}}$, $\frac{a}{b}\sqrt[n]{\frac{b^{n+2}}{a^{n+2}}}$, $\frac{x-y}{x+y}\sqrt[n]{\frac{(x+y)^{n-2}}{(x-y)^{n-2}}}$;

(2) $2x^ny^{-2}\sqrt[3]{5x^{-n}y^3}$, $\frac{\sqrt[4]{a^3b^2}}{a^{-2}b^{-1}}$, $-\left(\frac{a}{b}\right)^{-1}\sqrt[3]{\frac{a^2}{b^2}}$.

335. (1) $(m-3)\sqrt{\frac{1}{m-3}}$ for $m > 3$; (2) $(m-3)\sqrt{\frac{1}{3-m}}$ for $m < 3$;

(3) $(x-y)\sqrt{\frac{1}{y^2-x^2}}$ for $0 < x < y$; (4) $(8-x)\sqrt{\frac{1}{x-8}}$ for $x > 8$.

17.3. Reducing the Index of the Root and the Exponent of the Radicand. Reducing Radicals to the Same Index

336. 1. Compute (orally):

$$\sqrt[15]{125^5},\ \sqrt[2n]{9^n},\ \sqrt[8]{1.21^4},\ \sqrt[6]{\left(\frac{4}{9}\right)^3}.$$

2. Reduce the indices of the given radicals and the exponents of the radicands (orally):

$$\sqrt[4]{x^2},\ \sqrt[16]{a^8},\ \sqrt[6]{4m^2n^4},\ \sqrt[3]{27x^3y^{12}},\ \sqrt[10]{x^{10}}.$$

337. Reduce the indices of the given radicals and the exponents of the radicands:

(1) $\sqrt[9]{\frac{27x^3y^6}{8z^3t^9}}$, $\sqrt[12]{64x^9y^{3m}}$, $\sqrt[2n]{a^nb^{mn}}$, $\sqrt[2n]{\frac{x^{4n}}{y^{2n}}}$, $\sqrt[-4]{a^{-8}c^{-2}}$;

(2) $\sqrt[4]{(1-\sqrt{5})^2}$, $\sqrt[4]{(3-\sqrt{10})^2}$, $\sqrt[6]{(2-\sqrt{5})^2}$, $\sqrt[9]{(5-3\sqrt{5})^3}$;

(3) $\sqrt[6]{(3-m)^2}$ for $m > 3$, $\sqrt[8]{(2-x)^4}$ for $x > 2$.

338. Compute to within 0.1:

$$\sqrt[4]{36},\ \sqrt[6]{125},\ \sqrt[8]{81},\ \sqrt[10]{32},\ \sqrt[4]{2.56},\ \sqrt[6]{\frac{8}{27}},\ \sqrt[8]{10{,}000}.$$

339. Bring the following two (or three) radicals to the same index:

(1) $\sqrt{a}$ and $\sqrt[6]{b}$, $\sqrt[3]{2m^2}$ and $\sqrt[4]{3n^3}$, $\sqrt[n]{a^k}$ and $\sqrt[3n]{b^p}$, $\sqrt[8]{\frac{5x}{y^2}}$ and $\sqrt[6]{\frac{3x^2}{y}}$;

(2) $\sqrt[n]{3x^2y^2}$ and $\sqrt{xy^3}$, $\sqrt[mn]{\frac{2x^3}{ay^4}}$ and $\sqrt[m^2]{\frac{3a^2y}{x^2}}$;

(3) $\sqrt{\frac{a}{b}}$, $\sqrt[3]{\frac{a^2}{b^2}}$ and $\sqrt[4]{\frac{a^3}{b^3}}$, $\sqrt[12]{a^2b^3}$, $\sqrt[4]{a}$ and $\sqrt[8]{b^3}$.

340. 1. Which number is greater: $\sqrt{8}$ or $\sqrt[3]{25}$; $\sqrt[3]{3}$, $\sqrt[4]{4}$ or $\sqrt[12]{75}$?
2. Arrange the following numbers in ascending order:

(a) $\sqrt[3]{4}$, $\sqrt[4]{6}$, $\sqrt[12]{280}$; (b) $\sqrt{8}$, 3, $\sqrt[3]{25}$.

3. Determine the signs of the differences:

$$\sqrt{2}-\sqrt[3]{3},\ \sqrt[5]{4}-\sqrt[4]{3},\ \sqrt[3]{31}-\sqrt{10}.$$

4. Compare the given ratios $\sqrt[3]{4}:\sqrt[4]{3}$, $\sqrt[3]{3}:\sqrt{2}$, $\sqrt[4]{4}:\sqrt{2}$ with unity.

17.4. Reducing Radicands to the Integral Form

341. Using two methods compute

$$\sqrt{\frac{6}{7}} \text{ and } \sqrt{5\frac{5}{6}}.$$

Which of the methods is the most compact? Check to see which of the results is more accurate.

Reduce the radicands to the integral form:

342. $\sqrt{\frac{3}{5}}$, $\sqrt{\frac{7}{8}}$, $\sqrt{1\frac{2}{7}}$, $\sqrt{2\frac{4}{5}}$, $\sqrt[3]{-\frac{2}{9}}$, $\sqrt[3]{\frac{12}{25}}$, $\sqrt[4]{\frac{5}{27}}$, $2\frac{1}{2}\sqrt[4]{1\frac{7}{25}}$, $6\sqrt[5]{\frac{4}{27}}$, $-4\sqrt[6]{1\frac{3}{32}}$.

343. (1) $4\sqrt{\frac{x}{2}}$, $3m\sqrt{\frac{5}{3m}}$, $b\sqrt[5]{\frac{a^2}{b^3}}$, $4a^2\sqrt{\frac{b}{2a}}$;

(2) $\frac{3a}{4b}\sqrt[3]{\frac{2b}{3a}}$, $\frac{m^2}{n^2}\sqrt[3]{\frac{1}{m^5p^7}}$, $15mn\sqrt[4]{\frac{a^3m}{9m^2n^3}}$.

344. (1) $(a-b)\sqrt{\frac{2}{a-b}}$, $(x+y)\sqrt[3]{\frac{x-y}{(x+y)^2}}$, $\sqrt[3]{\frac{x}{y^2}+\frac{y}{x^2}+\frac{z}{xy}}$;

(2) $abc\sqrt[n]{\frac{a}{bc^{n-1}}}$, $\sqrt[2n]{\frac{m^{-n}x^n}{2}}$, $\frac{y}{x}\sqrt[m]{\frac{xy}{y^3(x-y)^{m+2}}}$.

17.5. Reducing Roots to the Simple Form

Reduce the given roots to the simple form:

345. (1) $9\sqrt{8\frac{1}{3}}$; (2) $1\frac{3}{7}\sqrt{19\frac{3}{5}}$; (3) $1\frac{1}{2}\sqrt[3]{1\frac{7}{9}}$;

(4) $\frac{6}{5}\sqrt[3]{-3\frac{17}{36}}$.

346. (1) $\frac{3ab^2}{2}\sqrt[3]{\frac{8}{ab}}$; (2) $\frac{4m}{n}\sqrt[4]{\frac{n^6p^3}{4m^2}}$; (3) $\frac{2ab^2}{c}\sqrt[5]{\frac{5a}{8b^4c^5}}$.

347. (1) $\sqrt{4x^2y^3-32x^3y^2}$; (3) $\sqrt{x^3-y^3+x^2y-xy^2}$;

(2) $\frac{m}{n}\sqrt{n^3-mn^2}$; (4) $\sqrt[3]{8x^6y^3-24x^9y^6}$.

348. (1) $a^2\sqrt{\frac{b}{a^4}-\frac{1}{a^3}}$; (2) $\frac{2b}{3a}\sqrt[4]{\frac{81a^4}{8b^6}-\frac{243a^5}{2b^4}}$;

(3) $\sqrt{3x^2+6x+3}$; (4) $\frac{m}{n}\sqrt[3]{\frac{n^5}{m^5}-\frac{n^3}{m^3}}$.

349. (1) $\frac{a+b}{2}\sqrt[3]{\frac{a^5b^6-a^6b^5}{b^2+2ab+a^2}}$; (2) $\sqrt{\frac{1}{x^3+x^2-x-1}}$;

(3) $\frac{a^2}{2-a}\sqrt{\frac{1}{a}+\frac{4(1-a)}{a^3}}$ if $a>2$;

(4) $x^3\sqrt{\frac{1}{x^2}-\frac{2x-1}{x^4}}$ if $x < 1$, except for $x = 0$.

350. (1) $5a^n\sqrt[3]{\frac{bc^4}{25a^{3n+2}}}$;

(2) $\frac{x-y}{x+y}\sqrt[m+n]{\frac{(x+y)^{m+n}}{a^m(x-y)^{2m+2n}}}$;

(3) $\frac{a}{a-b}\sqrt[4]{(a^2-2ab+b^2)(3a-3b)(a^4-a^2b^2)}$ if $a > b$.

17.6. Similarity of Roots

Prove that the following roots are similar:

351. (Orally). (1) $\sqrt{8}$ and $\sqrt{32}$, $\sqrt{28}$ and $3\sqrt{63}$;

(2) $\sqrt{72}$ and $\sqrt{162}$, $\sqrt[3]{40}$ and $2\sqrt[3]{135}$.

352. (1) $\sqrt[3]{80}$ and $5\sqrt[3]{270}$; (2) $5\sqrt[5]{6}$ and $\sqrt[5]{\frac{3}{16}}$.

353. (1) $\sqrt[3]{\frac{72}{343}}$ and $\sqrt[3]{-41\frac{2}{3}}$; (2) $\sqrt[4]{\frac{1}{125}}$ and $\sqrt[4]{0.3125}$.

354. (1) $\sqrt[3]{54}$, $-\sqrt[3]{16}$ and $2\sqrt[3]{432}$; (2) $\sqrt[3]{256}$, $\sqrt[3]{500}$ and $\sqrt[3]{108}$.

355. (1) $2\sqrt{a^3bc}$, $3\sqrt[4]{16a^6b^2c^6}$ and $5\sqrt{ab^3c^3}$;

(2) $\sqrt[3]{\frac{4}{xy}}$, $\sqrt[3]{\frac{x^2y^2}{54}}$ and $\sqrt[3]{4x^5y^5}$.

356. (1) $\frac{a}{b}\sqrt[6]{\frac{b^6}{a^4}}$, $\frac{a^2}{b}\sqrt[3]{\frac{a^4}{b^2}}$ and $\frac{b}{a^2}\sqrt[3]{\frac{a^{10}}{b^5}}$;

(2) $\sqrt[3]{\frac{x}{y}}$, $-x^2y\sqrt[3]{\frac{1}{x^2y}}$ and $\sqrt[3]{\frac{y^2}{x^2}}$.

357. (1) $\sqrt{\frac{4x^3}{y}}$, $\frac{1}{2}\sqrt{\frac{9x-6x^2+x^3}{y}}$ and $\sqrt{\frac{x^{-1}(1+y)^{-2}}{y^{-3}(1+x)^{-2}}}$;

(2) $\sqrt{\left(\frac{a}{b^2}-\frac{b}{a^2}\right)\frac{1}{a-b}}$, $\sqrt{\frac{(a+b)^4-a^2b^2}{(a+b)^2+ab}}$ and

$\sqrt{\frac{(a^2+b^2)^2-a^2b^2}{a^4b^2-a^3b^3+a^2b^4}}$;

(3) $\sqrt{(a-b)^3}$, $\sqrt{\frac{(a^3+b^3)^2}{a-b}}$ and $\sqrt{a^5-a^4b}$.

358. (1) $\sqrt[3]{8a^5-16a^3b^2}$, $ab\sqrt[3]{\frac{2b^2}{a^3}-\frac{1}{a}}$ and $\sqrt[3]{\frac{2}{a^3b}-\frac{1}{ab^3}}$;

(2) $\sqrt[n]{2a^{n+1}b^{n+2}}$ and $\sqrt[n]{\frac{a^{2n+1}}{b^{n-2}2^{n-1}}}$;

(3) $\sqrt[n]{a^{4+n}b^{4+n}}$ and $\sqrt[n]{\frac{b^4}{a^{3n-4}}}$.

18. POWERS WITH FRACTIONAL EXPONENTS. OPERATIONS INVOLVING RADICALS

18.1. Addition and Subtraction

359. Simplify and then compute accurate to 0.1:

(1) $\sqrt{200}-\sqrt{45}+3\sqrt{18}+2\sqrt{20}-\sqrt{49}$;

(2) $0.2\sqrt{600}-2\sqrt{0.216}+24\sqrt{1\frac{1}{2}}-18\sqrt{4\frac{1}{6}}$.

360. Perform the indicated operations:

(1) $\sqrt{\frac{1}{2}}-2\sqrt[3]{250}+\sqrt[3]{54}-\frac{3}{4}\sqrt{\frac{2}{9}}+2\sqrt{6\frac{3}{4}}$;

(2) $\sqrt{\frac{1}{5}}+\sqrt{1.8}-\frac{4}{5}\sqrt{5}-0.5\sqrt{180}+2\sqrt{\frac{1}{20}}$.

361. (1) $\left(5\sqrt[4]{0.01}+2\sqrt{0.08}\right)-\left(8\sqrt[4]{20\frac{1}{4}}+\sqrt{2.5}\right)$;

(2) $\left(9\sqrt[3]{\frac{1}{81}}-26\frac{2}{3}\sqrt[6]{0.000081}\right)-\left(0.9\sqrt[3]{-41\frac{2}{3}}-\frac{7}{6}\sqrt[3]{9000}\right)$.

362. (1) $\sqrt[3]{27a^4}+3\sqrt[3]{8a}-\frac{3}{a}\sqrt[3]{125a^7}$;

(2) $\sqrt[5]{a^5b^2}+\sqrt[5]{32b^7}-3a\sqrt[5]{b^2}$;

(3) $b\sqrt[3]{a^3b}+a\sqrt[3]{b^4}-2\sqrt[3]{a^3b^4}$;

(4) $\frac{1}{2m}\sqrt[4]{32m^5}+2\frac{1}{3}\sqrt[4]{162m}-\sqrt[4]{2m}$.

363. (1) $(\sqrt{9ab}-\sqrt{121ab})-(5\sqrt{4ab}-3\sqrt{81ab})$;

(2) $4\sqrt[3]{a^2b^5}-3b^2\sqrt[3]{\frac{a^2}{b}}+\frac{4b}{a^2}\sqrt[3]{-a^8b^2}-$

$-6ab\sqrt[3]{\frac{b^2}{a}}-\frac{3}{2}ab^2\sqrt[3]{-\frac{8}{ab}}$;

(3) $\frac{1}{a}\sqrt{7.8125a^2x}-\frac{b^2}{c^2}\sqrt{\frac{5c^4x}{b^4}}+3a^2\sqrt{\frac{5x}{a^2}}+a^2x\sqrt{\frac{0.45}{a^2x^2}}$.

364. (1) $\frac{3m^3}{m+n}\sqrt{\frac{1}{m^2}-\frac{n^2}{m^4}}-2m\sqrt{\frac{m^2-n^2}{(m+n)^2}}-$

$-3(mn-n^2)\sqrt{\frac{1}{m^2-n^2}}+4n\sqrt{\frac{m^4-n^4}{(m^2+n^2)(m+n)^2}}$;

(2) $(2xy-4y^2)\sqrt{\frac{1}{x-2y}}-4x^2\sqrt{\frac{1}{16x}-\frac{y}{8x^2}}+$

$+(x^2+y^2)\sqrt{\frac{x-2y}{y^2-2xy+x^2}}-\frac{xy}{x-y}\sqrt{4x-8y}$;

(3) $(a^2-2b^2)\sqrt{\frac{a+b}{a-b}}-\sqrt{(a+b)^3(a-b)}+\frac{b}{a-b}\sqrt{a^2b^2-b^4}$.

365. (1) $\sqrt[3]{(1+x)^3y}+\sqrt[3]{(1-x)^3y}-\sqrt[3]{(x-y)^3}-\sqrt[3]{8y}+$

$+\sqrt[3]{x^3+3x^2y+3xy^2+y^3}$;

(2) $\sqrt{2x^2+4x+2}-\sqrt{9y^2-18y+9}+$

$+\sqrt{2x^2-4x+2}-\sqrt{9y^2+18y+9}$;

(3) $a\sqrt[3]{\frac{1-a^2}{a^2}}-2a\sqrt[3]{\frac{a}{(1-a^2)^2}}-\sqrt[3]{a^6-a^4}+\sqrt[3]{\frac{(1+a)a^4}{(1-a)^2}}$.

Solve the following equations:

366. (1) $3\sqrt{2x}-4\sqrt{2x}=24-3\sqrt{2x}$;

(2) $2\sqrt{3x}-5\sqrt{27x}+2\sqrt{48x}+10=0$;

(3) $\frac{1}{2}\sqrt{4z}+\frac{1}{3}\sqrt{9z}+\frac{1}{5}\sqrt{25z}=9$;

(4) $\sqrt{\frac{x}{2}}+\sqrt{\frac{2x}{9}}+\sqrt{\frac{x}{8}}=\frac{1}{12}$.

367. (1) $2\sqrt[3]{8x-40}+\sqrt[3]{x-5}-\frac{2}{3}\sqrt[3]{27x-135}=36$;

(2) $\sqrt{25x+25}-\sqrt{9x+9}+\sqrt{4x+4}=16-\sqrt{x+1}$;

(3) $\sqrt{\frac{1}{x}+1}+\sqrt{\frac{4}{x}+4}=4-\sqrt{\frac{9}{x}+9}$.

18.2. Multiplication and Division of Radicals Having the Same Indices

Multiply:

368. $\sqrt{5}\times\sqrt{20}$, $\sqrt[4]{2}\times\sqrt[4]{8}$, $\sqrt[3]{4}\times2\sqrt[3]{2}$, $\sqrt{\frac{2}{5}}\times\sqrt{\frac{8}{5}}$,

$3\sqrt[3]{36}\times\sqrt[3]{-6}$, $\sqrt[3]{\frac{1}{9}}\times\sqrt[3]{\frac{8}{3}}$.

369. $2\sqrt[4]{3a}\times5\sqrt[4]{27a^3}$, $\sqrt{ab^3}\times\sqrt{a^3b}$,

$\sqrt{a+1}\times\sqrt{(a+1)^3}$, $\sqrt{\frac{a}{b}}\times\sqrt{\frac{b}{a}}$, $\sqrt[3]{a-b}\times\sqrt[3]{b-a}$,

$\sqrt[3]{-m-n}\times\sqrt[3]{(m+n)^2}$.

370. (1) $\frac{1}{3}\sqrt[3]{18}\times2\sqrt[3]{3}$; (2) $\frac{3}{4}\sqrt[4]{\frac{25a^2}{4}}\times\frac{4}{5}\sqrt{\frac{0.4}{a}}$;

(3) $5\sqrt[6]{32}\times0.2\sqrt[6]{128}$; (4) $a^2\sqrt[3]{5a}\times a\sqrt[3]{25a^2}$;

(5) $\sqrt{0.2}\times\sqrt{96}\times10\sqrt{27}\times\sqrt{\frac{1}{3}}$;

(6) $\sqrt[3]{-1080}\times\sqrt[3]{50}\times\sqrt[3]{40}$.

371. (1) $\sqrt[4]{\left(1-\frac{1}{2}\right)}\times\sqrt[4]{\left(2-\frac{1}{2}\right)^2}\times\sqrt[4]{\left(5-\frac{1}{2}\right)}$;

(2) $\sqrt{\frac{1}{2}-\frac{1}{3}}\times\sqrt{\frac{1}{3}-\frac{1}{4}}\times\sqrt{\frac{1}{4}-\frac{1}{5}}\times\sqrt{\frac{1}{5}-\frac{1}{6}}\times\sqrt{3}$;

(3) $\frac{1}{2}\sqrt{\frac{5xy^5}{4}}\times1.2\sqrt{0.5xy}\times\sqrt{5x^5y^5}\times4x\sqrt{2x^3y}$;

(4) $\sqrt[n]{\frac{x^{n-3}}{y^{n-4}}}\times\sqrt[n]{\frac{x^2}{y^3}}\times\sqrt[n]{\frac{x^3}{y^2}}$.

372. (1) $(2\sqrt{12}-\sqrt{75})\sqrt{3}$;

(2) $(5\sqrt{0.02}+\sqrt{8})\sqrt{2}$;

(3) $\left(3\sqrt{\frac{2}{3}}-6\sqrt{\frac{3}{8}}+4\sqrt{\frac{3}{2}}\right)\sqrt{\frac{2}{3}}$;

(4) $\left(15\sqrt[3]{36}-6\sqrt[3]{\frac{9}{4}}+18\sqrt[3]{\frac{16}{81}}\right)\frac{4}{3}\sqrt[3]{\frac{4}{9}}$.

373. (1) $(14\sqrt{ab^3}+21\sqrt{a^3b}+42\sqrt{a^5b^5})\frac{1}{7}\sqrt{ab}$;

(2) $\left(\frac{1}{3}\sqrt{mn}+\frac{1}{12}\sqrt{\frac{m^3}{n}}-\frac{1}{18}\sqrt{\frac{n^5}{m}}\right)(-3\sqrt{mn})$;

(3) $\left(\frac{1}{y^2x}\sqrt[3]{\frac{x^2}{y}}+\frac{1}{x^2y}\sqrt[3]{\frac{y^2}{x}}-2\sqrt[3]{\frac{1}{x^4y^4}}\right)\sqrt[3]{x^2y^2}$.

374. (1) $(4+\sqrt{6})(5\sqrt{2}-2\sqrt{3})$;

(2) $(6\sqrt{3x}-3\sqrt{5})(6\sqrt{3x}+3\sqrt{5})$;

(3) $(5+\sqrt{15})(5-\sqrt{15})$;

(4) $(\sqrt[3]{a^2b}-\sqrt[3]{ab^2})(\sqrt[3]{a}+\sqrt[3]{b})$.

375. (1) $(4\sqrt{0.2}-\sqrt{2}-\sqrt{1.2})\left(\frac{3}{4}\sqrt{3}+\sqrt{2}+\sqrt{3\frac{1}{3}}\right)$;

(2) $(\sqrt[3]{16}+4\sqrt[3]{2}+4\sqrt[3]{54})\left(3\sqrt[3]{\frac{1}{2}}+5\sqrt[3]{4}\right)$;

(3) $\left(2\sqrt[3]{\frac{3}{8}}-5\sqrt[3]{\frac{4}{3}}-4\sqrt[3]{\frac{2}{9}}\right)\left(\sqrt[3]{36}+\sqrt[3]{72}-6\sqrt[3]{\frac{9}{4}}\right)$.

376. (1) $(\sqrt{7}+\sqrt{3})(\sqrt{7}-\sqrt{3})$;

(2) $(\sqrt{x}+\sqrt{x+1})(\sqrt{x}-\sqrt{x+1})$;

(3) $(\sqrt[3]{25}+\sqrt[3]{16})(\sqrt[3]{25}-\sqrt[3]{16})$;

(4) $(\sqrt[4]{10}-\sqrt[4]{5})(\sqrt[4]{10}+\sqrt[4]{5})$.

377. (1) $\sqrt[3]{4+2\sqrt{2}}\times\sqrt[3]{4-2\sqrt{2}}$;

(2) $\sqrt[5]{17+\sqrt{46}}\times\sqrt[5]{17-\sqrt{46}}$;

(3) $(\sqrt{9}-\sqrt{15}+\sqrt{25})(\sqrt{3}+\sqrt{5})$;

(4) $(\sqrt{11}-\sqrt{3})(11+\sqrt{33}+3)$;

(5) $(\sqrt[3]{9}-\sqrt[3]{6}+\sqrt[3]{4})(\sqrt[3]{3}+\sqrt[3]{2})$;

(6) $(\sqrt[3]{25}+\sqrt[3]{10}+\sqrt[3]{4})(\sqrt[3]{5}-\sqrt[3]{2})$.

378. (1) $(\sqrt[3]{a^2}+\sqrt[3]{ab}+\sqrt[3]{b^2})(\sqrt[3]{a}-\sqrt[3]{b})$;

(2) $(\sqrt[3]{x^2}+\sqrt[3]{y^2})(\sqrt[3]{x^4}-\sqrt[3]{x^2y^2}+\sqrt[3]{y^4})$;

(3) $(x^2-x\sqrt{a}+a)(x+\sqrt{a})$;

(4) $(a-\sqrt{a}\sqrt[4]{b}+\sqrt{b})(\sqrt{a}+\sqrt[4]{b})$.

379. Transform the following expressions into products using the formulas for short-cut multiplication:

(1) $\sqrt{a^3}-\sqrt{b^3}$; (2) $\sqrt[3]{x^2}-1$; (3) $\sqrt{m}-\sqrt[4]{n}$;

(4) $a+b$; (5) a^2+b^2; (6) a^2+ab+b^2.

380. Simplify:

(1) $\dfrac{1}{x-\sqrt{x^2-1}}-\dfrac{1}{x+\sqrt{x^2-1}}$;

(2) $\dfrac{m\sqrt{m^2+a}+m^2+a}{m+\sqrt{m^2+a}}$;

(3) $\dfrac{\left(\dfrac{m}{\sqrt{m^2+1}}-1\right)(\sqrt{m^2+1}+m)+(m-\sqrt{m^2+1})\left(\dfrac{m}{\sqrt{m^2+1}}+1\right)}{(\sqrt{m^2+1}+m)(\sqrt{m^2+1}-m)}$.

381. Prove the following identities:

(1) $\left(\sqrt{ab}-\sqrt{\dfrac{b}{a}}-\sqrt{\dfrac{a}{b}}+\sqrt{\dfrac{1}{ab}}\right)\sqrt{ab}=(a-1)(b-1)$;

(2) $(\sqrt[4]{a}-\sqrt[4]{b})(\sqrt[4]{a}+\sqrt[4]{b})(\sqrt{a}+\sqrt{b})=a-b$.

382. Simplify:

(1) $\sqrt[3]{\dfrac{3x^{-2}y^5}{5x^4y^{-2}}}\sqrt[3]{\left(\dfrac{6x^{-2}}{5y^3}\right)^{-2}}\sqrt[3]{-120x^5y^2}$;

(2) $(\sqrt[3m]{2^n}-\sqrt[3n]{3^m})(\sqrt[3m]{2^{2n}}+\sqrt[3mn]{2^{n^2}3^{m^2}}+\sqrt[3n]{3^{2m}})$.

383. Solve the following equations:

(1) $(6+\sqrt{x})(5-\sqrt{x}) = 28-x$;

(2) $(2+3\sqrt{2x})(3-2\sqrt{2x}) = 5\sqrt{2x}-18$;

(3) $\dfrac{5\sqrt[3]{4x}+2}{7\sqrt[3]{4x}+20} = -\dfrac{4}{3}$; (4) $\dfrac{\sqrt[5]{2x-3}-1}{5+2\sqrt[5]{2x-3}} = \dfrac{2}{11}$.

384. 1. Pick out the pairs of reciprocal numbers:

(a) $\dfrac{\sqrt{6}}{3}$ and $\dfrac{\sqrt{6}}{2}$; (b) $6-\sqrt{3}$ and $6+\sqrt{3}$;

(c) $2+\sqrt{3}$ and $2-\sqrt{3}$; (d) $6+\sqrt{3}$ and $\dfrac{1}{33}(6-\sqrt{3})$;

(e) $\sqrt{7-4\sqrt{3}}$ and $\sqrt{7+4\sqrt{3}}$.

2. Find the mistake:

$$\sqrt{-9}\times\sqrt{-25} = \sqrt{(-9)(-25)} = \sqrt{225} = 15.$$

3. Are there any mistakes in the following transformations?

$$-3\sqrt[3]{2} = \sqrt[3]{(-3)^3}\times\sqrt[3]{2} = \sqrt[3]{-27}\times\sqrt[3]{2} = \sqrt[3]{-54}.$$

Carry out the operation of division:

385. (1) $15\sqrt{2} : 5$; (2) $2\sqrt[4]{5} : 0.5$; (3) $\sqrt[5]{3} : 2$;

(4) $16\sqrt{3} : 4\sqrt{3}$; (5) $\sqrt{a} : 3\sqrt{a}$; (6) $\sqrt[3]{16} : \sqrt[3]{2}$;

(7) $\sqrt[4]{0.01} : \sqrt[4]{100}$; (8) $\sqrt[3]{-0.1} : \sqrt[3]{-0.008}$.

386. (1) $\sqrt{\dfrac{1}{12}} : \sqrt{\dfrac{1}{9}}$; (2) $\sqrt{10} : \dfrac{1}{2}\sqrt{2.5}$;

(3) $\dfrac{5}{2}\sqrt[3]{192} : 5\sqrt[3]{\dfrac{3}{2}}$; (4) $2\sqrt[4]{\dfrac{4}{25}} : \dfrac{1}{3}\sqrt[4]{\dfrac{64}{125}}$;

(5) $1.25\sqrt[3]{3} : 0.25\sqrt[3]{\dfrac{8}{9}}$; (6) $10\sqrt[4]{0.32} : 2\sqrt[4]{200}$.

387. (1) $3\sqrt{ab^3} : \sqrt{a^3b^2}$; (2) $\sqrt[3]{-\dfrac{2}{49}a^5b^2} : \sqrt[3]{7a^2b^5}$;

(3) $3.6\sqrt{ab} : 9\sqrt{\dfrac{1}{ab}}$; (4) $\dfrac{x}{a}\sqrt[3]{\dfrac{a^2}{x}} : \dfrac{1}{2}a\sqrt[3]{\dfrac{8a}{x^4}}$;

(5) $\frac{1}{2}\sqrt{\frac{8a}{3b^2}} : \frac{4}{3}\sqrt{\frac{3b^2}{2a^3}}$;

(6) $\frac{m}{n}\sqrt[5]{\frac{m^3}{n^2}} : \frac{m^2}{n^3}\sqrt[5]{\frac{n^2}{m^7}}$.

388. Compute to within 0.1:

(1) $\sqrt{10} : \sqrt{4}$; (2) $5\sqrt{0.05} : \sqrt{10}$;

(3) $4\sqrt[3]{12} : \frac{1}{2}\sqrt[3]{3}$; (4) $\sqrt[3]{0.54} : \sqrt[3]{\frac{20}{3}}$;

(5) $\frac{\sqrt{18}}{(\sqrt{7}+2)(\sqrt{7}-2)}$; (6) $\frac{80}{(\sqrt[3]{5}-1)(1+\sqrt[3]{5}+\sqrt[3]{25})}$.

389. (1) $(18\sqrt{2}+12\sqrt{3}) : 0.3$;

(2) $(12\sqrt{45}-6\sqrt{20}) : 3\sqrt{5}$;

(3) $\left(\sqrt[3]{\frac{1}{4}}-\sqrt[3]{2}\right) : \sqrt[3]{2}$;

(4) $(10\sqrt[3]{0.08}-\sqrt[3]{-10}) : \sqrt[3]{10}$.

390. (1) $\left(\sqrt{ab}-\sqrt{\frac{a}{b}}\right) : \sqrt{\frac{a}{b}}$;

(2) $\left(9\sqrt{x^3y^3}+\sqrt{xy}-\frac{1}{4}\sqrt{\frac{1}{xy}}\right) : \sqrt{xy}$;

(3) $\left(4\sqrt[4]{x^3y}+3\sqrt[4]{\frac{1}{2}xy^3}-\sqrt[4]{\frac{1}{x}}\right) : \frac{1}{xy}\sqrt[4]{x^3y^2}$.

391. (1) $(p^2\sqrt{p}+p^3-p\sqrt{q}-\sqrt{pq}) : (p+\sqrt{p})$;

(2) $(2xy+3x\sqrt{x}-2y\sqrt{y}-3\sqrt{xy}) : (x-\sqrt{y})$;

(3) $(2a\sqrt{ab}+a^2\sqrt{b}+2b\sqrt{ab}+ab\sqrt{b}) : (2\sqrt{ab}+a\sqrt{b})$.

392. (1) $(x-y) : (\sqrt{x}+\sqrt{y})$;

(2) $(4\sqrt[5]{x^2}-9\sqrt[5]{y^2}) : (2\sqrt[5]{x}-3\sqrt[5]{y})$;

(3) $(x\sqrt{x}+y\sqrt{y}) : (\sqrt{x}+\sqrt{y})$;

(4) $(x\sqrt{8x}-y\sqrt{27y}) : (\sqrt{2x}-\sqrt{3y})$.

393. (1) $\left(\sqrt[3]{4a^3}-3\sqrt[3]{4b^3}+\sqrt[3]{a^2b}\right):\left(\sqrt[3]{2b}-\sqrt[3]{a}\right)$;

(2) $\left(2a\sqrt[3]{9ab^2}+3ab-3b\sqrt[3]{3a^2b}\right):\left(\sqrt[3]{3a^2b}+\sqrt[3]{9ab^2}\right)$.

394. Factorize:

(1) $\sqrt{15}-\sqrt{21}$; (2) $3-\sqrt[3]{3}$; (3) $\sqrt[3]{a^2x}+\sqrt[3]{b^2x}$;

(4) $\sqrt[3]{6a^2b}-\sqrt[3]{9ab^2}$; (5) $\sqrt[4]{mn}-\sqrt{m}$; (6) $a-b-\sqrt{a-b}$;

(7) $\sqrt{a^2-b^2}-\sqrt{a+b}$; (8) $\sqrt{ab+ac}-\sqrt{b^2+bc}$;

(9) $\sqrt{m^3+n^3}-\sqrt{m^2-mn+n^2}$; (10) $\sqrt{a^3}-6\sqrt{b^3}+3\sqrt{a^2b}-2\sqrt{ab^2}$;

(11) $x-4\sqrt{x}+3$; (12) $a+7\sqrt{a}+10$.

395. Factorize:

(1) $a-b$; (2) $a-4$; (3) $2-a^2$;

(4) $ab-2$; (5) $a-b^2c$; (6) a^3b-5.

396. Reduce the following fractions:

(1) $\dfrac{\sqrt{20}+\sqrt{14}-\sqrt{6}}{\sqrt{50}+\sqrt{35}-\sqrt{15}}$; (2) $\dfrac{2\sqrt{2mn}-m\sqrt{2}}{2n-\sqrt{mn}}$;

(3) $\dfrac{x\sqrt[3]{y}+\sqrt[3]{x^2y^2}}{\sqrt[3]{x^2y^2}+y\sqrt[3]{x}}$.

18.3. Powers with Fractional Exponents. Multiplying and Dividing Roots of Different Indices

Replace the given radicals by powers with fractional exponents:

397. (Orally). $\sqrt{3}$, $\sqrt[3]{a^2}$, $\sqrt[n]{b}$, $\sqrt[4]{a^2b}$, $\sqrt[3]{a^3-b^3}$, $\sqrt[4]{b^{-5}}$, $\sqrt[m]{c^{-n}}$, $\sqrt[5]{(m+n)^{-3}}$.

398. (1) $\sqrt[m]{\dfrac{x^2}{y^m}}$; (2) $\sqrt[mn]{\dfrac{1}{a^{pn}}}$; (3) $\sqrt[3]{\dfrac{1}{2}}$;

(4) $\dfrac{1}{\sqrt{a}}$; (5) $\dfrac{2}{\sqrt[3]{x-y}}$; (6) $\sqrt[5]{4a^2+b^2+4ab}$.

Replace the powers with fractional exponents by radicals:

399. (1) $a^{\frac{1}{3}}$, $b^{\frac{3}{4}}$, $a^{\frac{1}{n}}$, $x^{\frac{n}{3}}$, $(2a)^{\frac{3}{4}}$, $x^{0.2}$, $a^{-0.25}$;

(2) $m^{-1.5}$, $(-x)^{-\frac{1}{3}}$, $\left(\frac{1}{4}x^2y\right)^{-\frac{1}{2}}$, $(x-y)^{\frac{m}{k}}$,

$-(m+n)^{-\frac{2}{5}}$.

400. (1) $(4a)^{\frac{3}{2}}$, $\left(\frac{1}{8}x^3y\right)^{\frac{2}{3}}$, $5a^{\frac{2}{3}}(a+b)^{\frac{1}{2}}$, $x^{\frac{1}{3}}+y^{\frac{1}{3}}$;

(2) $(a+b)^{-\frac{m}{n}}$, $\left(a^{\frac{1}{3}}-b^{\frac{1}{3}}\right)^{\frac{1}{n}}$,

$\left(m^{\frac{1}{2}}+n^{\frac{1}{2}}\right)^{-\frac{1}{2}}$, $\frac{1}{(a+b)^{-0.8}}$.

401. Calculate orally:

(1) $16^{\frac{1}{2}}$, $8^{\frac{2}{3}}$, $16^{\frac{5}{4}}$, $64^{-\frac{1}{2}}$, $2^{-2}\times16^{\frac{1}{2}}$;

(2) $(-64)^{\frac{2}{3}}$, $\left(2\frac{1}{4}\right)^{\frac{1}{2}}$, $25^{-\frac{1}{2}}$, $\left(\frac{1}{4}\right)^{-\frac{1}{2}}$, $(0.64)^{-\frac{1}{2}}$;

(3) $\left(\frac{1}{27}\right)^{-3}$, $\left(4^{\frac{1}{4}}\right)^2$, $\left(3^{\frac{4}{3}}\right)^{\frac{3}{2}}$, $2^{-3}\times64^{\frac{1}{2}}$.

Compute:

402. (1) $8^{\frac{1}{3}}:2^{-1}$, $4^{-1}\times8^{\frac{2}{3}}$, $3^{-2}\times81^{\frac{1}{4}}$, $\left(3\frac{3}{8}\right)^{-\frac{2}{3}}$;

(2) $64^{-\frac{2}{3}}\times\left(\frac{4}{9}\right)^{-\frac{1}{2}}$, $\left(125^{\frac{2}{3}}-16^{\frac{1}{2}}+343^{\frac{1}{3}}-3\right)^{\frac{1}{2}}$;

(3) $16^{0.5}-\left(\frac{1}{16}\right)^{-0.75}+\left(\frac{1}{2}\right)^{-4}$, $5^{\frac{3}{5}}\times125\times25^{-0.3}\times5^{-2.5}$.

403. (1) $\left(\frac{1}{16}\right)^{-0.75}+(-0.027)^{\frac{4}{3}}+\left(-1\frac{61}{64}\right)^{-\frac{2}{3}}$;

(2) $\left(3^{\frac{1}{2}}-5^{\frac{1}{2}}\right)^2:\left[\left(2-15^{\frac{1}{4}}\right)\left(2+15^{\frac{1}{4}}\right)\right]$;

(3) $\left(16^{-\frac{1}{4}}-3^{-\frac{1}{2}}\right)\left[16^{-0.25}+\left(3\sqrt{3}\right)^{-\frac{1}{3}}\right]$;

(4) $\left(81^{-\frac{1}{4}}+2^{-1}\right)\left(81^{-0.5}-\frac{1}{2}\times9^{-0.5}+2^{-2}\right)$.

404. (1) $\left[\sqrt[1.5]{10^{-4.5}}\times\left(\frac{1}{81}\right)^{-\frac{1}{2}}+10{,}000^{-\frac{3}{4}}\right]^{\frac{1}{2}}$;

(2) $\left[\left(\frac{3}{4}\right)^{-0.5}\right]^{0}+7.5\left(\sqrt[-\frac{4}{3}]{4}\right)^{2}+(-2)^{-4}-81^{0.25}$;

(3) $0.008^{-\frac{1}{3}}-\left(\frac{1}{6}\right)^{-2}+256^{\frac{3}{8}}\quad 2^{-1}+6.5^{0}$;

(4) $\left[\left(5^{\frac{1}{3}}-2^{\frac{1}{3}}\right)^{-1}\times 3-\frac{7}{5^{\frac{1}{3}}+2^{\frac{1}{3}}}\right]\times 2\times 10^{-\frac{1}{3}}$.

405. 1. How is the expression $5^{\sqrt{2}}$ understood?

2. Compare the values of the following pairs of numbers: $2^{\sqrt{2}}$ and $2^{\sqrt{3}}$; 3^{2} and $3^{\sqrt{3}}$.

Carry out the indicated operations:

406. (1) $a^{\frac{1}{2}}\times a^{\frac{1}{3}}$; (2) $a^{\frac{1}{3}}\times a^{-\frac{1}{3}}$;

(3) $x^{\frac{2}{3}}:x^{-\frac{1}{3}}$; (4) $a:a^{\frac{2}{3}}$;

(5) $(a^{2}b^{4})^{\frac{3}{2}}$; (6) $x^{\frac{4}{3}}:\sqrt[3]{x}$.

407. (1) $\left(2a^{-\frac{1}{2}}\sqrt{x^{-\frac{1}{3}}a\sqrt[4]{x^{3}}}\right)^{8}$;

(2) $\left(\frac{8a^{-3}}{125b^{6}}\right)^{-\frac{1}{3}}$; (3) $\sqrt[3]{a^{-2}\sqrt{a}}$;

(4) $\left[\left(2^{\frac{1}{m-n}}\right)^{\frac{m^{2}-n^{2}}{m}}\right]^{\frac{m}{m+n}}$.

408. (1) $\left(2a^{\frac{1}{2}}+b^{\frac{1}{2}}\right)^{2}$; (2) $\left(3a^{\frac{2}{3}}-b^{-1}\right)\left(3a^{\frac{2}{3}}+b^{-1}\right)$;

(3) $\left(x^{\frac{1}{3}}+x^{\frac{2}{3}}-x^{\frac{5}{6}}\right)\left(x^{\frac{1}{3}}+x^{\frac{1}{6}}\right)$;

(4) $\left(a^{\frac{3}{2}}-a^{\frac{1}{6}}b^{\frac{1}{2}}+b^{\frac{3}{2}}\right)^{2}$;

(5) $\left(3^{\frac{3}{2}}+a^{\frac{3}{2}}\right):\left[3-(3a)^{\frac{1}{2}}+a\right]$;

(6) $\dfrac{m^{\frac{3}{2}}-n^{\frac{3}{2}}}{m^{\frac{1}{2}}-n^{\frac{1}{2}}}$.

409. Simplify the following:

(1) $\left(\frac{1}{2}x^{0.25}+x^{0.75}\right)^2-x^{1.5}(1+x^{-0.5})$;

(2) $\left(\frac{2a^{\frac{1}{3}}}{a^{\frac{4}{3}}-4a^{\frac{1}{3}}}+\frac{1}{a^{\frac{1}{2}}-4a^{-\frac{1}{2}}}\right)^{-2}-(a^2+8a+16)^{\frac{1}{2}}$;

(3) $\left(m+\frac{n^{1.5}}{m^{0.5}}\right)^{\frac{2}{3}}\left(\frac{m^{0.5}-n^{0.5}}{m^{0.5}}+\frac{n^{0.5}}{m^{0.5}-n^{0.5}}\right)^{-\frac{2}{3}}$;

(4) $\frac{1}{2}\left(\frac{5^{\frac{1}{2}}+1}{1+5^{0.5}+a^{0.5}}+\frac{5^{\frac{1}{2}}-1}{1-5^{0.5}+a^{0.5}}\right)\left(a^{\frac{1}{2}}-4a^{-\frac{1}{2}}+2\right)\times 5^{-\frac{1}{2}}$.

Carry out the indicated operations:

410. (1) $\sqrt[3]{3}\times\sqrt[6]{3}$, $\sqrt{\frac{1}{2}}\times\sqrt[3]{4}$, $\sqrt[4]{8}:\sqrt[8]{16}$, $5:\sqrt[3]{5}$;

(2) $\sqrt[6]{a^5}:\sqrt{a}$, $\sqrt{3}\times\sqrt[3]{4}\times\sqrt[4]{9}$, $\sqrt[6]{54}\times\sqrt{6}\times\sqrt[3]{2}$.

411. (1) $\sqrt{a^3}\times\sqrt[3]{a^2}\times\sqrt[4]{a}$; (2) $a^2\sqrt[3]{\frac{a^5}{b^2}}\times b\sqrt{\frac{a^3}{b}}:\sqrt[4]{\frac{b^7}{a^3}}$;

(3) $\frac{1}{2}\sqrt[3]{\frac{2x}{(y-x)^2}}\times\sqrt{\frac{y-x}{x^3}}\times\sqrt[3]{\frac{4(y-x)}{x^2}}$;

(4) $ab\sqrt[6]{81a^5b}\times\frac{1}{3}a^2\sqrt{3ab}\times b\sqrt[3]{9ab^2}$.

412. (1) $\sqrt[12]{a^{11}}:\sqrt[4]{a^3}$; (2) $\sqrt{1\frac{2}{3}}:\frac{1}{6}\sqrt[6]{3\frac{3}{8}}$;

(3) $\sqrt{a}:a^{-\frac{1}{2}}$; (4) $4\sqrt[3]{12}:\frac{1}{2}\sqrt{\frac{2}{3}}$;

(5) $15a^2b:5a\sqrt{b}$; (6) $ab^2:\sqrt[4]{ab^2}$.

413. (1) $\left(\sqrt{3}+\sqrt[3]{9}-\sqrt[6]{81}\right)\times\sqrt[4]{27}$;

(2) $\left(\sqrt{8}-2\sqrt[6]{2}+\sqrt[3]{4}\right)\times\sqrt[6]{32}$;

(3) $\left(4\sqrt{27}-6\sqrt[3]{3}\right):\sqrt{3}$;

(4) $\left(2\sqrt{18}-4\sqrt[3]{9}+6\sqrt[4]{243}\right):2\sqrt[4]{3}$.

414. (1) $\left(\sqrt[5]{a^2}+\sqrt[3]{a^4}-a\sqrt{a^3}\right):\left(-3a\sqrt[3]{a^2}\right)$;

(2) $\left(a^2\sqrt[4]{b}-a\sqrt[3]{b}\right):a\sqrt{b}$;

(3) $\left(\sqrt[8]{8a^6b^9}+ab\sqrt[6]{8a^4b^5}-ab^2\sqrt[4]{2a^4b}\right):\sqrt[4]{2a}$;

(4) $\left(\frac{x}{2}\sqrt[3]{\frac{m^2}{n^2}}+\frac{y}{2}\sqrt[6]{\frac{m^5}{n^5}}\right):\frac{xy}{4}\sqrt{\frac{m}{n}}$.

415. (1) $\left(\sqrt{m}+\frac{2}{m}\sqrt[4]{mn}\right)\left(\sqrt{m}-2\sqrt[4]{\frac{n}{m^3}}\right)$;

(2) $\left(\sqrt[3]{a^2}-\sqrt[3]{b^2}\right)\left(\sqrt{a^{\frac{4}{3}}}+\sqrt{b^{\frac{4}{3}}}\right)$;

(3) $\left(2\sqrt{ab}-\sqrt[3]{ab^2}+3\sqrt[4]{ab^3}\right)\times\left(\sqrt[12]{ab^4}-4\sqrt{ab}\right)$.

416. (1) $\left(\sqrt[4]{8a^3}-2\sqrt{2}\right):\left(\sqrt[4]{2a}-\sqrt{2}\right)$;

(2) $\left(15x-3x^{\frac{1}{3}}-2x^{-\frac{1}{3}}+8x^{-1}\right):\left(5x^{\frac{2}{3}}+4\right)$;

(3) $\left(a\sqrt{2ab}-ab\sqrt{ab}\right):\left(\sqrt[6]{2a^3b}-\sqrt{ab}\right)$.

417. (1) $\left(m\sqrt[4]{18m^3n^5}+3m^2n\right):\left(\sqrt[4]{2mn^3}+\sqrt{3mn}\right)$;

(2) $\left(2x\sqrt[6]{xy^5}+y\sqrt[6]{x^5y}-3xy\right):\left(\sqrt{xy}-2\sqrt[3]{x^2y}\right)$;

(3) $\left(3x\sqrt[3]{xy^2}-2\sqrt[12]{x^{11}y^{10}}-y\sqrt{x}\right):\left(3\sqrt[3]{x^2y}+\sqrt[4]{xy^2}\right)$.

18.4. Raising Radicals to a Power and Extracting Roots of Radicals

418. Raise the given numbers to the indicated powers (orally): $\left(\sqrt[n]{m}\right)^n$, $\left(4\sqrt{3}\right)^2$, $\left(\sqrt[6]{3}\right)^{12}$, $\left(\sqrt[4]{2}\right)^6$, $\left(\sqrt[4]{a^3}\right)^3$, $\left(-\sqrt[3]{b^2}\right)^6$.

419. 1. Which number is greater: (a) $\sqrt{2}$ or $\sqrt[6]{6}$, (b) $\sqrt[4]{10}$ or $\sqrt[6]{32}$?

2. What sign the differences $\sqrt[4]{63}-2\sqrt{2}$, $\sqrt[6]{6}-\sqrt[8]{8}$ have?

Perform the indicated operations:

420. (1) $\left(m\sqrt[3]{m^2n}\right)^3$, $\left(\frac{1}{a}\sqrt[4]{ab}\right)^8$, $\left(-a\sqrt[5]{a^3b^2}\right)^2$;

(2) $\left(2ab^2\sqrt[3]{\frac{1}{a^2b^4}}\right)^6$, $\left[\frac{m+n}{m-n}\sqrt[3]{\left(\frac{m-n}{m+n}\right)^2}\right]^4$;

(3) $\left[\frac{(a+b)^{-2}}{a}\sqrt[4]{\frac{(a+b)^3}{a^{-1}}}\right]^4$, $\left(\sqrt[n]{x}\right)^{n+2}$, $\left(\sqrt[n+1]{a^n}\right)^3$;

(4) $\left(-0.1\sqrt[6]{2a^4b^3}\right)^3$, $\left(a^{-2}b\sqrt{4a^{-2}b^{2n}}\right)^{-2}$;

(5) $\left(\frac{2}{3a}\sqrt[3]{-\frac{3}{2}a}\right)^6$, $\left(\frac{a^{-1.5}}{\sqrt{(a-b)^{-1}}}\right)^2$.

421. (1) $(3+\sqrt{3})^2$, $(4-\sqrt{5})^2$, $(2\sqrt{2}-3\sqrt{3})^2$;

(2) $(2\sqrt{a}-3\sqrt{b})^2$, $(\sqrt{3}+\sqrt{5}+\sqrt{7})^2$, $(\sqrt{2}-\sqrt{3}+\sqrt{5})^2$;

(3) $(\sqrt[3]{a}-2\sqrt[3]{b})^2$, $\left(3\sqrt[3]{\frac{x^2}{y}}+\frac{1}{2}\sqrt[3]{\frac{x}{y^2}}\right)^2$.

422. (1) $(2^{\frac{1}{2}}-4^{\frac{1}{3}})^2$, $(m^{-\frac{1}{2}}+n^{-\frac{1}{2}})^2$, $(a^{\frac{3}{2}}+b^{\frac{3}{2}})^2$;

(2) $(\sqrt[3]{6}+\sqrt[3]{-4})^2$, $(\sqrt[3]{a}+\sqrt[3]{b})^3$, $(\sqrt[3]{a^2}-\sqrt[3]{a})^3$;

(3) $(\sqrt[3]{5}+2\sqrt{2})^2$, $(1-3\sqrt{2})^3$, $\left(a\sqrt{a}+\sqrt{\frac{a}{2}-\sqrt{2a}}\right)^2$.

423. 1. Simplify:

$$(\sqrt[5]{\sqrt{18}+\sqrt{2}})^2,\quad (\sqrt[3]{\sqrt{75}-\sqrt{12}})^{-2}.$$

2. Show that $\sqrt{3+\sqrt{5}}=\frac{\sqrt{5}+1}{\sqrt{2}}$.

424. (1) $\left(\sqrt{2+\sqrt{3}}+\sqrt{2-\sqrt{3}}\right)^2$;

(2) $\left(\sqrt{3+\sqrt{5}}-\sqrt{3-\sqrt{5}}\right)^2$;

(3) $\left(\sqrt{a+\sqrt{b}}+\sqrt{a-\sqrt{b}}\right)^2$;

(4) $\left(\sqrt{a+\sqrt{b}}-\sqrt{a-\sqrt{b}}\right)^2$;

(5) $\left(\sqrt{12+2\sqrt{6}}+\sqrt{12-2\sqrt{6}}\right)^2$;

(6) $\left(\sqrt{4-\sqrt{7}}-\sqrt{4+\sqrt{7}}\right)^2$.

425. 1. Prove that $\sqrt{10+\sqrt{24}+\sqrt{40}+\sqrt{60}}=\sqrt{2}+\sqrt{3}+\sqrt{5}$.

2*. Without extracting the roots, determine which is greater:

$$\sqrt{11}-\sqrt{5}\ \text{ or }\ \sqrt{19}-\sqrt{11}.$$

426*. 1. The Bertrand[1] problem. Prove that

$$\sqrt[3]{20+14\sqrt{2}}+\sqrt[3]{20-14\sqrt{2}}=4.$$

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[1] J. L. Bertrand (1822-1900), French mathematician.

2. The second Bertrand problem. Prove that

$$\sqrt[3]{6+\sqrt{\frac{847}{27}}}+\sqrt[3]{6-\sqrt{\frac{847}{27}}}=3.$$

427. Solve the following equations:

(1) $(5\sqrt{x}-2)^2-(3\sqrt{x}+1)^2=(4\sqrt{x}-2)^2-4$;

(2) $(5\sqrt{3x}-2)^2+(12\sqrt{3x}-25)^2=(13\sqrt{3x}-21)^2-34$.

428*. 1. Without using the formula for a complex radical transform the radical $\sqrt{9+\sqrt{56}}$ into a sum of two radicals.

2. Derive the formula $\sqrt{a\pm\sqrt{b}}=\sqrt{\frac{a+\sqrt{a^2-b}}{2}}\pm\sqrt{\frac{a-\sqrt{a^2-b}}{2}}$.

429. Simplify the following complex radicals:

(1) $\sqrt{7+\sqrt{24}}$; (2) $\sqrt{7-\sqrt{24}}$; (3) $\sqrt{2-\sqrt{3}}$;

(4) $\sqrt{5+\sqrt{24}}$; (5) $\sqrt{5-\sqrt{24}}$; (6) $\sqrt{(m+n)-2\sqrt{mn}}$;

(7) $\sqrt{4x+4\sqrt{xy}+y}$.

430. Extract the roots:

(1) $\sqrt[4]{4096}$, $\sqrt[8]{6561}$, $\sqrt[10]{1024}$, $\sqrt[12]{531{,}441}$;

(2) $\sqrt{\sqrt{2}}$, $\sqrt[3]{2\sqrt{6}}$, $\sqrt{5\sqrt[4]{125}}$, $\sqrt{\frac{1}{2}\sqrt[3]{4}}$;

(3) $\sqrt{\sqrt[3]{x^2}}$, $\sqrt[5]{\sqrt[3]{a^5}}$, $\sqrt[4]{\sqrt[3]{a^4b^8}}$, $\sqrt{\sqrt[5]{9a^4b^2}}$;

(4) $\sqrt[4]{x^2\sqrt[3]{x}}$, $\sqrt[4]{\frac{x}{y^2}\sqrt{\frac{y^2}{x}}}$, $\sqrt[3]{\left(\frac{a-1}{a+1}\right)^2\times\sqrt[4]{\left(\frac{a+1}{a-1}\right)^3}}$.

431. Extract the roots:

(1) $\sqrt[4]{\left(1+\frac{1}{x}\right)\sqrt{\left(\frac{x}{1+x}\right)^3}}$, $\sqrt[3]{-\sqrt[5]{-a^{30}}}$, $\sqrt[3]{-2x^2y\sqrt[4]{5x^3}}$;

(2) $\sqrt{x\sqrt{x\sqrt{x}}}$, $\sqrt[4]{2\sqrt[3]{2\sqrt{2}}}$, $\sqrt[3]{x^2\sqrt[3]{x^2\sqrt[3]{x^2}}}$;

(3) $\sqrt{\frac{a}{b}\sqrt{\frac{b}{a}\sqrt{\frac{a}{b}}}}$, $\sqrt[3]{\frac{2}{3}\sqrt[3]{\frac{3}{2}\sqrt{\frac{2}{3}}}}$;

(4) $\sqrt[3]{\frac{2}{3}\sqrt{\frac{9}{2}\sqrt{\frac{1}{4}}}}$, $\sqrt[3]{ab\sqrt{\frac{1}{ab}\sqrt[4]{a^3b^3}}}$.

432*. Simplify the following expressions:

(1) $\sqrt{4\sqrt{2}+2\sqrt{6}}$; (2) $\sqrt[4]{17+\sqrt{288}}$;

(3) $\sqrt[4]{28-16\sqrt{3}}$; (4) $\sqrt{17-4\sqrt{9+4\sqrt{5}}}$;

(5) $2\sqrt{3+\sqrt{5-\sqrt{13+\sqrt{48}}}}$;

(6) $\sqrt[3]{7+5\sqrt{2}}-\sqrt[3]{7-5\sqrt{2}}$;

(7) $\sqrt[3]{6\sqrt{3}-10}-\sqrt[3]{6\sqrt{3}+10}$.

433. 1. Extract the square root of the given binomial presenting the radicand in the form of the squared binomial:

(a) $\sqrt{3-2\sqrt{2}}$; (b) $\sqrt{7+4\sqrt{3}}$; (c) $\sqrt{10-2\sqrt{21}}$;

(d) $\sqrt{7+2\sqrt{10}}$; (e) $\sqrt{13-7.5\sqrt{3}}$.

2. Show the validity of the equality

$$\sqrt{2x^2-2\sqrt{x^4-16}}=\sqrt{x^2+4}-\sqrt{x^2-4}.$$

For what values of x does this equality hold true?

434*. 1. Prove the identity

$$\sqrt{2}\,(2x+\sqrt{x^2-y^2})\sqrt{x-\sqrt{x^2-y^2}}=\sqrt{(x+y)^3}-\sqrt{(x-y)^3}.$$

2. Simplify the expression $\sqrt{a+\sqrt{x}}+\sqrt{a-\sqrt{x}}$, where $x=4(a-1)$.

19. RATIONALIZING THE DENOMINATOR OR NUMERATOR OF A FRACTION. PERFORMING ALL OPERATIONS WITH FRACTIONS CONTAINING RADICALS

Rationalize the denominators of the following fractions:

435. (1) (Orally). $\frac{2}{\sqrt{2}}$, $\frac{6}{\sqrt{3}}$, $\frac{3}{\sqrt[3]{9}}$, $\frac{4}{\sqrt[4]{8}}$, $\frac{1}{\sqrt{x-y}}$,

$\frac{a+b}{\sqrt{a+b}}$, $\frac{3mn}{\sqrt{mn}}$, $\frac{2-x}{\sqrt[3]{(2-x)^2}}$;

(2) $\frac{3}{2\sqrt{6}}$, $\frac{-4}{\sqrt{12}}$, $\frac{6}{\sqrt[3]{32}}$, $\frac{9-x^2}{\sqrt[4]{(3-x)^3}}$, $\frac{1}{\sqrt[n]{x^{n-2}}}$;

(3) $\frac{m}{\sqrt[n]{m}}$, $\frac{2t}{\sqrt[n+1]{t^{n-1}}}$, $\frac{\sqrt{x-1}}{\sqrt{x+1}}$, $\frac{\sqrt[5]{a^2}}{\sqrt[10]{a^7}}$;

(4) $\frac{m\sqrt{n}}{n\sqrt[4]{mn}}$, $\frac{7}{\sqrt[5]{7^3}}$, $\frac{a+b}{\sqrt[n]{(a+b)^{n-1}}}$;

(5) $\frac{x+3}{\sqrt{x^2-9}}$, $\frac{a-b}{2\sqrt{a^2-b^2}}$, $\frac{\sqrt{a^n-1}}{\sqrt{a^n+1}}$.

436. (1) $\frac{1}{1+\sqrt{2}}$; (2) $\frac{1}{1-\sqrt{2}}$; (3) $\frac{\sqrt{3}}{\sqrt{3}-5}$;

(4) $\frac{a}{\sqrt{a}-a}$; (5) $\frac{m}{\sqrt{m}+1}$; (6) $\frac{x-1}{\sqrt{x}-1}$;

(7) $\frac{4\sqrt{5}}{\sqrt{5}-\sqrt{3}}$; (8) $\frac{a-b}{\sqrt{a}+\sqrt{b}}$;

(9) $\frac{m^2-n^2}{\sqrt{m}-\sqrt{n}}$; (10) $\frac{3\sqrt{3}-2\sqrt{2}}{3\sqrt{3}+2\sqrt{2}}$;

(11) $\frac{m\sqrt{m}+n\sqrt{n}}{m\sqrt{m}-n\sqrt{n}}$; (12) $\frac{8\sqrt{3}-3\sqrt{5}}{9\sqrt{3}-4\sqrt{5}}$.

437. (1) $\frac{1}{\sqrt[3]{2\sqrt{6}-5}}$; (2) $\frac{\sqrt{4\sqrt{5}-2\sqrt{11}}}{\sqrt{4\sqrt{5}+2\sqrt{11}}}$;

(3) $\frac{n}{\sqrt{m+\sqrt{m^2-n^2}}}$; (4) $\frac{\sqrt{x-3}-\sqrt{x+3}}{\sqrt{x-3}+\sqrt{x+3}}$;

(5) $\frac{\sqrt{1-a^n}+\sqrt{1+a^n}}{\sqrt{1-a^n}-\sqrt{1+a^n}}$; (6) $\frac{n(m+n+\sqrt{m^2-n^2})}{m+n-\sqrt{m^2-n^2}}$.

438. (1) $\frac{1}{(2-\sqrt{2})^3}$; (2) $\frac{a-b}{(\sqrt{a}-\sqrt{b})^2}$; (3) $\frac{2}{(\sqrt{2}-1)^2}$;

(4) $\frac{81}{(\sqrt{6}-\sqrt{3})^4}$; (5) $\frac{2}{(\sqrt{7}+\sqrt{6})^4}$;

(6) $\frac{1}{(2+\sqrt{3})^5}$; (7) $\frac{13}{2+\sqrt[4]{3}}$; (8) $\frac{1}{\sqrt[4]{3}-\sqrt{2}}$.

439. (1) $\frac{11}{2+\sqrt{3}+\sqrt{5}}$; (2) $\frac{6}{3+\sqrt{2}-\sqrt{3}}$;

(3) $\frac{\sqrt{2}}{\sqrt{8}+\sqrt{5}-\sqrt{3}}$; (4) $\frac{\sqrt{m}}{\sqrt{m}+\sqrt{n}+\sqrt{p}}$;

(5) $\frac{\sqrt{a}-\sqrt{b}}{\sqrt{a}-\sqrt{b}+\sqrt{c}}$; (6) $\frac{1}{2-\sqrt{2}+\sqrt{3}-\sqrt{6}}$;

(7) $\frac{1}{\sqrt{10}+\sqrt{15}+\sqrt{14}+\sqrt{21}}$;

(8) $\frac{2\sqrt{14}\sqrt{3\sqrt{3}-\sqrt{2}-\sqrt{6}+3}}{\sqrt{3\sqrt{3}-\sqrt{2}+\sqrt{6}-3}}$.

440*. (1) $\frac{1}{2-\sqrt[3]{3}}$; (2) $\frac{1}{\sqrt[8]{3}+\sqrt[8]{2}}$;

(3) $\frac{1}{\sqrt[3]{2+\sqrt{5}}-\sqrt[3]{2-\sqrt{5}}}$; (4) $\frac{1}{2+\sqrt[4]{12}+\sqrt{3}}$;

(5) $\frac{1}{\sqrt[3]{4}+\sqrt[3]{6}+\sqrt[3]{9}}$; (6) $\frac{1}{\sqrt{\sqrt{2}+\sqrt[3]{3}}}$;

(7) $\frac{1}{\sqrt[3]{2}-\sqrt{2}}$; (8) $\frac{1}{\sqrt[4]{2}+\sqrt[4]{4}+\sqrt[4]{8}+2}$;

(9) $\frac{1}{4\sqrt[3]{4}-2\sqrt[3]{2}-16}$.

441. Rationalize the numerators of the following fractions:

(1) $\frac{\sqrt{5}-1}{4}$; (2) $\frac{\sqrt{7}+3}{2}$; (3) $\frac{\sqrt{5}-\sqrt{3}}{4}$;

(4) $\frac{\sqrt{12}+\sqrt{10}}{6}$; (5) $\frac{x-\sqrt{x^2-1}}{x+\sqrt{x^2-1}}$;

(6) $\frac{a+b-2\sqrt{ab}}{(\sqrt{a}+\sqrt{b})^3}$; (7) $\frac{\sqrt[3]{x}+\sqrt[3]{y}}{x+y}$.

442. Rationalize first the numerator and then the denominator of each fraction, and show that both results are equal:

$$\frac{\sqrt{3}}{\sqrt{2}},\ \frac{\sqrt{2}}{\sqrt[3]{3}},\ \frac{2\sqrt[3]{5}}{3\sqrt{15}},\ \frac{\sqrt{7}-\sqrt{2}}{\sqrt{7}+\sqrt{2}},\ \frac{\sqrt{x}+\sqrt{xy}}{\sqrt{x}-\sqrt{y}}.$$

Simplify the following expressions:

443. (1) $\frac{2}{7+4\sqrt{3}}+\frac{2}{7-4\sqrt{3}}$; (2) $\frac{\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}-\frac{\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}$;

(3) $\frac{2}{\sqrt{10}+5}+\frac{5}{\sqrt{10}-2}-\frac{7}{\sqrt{10}}$;

(4) $\frac{\sqrt{2}+2}{\sqrt{2}-2}+\frac{\sqrt{2}-1}{\sqrt{2}+1}-\frac{\sqrt{2}+3}{\sqrt{2}}$.

444. (1) $\frac{5}{4+\sqrt{11}}-\frac{1}{3+\sqrt{7}}+\frac{6}{\sqrt{7}-2}-\frac{\sqrt{7}-5}{2}$;

(2) $\frac{(5-\sqrt{24})(\sqrt{75}+\sqrt{50})}{\sqrt{75}-\sqrt{50}}$; (3) $\frac{(1-\sqrt{5})^2}{(3-2\sqrt[4]{5})(3+2\sqrt[4]{5})}$;

(4) $\frac{(\sqrt{45}-\sqrt{20})(\sqrt{12}+\sqrt{75})\times\sqrt{3}}{\sqrt{5}+\sqrt{180}}$.

445. (1) $\left(\sqrt[6]{13-4\sqrt{3}}+\sqrt[3]{\frac{1}{8}-\frac{\sqrt{3}}{4}}\right)\sqrt[3]{1+2\sqrt{3}}$;

(2) $\frac{2(\sqrt{3}-1)^2}{3(\sqrt{3}-1)^2-2}$; (3) $5\sqrt{2+\sqrt{5}}\sqrt[6]{17\sqrt{5}-38}$;

(4) $2\sqrt[3]{1-\sqrt{2}}\sqrt[6]{2\sqrt{2}+3}$.

446. (1) $\frac{a^2+1+a\sqrt{a^2+1}}{a+\sqrt{a^2+1}}$;

(2) $\frac{\sqrt{1+x}}{\sqrt{1+x}-\sqrt{1-x}}+\frac{1-x}{\sqrt{1-x^2}+x-1}$;

(3) $\frac{a}{\sqrt[3]{a}-1}-\frac{\sqrt[3]{a^2}}{1+\sqrt[3]{a}}+\frac{1}{\sqrt[3]{a}+1}+\frac{1}{1-\sqrt[3]{a}}$;

(4) $\left(\frac{a+\sqrt{a^2-4}}{a-\sqrt{a^2-4}}-\frac{a-\sqrt{a^2-4}}{a+\sqrt{a^2-4}}\right):\frac{a\sqrt{a^2-4}}{4}$.

447. (1) $\frac{n+3-\sqrt{n^2-9}}{n+3+\sqrt{n^2-9}}+\frac{n+3+\sqrt{n^2-9}}{n+3-\sqrt{n^2-9}}$;

(2) $\frac{\sqrt{a}-\sqrt{b}+1}{a-\sqrt{ab}}+\frac{\sqrt{a}+\sqrt{b}}{2\sqrt{ab}}\left(\frac{\sqrt{a}}{b-\sqrt{ab}}+\frac{\sqrt{a}}{b+\sqrt{ab}}\right)$;

(3) $\left[\frac{1}{2}\left(\sqrt{a^3b^{-3}}-\sqrt{a^{-3}b^3}\right):\left(1+\frac{a^2+b^2}{ab}\right)\right]\times\frac{2\sqrt{ab}}{\sqrt{a-b}}$;

(4) $\left(\sqrt{a}+\sqrt[4]{a}+1\right)\left(\sqrt{a}-\sqrt[4]{a}+1\right)\left(a-\sqrt{a}+1\right)$.

448.

(1) $\left(\frac{\sqrt{2+a}}{\sqrt{2+a}-\sqrt{2-a}}+\frac{2-a}{\sqrt{4-a^2}-2+a}\right):\frac{\sqrt{\frac{4}{a^2}-1}+\frac{2-a}{a}}{\sqrt{2-a}}$;

(2) $\frac{(\sqrt{m}+\sqrt{n})^3+2m^2:\sqrt{m}-n\sqrt{n}}{m\sqrt{m}-n\sqrt{n}}-\frac{3\sqrt{mn}+3n}{m-n}$;

(3) $\left(\frac{a+\sqrt[4]{ax^3}}{\sqrt{a}+\sqrt[4]{ax}}-\sqrt[4]{ax}\right):\left(\sqrt[4]{a}-\sqrt[4]{x}\right)-\frac{\sqrt{a}-\sqrt{x}}{\sqrt[4]{a}-\sqrt[4]{x}}$;

(4) $\left[\frac{\left(\sqrt[4]{a^3}+\sqrt[4]{b^3}\right)\left(\sqrt[4]{a^3}-\sqrt[4]{b^3}\right)}{\sqrt{a}-\sqrt{b}}-\sqrt{ab}\right]\times\frac{2}{a+b}$.

449. (1) $\left(\sqrt[-2]{1+a}+\sqrt{1-a}\right):\left(\sqrt[-2]{1-a^2}+1\right)$;

(2) $\left(x+y\sqrt{\frac{y}{x}}\right)^{\frac{2}{3}}\left(\frac{\sqrt{y}}{\sqrt{x}-\sqrt{y}}+\frac{\sqrt{x}-\sqrt{y}}{\sqrt{x}}\right)^{-\frac{2}{3}}$.

450.

(1) $2a\sqrt{1+\frac{1}{4}\left(\sqrt{\frac{a}{b}}-\sqrt{\frac{b}{a}}\right)^2}:\left[\frac{1}{2}\left(\sqrt{\frac{a}{b}}-\sqrt{\frac{b}{a}}\right)+\right.$

$\left.+\sqrt{1+\frac{1}{4}\left(\sqrt{\frac{a}{b}}-\sqrt{\frac{b}{a}}\right)^2}\right]$;

(2) $\left[\frac{1}{\sqrt{x}-4\sqrt[-2]{x}}+\frac{2\sqrt[3]{x}}{\sqrt[3]{x^4}-\sqrt[3]{64x}}\right]^{-2}-\sqrt{x^2+8x+16}$.

451.

(1) $\left(\sqrt[-2]{\frac{m+n}{2\sqrt{mn}}-1}+\sqrt[-2]{\frac{m+n}{2\sqrt{mn}}+1}\right):\left(\sqrt[-2]{\frac{m+n}{2\sqrt{mn}}-1}-\sqrt[-2]{\frac{m+n}{2\sqrt{mn}}+1}\right)$;

(2) $\left(\frac{\sqrt[4]{ab}-\sqrt{ab}}{1-\sqrt{ab}}+\frac{1-\sqrt[4]{ab}}{\sqrt[4]{ab}}\right):\frac{\sqrt[4]{ab}}{1+\sqrt[4]{a^3b^3}}-\frac{1-\sqrt[4]{ab}-\sqrt{ab}}{\sqrt{ab}}$.

452. Evaluate the expression:

(1) $4x^3+2x^2-8x+7$ for $x=\frac{1}{2}(\sqrt{3}+1)$;

(2) $3x^2+4xy-3y^2$ for $x=\frac{\sqrt{5}+\sqrt{2}}{\sqrt{5}-\sqrt{2}}$, $y=\frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}+\sqrt{2}}$.

453. Prove that the fraction $\frac{\sqrt{a+x}-\sqrt{a-x}}{\sqrt{a+x}+\sqrt{a-x}}$ for $x=\frac{2ab}{b^2+1}$ attains the value equal to (a) $\frac{1}{b}$ if $b\leqslant -1$ and $b\geqslant 1$, $a>0$; (b) b if $-1\leqslant b\leqslant 1$ and $a>0$.

454. Prove that for $x=\frac{-b-\sqrt{b^2-4ac}}{2a}$ the expression ax^2+bx+c vanishes.

455. Prove that the expression $\sqrt{\frac{3}{4}-x}+\sqrt{2x}-\frac{3}{2}\sqrt{1-4x}$ vanishes for $x=\frac{1}{12}$.

456. 1. Prove that $\left(\frac{6+4\sqrt{2}}{\sqrt{2}+\sqrt{6+4\sqrt{2}}}+\frac{6-4\sqrt{2}}{\sqrt{2}-\sqrt{6-4\sqrt{2}}}\right)^2=8$.

2*. Prove the identity

$$\frac{(a+2)^2(a-3)+(a^2-4)\sqrt{a^2-9}}{(a-2)^2(a+3)+(a^2-4)\sqrt{a^2-9}}=\frac{(a+2)\sqrt{a-3}}{(a-2)\sqrt{a+3}} \quad \text{for } a\geqslant 3.$$

457*. Simplify:

(1) $\frac{1-mz}{1+mz}\sqrt{\frac{1+nz}{1-nz}}$ if $z=\frac{1}{m}\sqrt{\frac{2m}{n}-1}$;

(2) $\left(\frac{m-\sqrt{y}}{m+\sqrt{y}}\right)^{\frac{1}{2}}+\left(\frac{m+\sqrt{y}}{m-\sqrt{y}}\right)^{\frac{1}{2}}-\frac{4}{\sqrt{m^2-y}}$ if $y=4(m-1)$;

(3) $\sqrt{\frac{2}{(1-z^2)^{-\frac{1}{2}}+1}}-\sqrt{\frac{2}{(1-z^2)^{-\frac{1}{2}}-1}}$ if $z=2m^{\frac{1}{2}}(1+m)^{-1}$.

CHAPTER VII

Simplest Functions and Their Graphs

20. FUNCTIONAL DEPENDENCE AND METHODS OF ITS REPRESENTATION

458. 1. Construct the point (3, 3) in the (Cartesian) coordinates. On the same drawing mark the following points: (a) (4, 3), (5, 3), (7, 3), (0, 3), (−2, 3); (b) (3, 4), (3, 5), (3, 6), (3, 0) (3, −1). How are the points listed in (a) and (b) located with respect to the point (3, 3)?

2. In Cartesian coordinates xOy construct the points $(x, 1)$ and $(1, y)$, where x and y are any real numbers. How many such points can be constructed? How are they situated with respect to the coordinate axes?

3. Construct the point (a, b) and on the same drawing the points $(a \pm 1, b)$; $(a, b \pm 1)$; $(a \pm 2, b \pm 2)$.

459. Given the points with the following coordinates: (2, 2), (3, 1), (−1, 3), (−2, −3), (a, b). For each of them construct a point symmetrical about (a) the x-axis, (b) the y-axis, (c) the origin, (d) the bisector of the first and third quadrants.

460. Given the point (x, y). Construct the points: $(|x|, y)$, $(x, |y|)$, $(|x|, |y|)$. Consider all possible cases of location of the point (x, y) (in different quadrants and on the coordinate axes).

461. What can be said about the coordinates of the points lying on (1) the x-axis, y-axis, (2) a straight line parallel to the x-axis, (3) a straight line parallel to the y-axis, (4) the bisector of the first and third quadrants, (5) the bisector of the second and fourth quadrants?

462. Where is the point situated if (a) its abscissa is positive, and the ordinate is negative, (b) the abscissa equals zero, and the ordinate is positive, (c) its abscissa is equal to 2, and the ordinate is any real number, (d) its coordinates are equal to each other in magnitude?

463. Where are the points situated if their coordinates satisfy the following conditions:

(1) $-1 \leqslant x \leqslant 4$, and y is any real number; (2) $x \geqslant 0$, and $1 \leqslant y \leqslant 3$; (3) $-1 < x \leqslant 5$, $0 \leqslant y \leqslant 4$; (4) $y > x$; (5) $y < x$; (6) $y = x$?

464. Where are the points situated if (1) the distance of each of them from the origin is equal to 3, (2) the distance of each of them from the origin is more than 5, but less than 8, (3) the coordinates of each of them satisfy the equation $x^2+y^2 = 4$, the inequality $x^2+y^2 < 4$, the inequality $x^2+y^2 \geqslant 4$?

465. Find the length of the line segment AB if its end-points have the following coordinates:

(a) $A(0, -3)$, $B(0, 5)$; (b) $A(2, 7)$, $B(6, 4)$; (c) $A(x_1, y_1)$, $B(x_2, y_2)$.

466. The side of an equilateral triangle equal to 6 cm is parallel to the x-axis, and the vertex opposite this side has the coordinates (0, 0). Determine the coordinates of the other two vertices of the triangle.

467. Given the coordinates of three vertices of a square: $A(2, 6)$, $B(5, 6)$, and $C(5, 3)$. Find the coordinates of the remaining vertex D, the perimeter, and the area of the square.

468. Find the coordinates of the mid-point of the line segment AB if: (1) $A(3, 0)$, $B(7, 0)$; (2) $A(0, -2)$, $B(0, 2)$; (3) $A(-3, 2)$, $B(5, -2)$.

469. Two vertices of a rectangle lie on the y-axis and the other two at the points $A(-5, 3)$ and $B(-5, -5)$. Determine the perimeter and the area of the rectangle.

470. The sides of the rectangle $ABCD$ are parallel to the coordinate axes, its two opposite vertices are $A(2, 3)$ and $C(7, 6)$. Find the coordinates of the remaining vertices and the centre.

471. Given the points $A(-4, 2)$ and $B(3, 5)$. Compute the area of the trapezoid whose vertices are the given points and their projections on the x-axis.

472. Which of the below listed quantities characterizing a freely falling body are variables and which are constants: path, time, velocity, acceleration, weight, temperature?

473. A circle is circumscribed about a triangle ABC. The vertex

A moves in this circle, and the vertices B and C are fixed. Which quantities involved in this process remain constant and which quantities change? Consider also the case when BC is the diameter of the circumscribed circle.

474. Air is being forced into a cylinder. Which characteristics of this process remain constant and which characteristics change?

475. The period T of small-amplitude oscillations of a mathematical pendulum is computed by the formula $T = 2\pi\sqrt{\frac{l}{g}}$ where l is the length of the pendulum, g is acceleration due to gravity. Which of the quantities entering this formula are absolute constants, parameters and variables?

476. The results of daily measurements of temperature during the last ten days of March are tabulated as follows:

Date	21	22	23	24	25	26	27	28	29	30
Temperature, °C	−2	−3	1	1	6	8	10	9	6	8

1. Plot the graph showing the change in temperature during this period of time.

2. Find daily average temperature during the ten days, plot its graph (on the same drawing) and determine the fluctuations from the average temperature for each day.

477. The function y is given by the following table:

x	−3	−2	−1	0	2	3	4	5	6	7
y	4	2	0	−2	0	−4	−2	2	5	3

1. Plot the graph of this function.

2. Using the graph find the values of the function for $x = -0.5$, $x = 2.5$, $x = 4.5$.

3. At what points does the graph of the function intersect the x-axis?

4. Using the graph, find the greatest and the least values of the function on the intervals $[-3, 7]$, $[-2, 2]$, $[3, 6]$.

5. Using the graph, find the change of the ordinate due to the change of the abscissa from $x_1 = 3.5$ to $x_2 = 5$.

478. At 6 o'clock in the morning a tourist left the village for the town. During the first hour of his travel he covered the distance of 6 km, the next hour he walked up-slope and covered only 4 km. Then he had a rest during half an hour and continued walking at an average speed of 5 km/h and reached the town at 10 o'clock in the morning.

1. Compile a table expressing the dependence between the distance covered by the tourist and the time and plot the appropriate graph.

2. Using the graph determine: (a) the distance between the village and the town, (b) at what distance from the village and at what time had the tourist his rest.

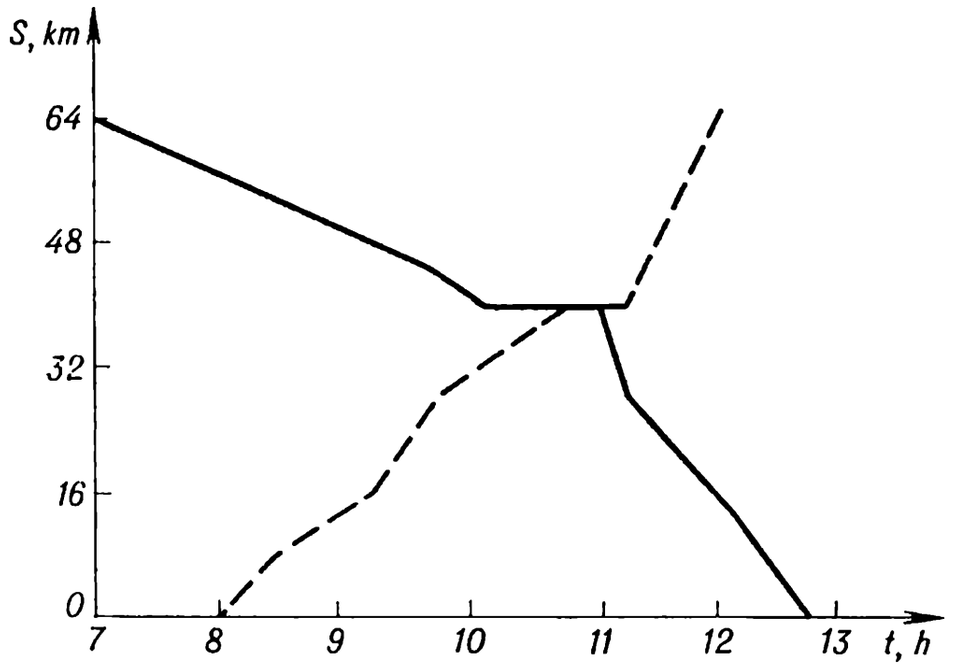


FIG. 1

479. Figure 1 illustrates graphically the motion of two cyclists travelling to meet each other. Using the graph, determine:

(1) the time of departure and arrival for each cyclyst;
(2) when and where the cyclists met;
(3) where and when the cyclists had their rest;
(4) how much time it took each cyclist to meet the other;
(5) the average speed of cycling for each of them.

480. A cyclist leaves the city at 8 o'clock in the morning and travels with a speed of 15 km/h. Three hours later a car takes the

same road to move along it at a speed of 60 km/h. After it covers 30 km it stops for 30 min and then continues driving with a new speed of 45 km/h. Determine graphically the place and time where and when the car catches up with the cyclist. How many kilometres will the car get ahead of the cyclist by 2 o'clock in the afternoon, if from 13 : 00 to 13 : 30 the cyclist has his rest?

481. A function is represented by its graph shown in Fig. 2. Draw it on millimetre graph paper, to an appropriate scale, take a few values of the independent variable and, with the aid of the graph, evaluate the function for the chosen values of the independent variable. Tabulate the values of the variable and the corresponding values of the function.

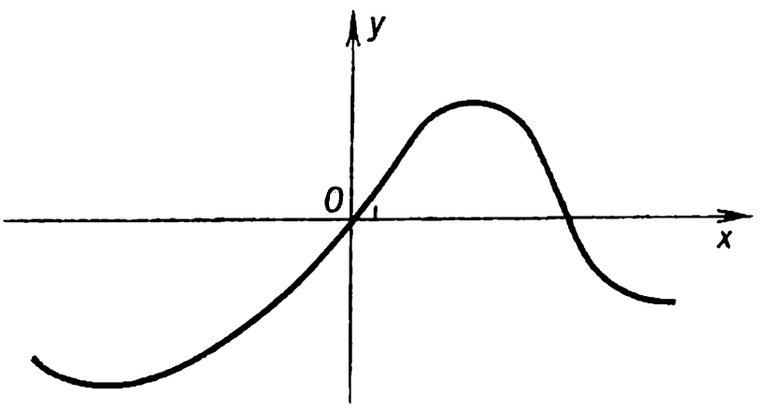


FIG. 2

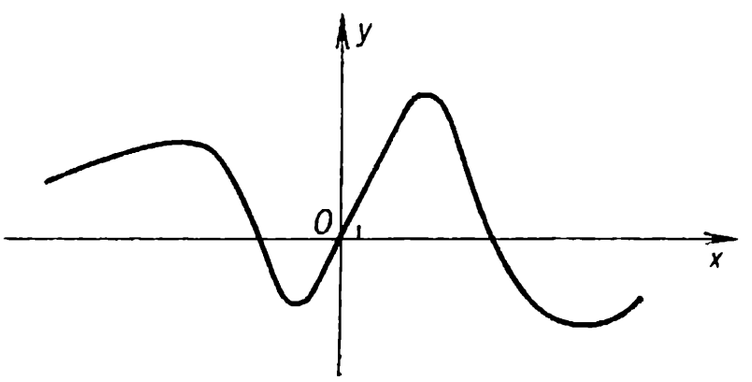


FIG. 3

482. A function is specified by its graph depicted in Fig. 3. Using the graph, determine:

(1) the values of the independent variable for which the function vanishes;

(2) the intervals within which the function is positive, negative;

(3) the intervals within which the function increases, decreases;
(4) the greatest and the least values of the function on the interval $[-5, 7]$.

483. A function is given by its graph illustrated in Fig. 4. As $x \to +\infty$ and $x \to -\infty$ its graph approaches indefinitely near to the x-axis without intersecting it. For this function determine:
(1) whether it has some roots;
(2) the domain of definition and the domain of variation;
(3) the intervals of increase and decrease;
(4) the greatest and the least values.

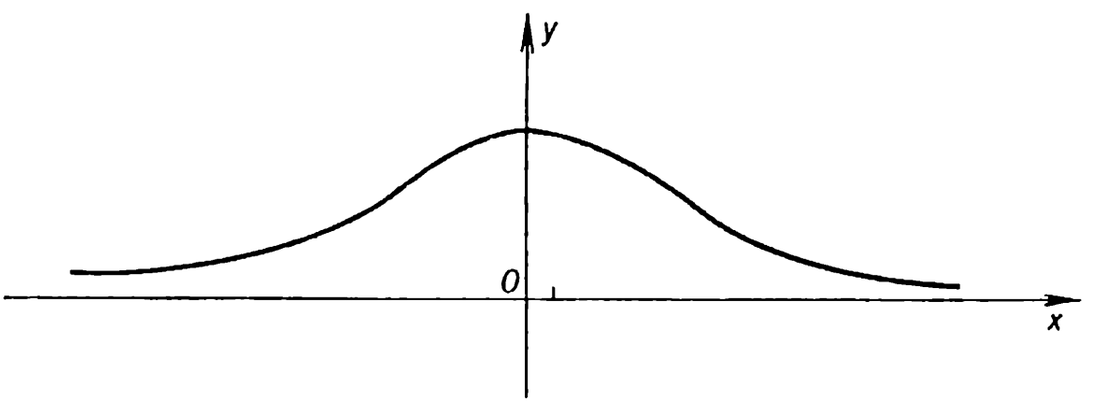


FIG. 4

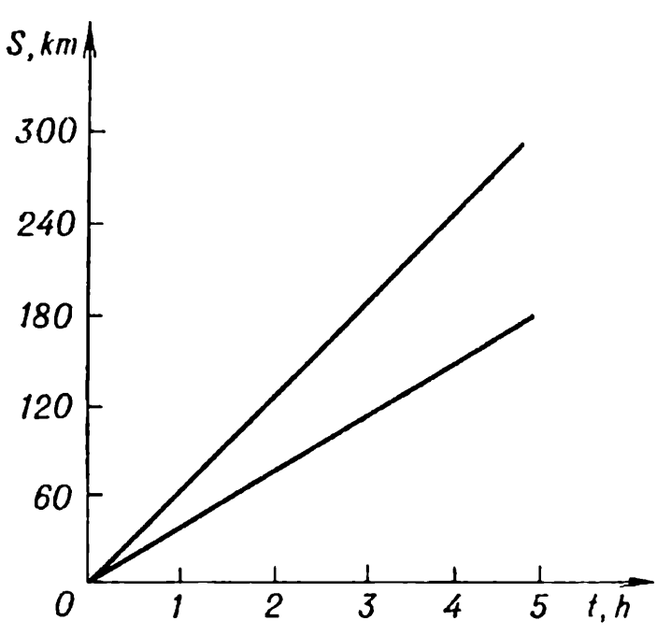


FIG. 5

484. Figure 5 gives the graphs of motion for two cars.
1. Using the graphs, find the speed of each car (in km/h).
2. Write the formula expressing the dependence of the distance covered by each car on the time spent.

3. In what respects do the given graphs differ from and resemble each other?

4. What is the difference between the motions of these cars?

485. The function y is represented in the tabular form:

x	-4	-3	-2	-1	0	1	2	3	4
y	17	10	5	2	1	2	5	10	17

1. Specify this function analytically.

2. Plot its graph and, using it, find:

(a) the least and the greatest values of the function;

(b) the intervals of decrease and increase of the function.

486. Two variables change so that (1) one of them remains equal to the sum of the square of the other variable and 4; (2) the square of one variable remains equal to the reciprocal of the cube of the other variable; (3) the product of the variables remains equal to the ratio of their sum to the sum of their squares. For each case, express analytically the functional dependence between the two variables.

487. 1. Express the sum of the interior angles of a polygon as a function of the number of its sides.

2. A tank of 1000 dm^3 capacity is being filled with petrol. The pump feeds 50 litres of petrol per minute. Express analytically the amount of petrol contained in the tank as a function of time if at the initial instant it was empty.

488. A tourist kept travelling during 9 hours. During the first five hours of his travel he walked at a speed of 4.5 km/h, then he had a 0.5-hour rest, the remaining time covering 4 km per hour.

1*. Express analytically the distance covered by the tourist as a function of time.

2. Construct the graph of the function thus obtained.

3. Find graphically and analytically the distance covered by the tourist during 3.5 hours; during 5 hours and 15 minutes; during 8 hours.

489. 1. $f(x) = 2x^2-3x+1$. Find $f(0), f(3), f(-3), f(a-1)$.

2. $f(x) = \sqrt{x^2-5x+4}$. Find $f(0)$, $f(1)$, $f(m+1)$.

3. $f(x) = \frac{|2-x|}{1+x}$. Find $f(0), f(3), f(-3), f(2), f(-2)$.

4. $f(z) = z^2$. Find $\frac{f(m)-f(n)}{m-n}$, $\frac{f(m+n)-f(m-n)}{2n}$.

490. (1) $f(x) = x^2+x+\frac{1}{x}+\frac{1}{x^2}$. Prove that $f(a) = f\left(\frac{1}{a}\right)$; (2) in the expression $f(x) = Ax^2+Bx+C$ find A, B and C if $f(0) = 1$, $f(1) = 0$, $f(3) = 5$; (3) $f(x) = 2^x+2^{-x}$, show that $f(-x) = f(x)$; (4) $f(x) = \frac{3x}{x^2+2}$, show that $f(-x) = -f(x)$.

491. (1) $f(x) = \begin{cases} x+1 & \text{if} \quad -\infty < x \leqslant 0 \\ 1 & \text{if} \quad 0 < x < +\infty. \end{cases}$

Find $f(-3), f(-1), f(0), f(1), f(5), f(10)$ and plot the graph of the function.

(2) $f(x) = \begin{cases} x^2-1 & \text{if} \quad x \leqslant 2 \\ x-1 & \text{if} \quad x > 2. \end{cases}$

Find $f(-1), f(1), f(0), f\left(\frac{1}{2}\right), f(3), f(5)$ and plot the graph of the function.

(3) $f(x) = \begin{cases} x+2 & \text{if} \quad -\infty < x \leqslant -2 \\ 4-x^2 & \text{if} \quad -2 < x < 2 \\ x-2 & \text{if} \quad 2 \leqslant x < \infty. \end{cases}$

Find $f(-3), f(-2), f(0), f(2), f(5)$ and plot the graph of the function.

492. Write each of the given functions

$$f(x) = \begin{cases} x & \text{if} \quad x \geqslant 0 \\ -x & \text{if} \quad x \leqslant 0 \end{cases} \quad \text{and} \quad \varphi(x) = \begin{cases} 2x-3 & \text{if} \quad x \geqslant \frac{3}{2} \\ 3-2x & \text{if} \quad x \leqslant \frac{3}{2} \end{cases}$$

with the aid of one formula using the sign of the absolute value.

493. A function is represented by the graph shown in Fig. 6. Mark two points on the x-axis: $x = a$, and $x = b$, $a < b$. Construct $f(a)$ and $f(b)$ on the drawing. What geometrical interpretation has the relation $\frac{f(b)-f(a)}{b-a}$?

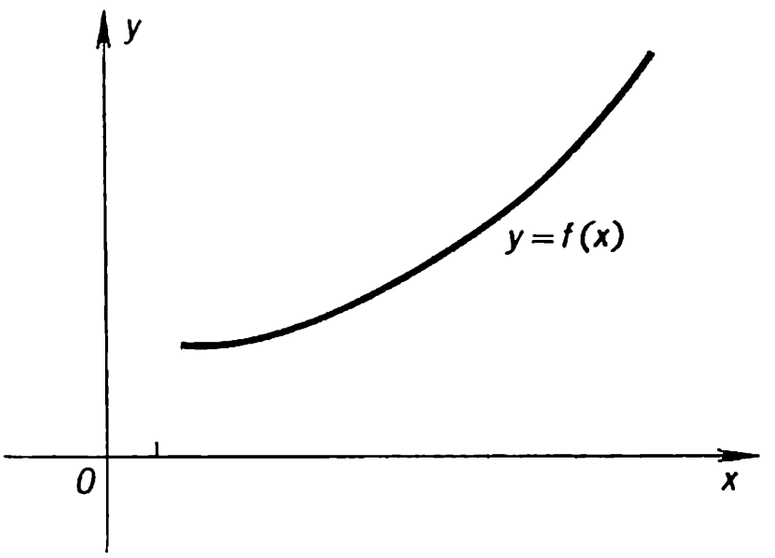


FIG. 6

494. Find the roots of the following functions:

(1) $y = x-2$; (2) $y = -2x^2+5x$; (3) $y = x^2-2x+1$;

(4) $y = x^2-6x+8$; (5) $y = x^2-4$; (6) $y = \frac{x+1}{x-1}$;

(7) $y = \sqrt{4x+3}$; (8) $y = \sqrt{x^2+9}$; (9) $y = \sqrt{\frac{x+8}{x}}$.

495. 1. Find the points of intersection of the coordinate axes and the graphs of the following functions:

(a) $y = (x-2)(x+3)(2x+8)(x^2+1)$;

(b) $y = 2+\frac{1}{1+x^2}$; (c) $y = \frac{x(x-3)(x+4)}{x^2+2}$;

(d) $y = 4-\frac{8}{4+x^2}$.

2. Determine the roots and the intervals of positive and negative values of the following functions:

(a) $y = 1+x$; (b) $y = 2+x-x^2$; (c) $y = x^3+3x$;

(d) $y = \frac{x^2-4}{x^2+4}$.

496. Find the domains of definition of the following functions:

(1) $y = 3x$; (2) $y = 2x^3-3x^2+4$; (3) $y = \frac{2}{x}$;

(4) $y = \frac{1}{x^2+1}$; (5) $y = \frac{x^2+4}{x^2-4}$;

(6) $y = \frac{3x}{x^2-4x+4}$; (7) $y = \frac{2}{x^3-x}$;

(8) $y = \frac{x-2}{x^4+x}$; (9) $y = \frac{3x+2}{x^4-8x}$;

(10) $y = \sqrt{2x+3}$; (11) $y = \sqrt[3]{x^3-1}$; (12) $y = \sqrt[4]{1-2x}$;

(13) $y = \frac{2x}{\sqrt{3x-4}}$; (14) $y = \sqrt{\frac{x-3}{x+2}}$;

(15) $y = \frac{\sqrt{x-1}}{x^2-4}$.

497. Find the domains of definition of the following functions:

(1) $y = \sqrt{-x} + \frac{1}{\sqrt{2+x}}$; (2) $y = \sqrt{1+x} - 3\sqrt[4]{5-x}$;

(3) $y = \sqrt{4-x} - \sqrt{x+2}$; (4) $y = \sqrt{x-2} + \sqrt{1-x}$;

(5) $y = \sqrt{4+x} - \sqrt{x+2} + \sqrt{15-x}$;

(6) $y = x\sqrt{x^2-2}$; (7) $y = \frac{x^2-3x+2}{|x^2-5x|+6}$;

(8) $y = \sqrt[3]{x^2-5x-8}$.

Find the domains of variation of the following functions:

498. (1) $y = 4x+1$; (2) $y = \frac{2}{x}$;

(3) $y = x^2+3$; (4) $y = x^3+2$;

(5) $y = (x-4)^2-8$; (6) $y = 5-(x+3)^2$;

(7) $y = |x-1|$; (8) $y = x-|x|$;

(9) $y = |2x^2-4x|+2$; (10) $y = \frac{3}{2+x^2}$;

(11)* $y = \frac{3}{2-x^2}$; (12) $y = \frac{x}{|x|}$.

499. (1) $y = x^2-8x+1$; (2) $y = x^2+x+1$;

(3) $y = -x^2+6x-5$; (4) $y = |x^2+3x|$;

(5) $y = |x^2+6x+5|$; (6)* $y = 3 - \frac{4}{(x-2)^2+2}$;

(7) $y = \frac{1}{|x|}$; (8) $y = \sqrt{\frac{x+3}{x-2}}$;

(9)* $y = \frac{2x^2-8x+14}{x^2-4x+6}$; (10) $y = |x-2| - |x+1|$;

(11) $y = \frac{2x}{x^2+1}$; (12) $y = -\frac{4x}{x^2+4}$.

500. 1. Give an example of an analytically specified function defined (a) throughout the number scale, (b) throughout the number scale except at $x = 2$, (c) throughout the number scale except for the points $x = \pm\sqrt{3}$, (d) in the half-interval $2 \leqslant x < \infty$, (e) in the interval $-2 < x < 2$; (f) in the interval $-2 \leqslant x \leqslant 2$.

2. Are the following functions identical?

$f(x) = \frac{x}{x^2}$ and $\varphi(x) = \frac{1}{x}$; $f(x) = \frac{x^2}{x}$ and $\varphi(x) = x$.

501. Which of the following functions are increasing and which are decreasing?

(1) $y = 3x$; (2) $y = 2x+3$; (3) $y = 2-3x$; (4) $y = \sqrt{x+1}$;

(5) $y = 1-\sqrt{x}$; (6) $y = \sqrt[3]{2x-3}$; (7) $y = \frac{1}{x+3}$; (8) $y = \frac{1}{1-x}$.

502. Find the intervals of increase and decrease for the following functions:

(1) $y = 2x$; (2) $y = -\frac{1}{2}x$; (3) $y = \frac{1}{x}$;

(4) $y = \frac{2}{x^2+1}$; (5) $y = (x-1)^2-3$; (6) $y = |x-2|$;

(7) $y = x^2+4x-2$; (8) $y = -x^2+3x+4$;

(9)* $y = |-x^2-3x+4|$;

(10) $y = 2^x$; (11) $y = 2^{2-4x}$; (12)* $y = \frac{5x+4}{x}$.

503. 1. Prove that a sum of two increasing (decreasing) functions is an increasing (decreasing) function.

2. Will a difference of two increasing (decreasing) functions be an increasing (decreasing) function? Give a graphical explanation.

504. Find the greatest values of the given functions:

(1) $y = 16-x^2$; (2) $y = 9-(2x-3)^2$;

(3) $y = 6x-x^2$; (4) $y = -3+4x-x^2$;

(5) $y = (3-x)(x+1)$; (6) $y = \sqrt{25-x^2}$;

(7) $y = \sqrt[3]{x^3-4}$; (8) $y = \frac{3}{x^2+x+1}$;

(9) $y = \frac{2}{\sqrt{4x^2+3}}$; (10)* $y = \frac{10}{|1+4x-x^2|+5}$;

(11)* $y = |x-2|-|x-1|$; (12) $y = \sqrt{-2x^2-3x+1}$.

505. Find the least value of the given function:

(1) $y = x^2-9$; (2) $y = (3x-4)^2+5$; (3) $y = x^2+6x$;

(4) $y = (x+2)(x-3)$; (5) $y = \sqrt{x^2+3}$;

(6) $y = \sqrt[3]{x^2-8}$; (7) $y = \sqrt{x^2-4x+13}$;

(8)* $y = |x^2+5x+6|$;

(9) $y = |5+4x-x^2|$; (10) $y = \frac{5}{3-x^2}$; (11) $y = \frac{48}{-\sqrt[3]{8-x^2}}$.

506. Break the number 18 into two addends so that their product is the greatest possible.

507. A rectangular plot adjacent to a house is to be fenced with metal gauze 100 m long. Find the dimensions of the rectangle for which the area of the plot will be the greatest possible.

508. Find out which of the given functions are even and which are odd:

(1) $f(x) = 2x^4-3x^2+5$; (2) $f(x) = 3x^3+x^5$;

(3) $f(x) = x^2+3x-1$; (4) $f(x) = x^{-3}+x^{-1}$;

(5) $f(x) = x^{-6}+2x^{-4}+5$; (6) $f(x) = \sqrt{x}$;

(7) $f(x) = \sqrt[3]{x}$; (8) $f(x) = \sqrt{x^2-1}$;

(9) $f(x) = \sqrt[3]{(x+1)^2}+\sqrt[3]{(x-1)^2}$;

(10) $f(x) = 2x^2-3x$; (11) $f(x) = x^4-\sqrt{x^4+2}$;

(12) $f(x) = \sqrt{x^2-1}+\frac{1}{x^2+1}$.

509. Determine which of the given functions are even and which are odd:

(1) $f(x) = (x^4+3)\sqrt{x^6-3}$; (2) $f(x) = x^3(3x+x^5)^3$;

(3) $f(x) = \frac{x^2-1}{x^2+1}$; (4) $f(x) = \frac{x^3}{\sqrt[3]{x^3+x}}$;

(5) $f(x) = 2^x$; (6) $f(x) = \frac{x+1}{x^2-3}$;

(7) $f(x) = \frac{x^4}{x^3+3}$; (8) $f(x) = 3^x+3^{-x}$;

(9) $f(x) = 3^x-3^{-x}$; (10) $f(x) = (x^2+x)^2$.

510. Prove that:

(1) if $f(x)$ and $\varphi(x)$ are even functions, then their sum, difference product and quotient are also even functions;

(2) if $f(x)$ and $\varphi(x)$ are odd functions, then their product and quotient are even functions;

(3) if one of the functions $f(x)$ and $\varphi(x)$ is even and the other is odd, then their product and quotient are odd functions.

511. Analyse the behaviour of the given functions as the argument increases and decreases without bound:

(1) $y = \frac{1}{x}$; (2) $y = \frac{2}{x-3}$; (3) $y = -\frac{7}{5-2x}$;

(4) $y = -\frac{5}{x+2}$; (5) $y = -\frac{1}{x^2+1}$;

(6) $y = \frac{1}{1-x^2}$; (7) $y = \frac{8}{\sqrt{1+x}}$;

(8) $y = \frac{1}{\sqrt{\frac{1}{x}+4}}$; (9) $y = \frac{2}{\sqrt{9-\frac{1}{x}}}$;

(10) $y = x^2-2$; (11) $y = 2x^2+3x-4$; (12) $y = -x^2+4x$.

512. Analyse the behaviour of the given function:

(1) $y = \frac{1}{x}$ as x approaches zero at infinity;

(2) $y = \frac{2}{x+3}$ as x approaches -3 without bound;

(3) $y = \frac{x^2+4}{x^2-4}$ as x approaches 2, -2 without bound.

513. Analyse the given functions (the domain of definition, evenness or oddness, the intervals of increase and decrease, the greatest and the least values, the points of intersection of the function with the coordinate axes, the behaviour of the function as the argument increases without bound), and plot their graphs:

(1) $y = x^2+4x+9$; (2) $y = 4x^2-12x+11$;

(3) $y = -x^2+4x-5$; (4) $y = -9x^2-6x+1$;

(5) $y = \frac{2}{x^2+1}$; (6) $y = \frac{1}{1-x}$;

(7)* $y = \frac{4x}{x^2+4}$; (8)* $y = \frac{4}{\sqrt{x^2+1}}$;

(9)* $y = |x^2-4|$; (10) $y = x^2-4|x|+4$.

21. DIRECT AND INVERSE PROPORTIONALITY

514. Indicate the directly and inversely proportional quantities:

1. The area of a triangle and its altitude with the base given.
2. The base of a triangle and its altitude with the area given.
3. The area of a square and its side.
4. A central angle and the circular arc corresponding to it.
5. The length of the circumference and its radius.
6. The length of a chord and the arc subtended by it.
7. The magnitude of a fraction and its numerator with the denominator constant.
8. The magnitude of a fraction and its denominator with the constant numerator.
9. The distance covered during given time and the velocity in a uniform motion.
10. The time during which a given distance is covered and the velocity in a uniform motion.

11. $y = \frac{1}{2}x$, $y = \frac{1}{2x}$, $y = \sqrt{2x}$, $y = (2x-1)$, $y = \frac{4}{x+2}$.

515. Using the terms 'direct proportionality' and 'inverse proportionality', read:

(1) the formula for the centrifugal force $F = mR\omega^2$ in a uniform circular motion;

(2) the formula for the electrical resistance $R = \varrho\frac{l}{s}$ of a conductor;

(3) Coulomb's law $F = \frac{q_1q_2}{4\pi\varepsilon_0\varepsilon r^2}$.

516. A vessel is filled with mercury. The pressure exerted by the mercury on the bottom of the vessel is determined by the

formula $P = kh$, where h is the height of the mercury column. What sense has the proportionality factor in this case?

517. It is known that the quantity y is directly proportional to the quantity x. What must one know to compute corresponding value of y given the value of x?

518. The cost of goods is proportional to its amount. Derive the formula expressing the dependence between these quantities and find out the meaning of the proportionality factor.

519. Construct on one drawing the graphs of the functions:

$$(1)\ y = 3x \text{ and } y = -3x; \quad (2)\ y = \frac{1}{3}x \text{ and } y = -\frac{1}{3}x.$$

520. 1. What angle do the graphs of the functions (a) $y = x$, b) $y = -x$ form with the x-axis? Compute the angle between the graphs of these functions.

2. Are the angles between the graphs of the functions $y = 2x$, $y = 7x$, $y = \frac{1}{3}x$, $y = -\frac{1}{3}x$, $y = 0.7x$ and the x-axis greater or less than 45°?

521. Using the graph of the function $y = x$, construct the graphs of the functions $y = 3x$, $y = \frac{1}{3}x$, $y = |x|$.

522. 1. A force P is applied to the origin, and its x- and y-coordinates are equal to 3 and 4, respectively. Find the equation of the straight line along which the force is directed.

2. A point moves simultaneously from the origin along the x-axis at a velocity of a, and along the y-axis at a velocity of b. Find the equation of the line in which the point moves.

523. The quantities x and y are inversely proportional. It is known that when $x = 1.5$, $y = 2.4$. Write the formula expressing the dependence.

524. At curved sections of the track a train tends to run off the rails, since any moving body tends to continue its rectilinear motion. To keep the train on the rails it is standard practice to lay the outer rail at a rounded section somewhat higher than the inner one. The elevation h of the outer rail, the maximum speed v of the train, the gauge l, the radius R of the rounding and the acceleration due to gravity g must be interconnected by the relation $h = \frac{lv^2}{gR}$. How does h depend on R? How will h change, if R increases without bound?

525. 1. Knowing that y changes inversely proportionally to x, fill in the following table:

x	1	0.5	2	3	4	6	-1	-2	-3	-4	-6
y	6										

2. Write the equation expressing the dependence of y on x.

3. Show that if the point with the coordinates (a, b) belongs to the graph, then the point (b, a) belongs to the same graph as well.

4. Are there any points on the graph at which $y = x$?

5. Prove that the curve is symmetric with respect to the origin and to the bisectors of the quadrants.

526. 1. Plot the graphs of the given functions to get one drawing: (a) $y = \frac{12}{x}$; (b) $y = -\frac{12}{x}$.

2. Graph the functions: $y = \frac{8}{x-2}$, $y = \frac{2}{x} + 3$, $y = -\frac{4}{x} - 2$, $y = \frac{2x-3}{x-1}$.

527. Take a piece of cardboard and cut out a stencil of the hyperbola $y = \frac{1}{x}$. Using this stencil, graph the following functions:

(1) $y = -\frac{1}{x}$; (2) $y = \frac{1}{|x|}$; (3) $y = \frac{1}{x} \pm 3$, $y = \frac{1}{x \pm 3}$;

(4) $y = \frac{2x-1}{x}$.

528. By deforming the graph of the function $y = \frac{1}{x}$, plot the graphs of the following functions:

(1) $y = \frac{4}{x}$; (2) $y = \frac{1}{3x}$; (3) $y = \frac{3}{2x}$; (4) $y = \frac{2}{|x|}$.

529. Without graphing the functions $y = \frac{1}{2x}$ and $y = \frac{3.2}{x}$, answer the following questions:

1. Do they intersect?

2. For what values of x are the points of the first hyperbola situated higher than those of the second one, and vice versa?

530. 1. The quantity x is directly proportional to the quantity y, and y is directly proportional to z. What dependence exists between x and z?

2. The quantity x is inversely proportional to the quantity y, and y is inversely proportional to z. What is the dependence between x and z?

3. The quantity x is inversely proportional to the quantity y, and y is directly proportional to z. What is the dependence between x and z?

4. The quantity x is inversely proportional to y, y is directly proportional to z, and z is directly proportional to u. What dependence exists between x and u?

22. LINEAR FUNCTION

531. The dependence between the length l of a metal bar and its temperature $t°$ is expressed by the formula $l = l_0(1+\alpha t°)$, where l_0 is the length of this bar at 0°, and α is a constant. How does l depends on $t°$? What is the physical meaning of the number α?

532. 1. How does the velocity v in a uniformly variable motion depend on time t in the cases $v_0 = 0$ and $v_0 \neq 0$?

2. Are the following assertions correct:

(a) every directly proportional dependence between y and x is a linear function y of x, (b) every linear function y of x is a directly proportional dependence of y on x?

533. On one and the same drawing construct the graphs of the given functions:

(a) $y = -\frac{1}{2}x$; (b) $y = -\frac{1}{2}x+3$.

1. Prove that the straight lines $y = -\frac{1}{2}x$ and $y = -\frac{1}{2}x+3$ are parallel.

2. Find the length of the y-intercept cut by the straight line $y = \frac{1}{2}x+3$.

534. 1. Using the graph $y = \frac{1}{2}x$, construct the following graphs:

(a) $y = \frac{1}{2}x+1$; (b) $y = \frac{1}{2}x-2$; (c) $y = \frac{1}{2}x-2.5$.

2. How many points is it sufficient to take to plot the graphs $y = kx$ and $y = kx+b$? How is it expedient to take the points?

3. Construct the graphs

(a) $y = 0.5x+2.5$; (b) $y = 0.5x-2.5$;

(c) $y = -0.5x+2.5$; (d) $y = -0.5x-2.5$

by plotting the points of their intersection with the coordinate axes.

535. 1. It is known that for $x = 2$ the linear function $y = 3x+b$ attains the value 7.5. Determine b.

2. The graph of the function $y = kx-4$ is known to pass through the point A $(-2, 5)$. Determine k.

536. (1) On one and the same drawing construct the graphs of the functions $y = 2x+3$ and $y = -2x+3$.

(2) What are the resemblance and the difference between the obtained graphs?

(3) Show that with an increase in x the function $y = 2x+3$ increases at a rate equal to 2, while the function $y = -2x+3$ decreases at a rate equal to -2.

537. On one and the same drawing construct the graphs of the following functions:

(1) $y = \frac{3}{2}x+3$; (2) $y = \frac{3}{2}x-3$; (3) $y = -\frac{3}{2}x+3$;

(4) $y = -\frac{3}{2}x-3$.

Find the coordinates of the points at which each graph intersects the x- and y-axes.

538. A cyclist rides uphill without using the pedals at a speed of $v = (12-1.5t)$ m/s. How much time will he ride until he stops? Illustrate the solution by a graph.

539. 1. Given two functions $y = 0.6x+5$ and $y = 2x-2$. Find the value of x for which both functions have one and the same numerical value. Solve the problem by computation and graphically.

2. Find the intervals within which the function $y = x-2$ (a) is positive, (b) is negative, (c) takes on values greater than 3, (d) takes on values not less than 4, (e) takes on values greater than 1, but less than 3.

540. 1. For what condition are the graphs of the linear functions $y = k_1x+b_1$ and $y = k_2x+b_2$ parallel?

2. Write the equation of a straight line parallel to the straight line $y = 0.7x-4$.

3. Find the condition for which the straight lines $a_1x+b_1y = c_1$ and $a_2x+b_2y = c_2$ (a) intersect, (b) are parallel, (c) coincide.

541. Write the equations of the sides of a square whose diagonals serve as the coordinate axes. The length of the side of the square equals a.

542. Add to the equation $x-y = 3$ a second equation such that the obtained system of equations (a) has a solution, (b) has no solution, (c) has a set of solutions.

543. 1. The graph of the function $y = kx+b$ passes through the points $M_1(2, 3)$ and $M_2(-5, -4)$. Find the values of the parameters k and b and plot the graphs.

2. Given a straight line $ax+by = c$, passing through the points $M_1(1, 1)$ and $M_2(-5, 5)$. Find the values of the parameters a, b, c and construct the straight line.

23. POWER FUNCTION WITH A RATIONAL EXPONENT

544. 1. Find the expression for the radius R of a circle depending on (a) the length of its circumference, (b) the area of the circle.

2. Express (a) the volume of a cube as a function of the area of its face, (b) the surface area of a cube as a function of its volume, (c) the area of a circle as a function of the length of its circumference, (d) the perimeter of a square as a function of its area.

545. 1. Construct on one drawing the graphs of the functions $y = \frac{6}{x}$ and $y = \frac{6}{x^3}$. What are the resemblance and the difference between these functions? (Compare the behaviour of the functions as $x \to 0$ and $x \to \infty$.)

2. Construct on one drawing the graphs of the functions $y = \frac{4}{x^2}$ and $y = \frac{4}{(x-2)^2}$. What are the resemblance and the difference between the graphs?

546. (1) Construct on one drawing the graphs of the functions $y = \frac{1}{2}x^2$ and $y = \frac{1}{2}x^4$.

(2) In what respects do the graphs resemble each other and differ?

547. 1. Which points are common (a) for all parabolas $y = x^n$, (b) for the parabolas $y = x^n$ with odd exponents, (c) for the parabolas $y = x^n$ with even exponents? In all the cases n is a natural number.

2. For what values of n does the parabola $y = x^n$ have an axis of symmetry? What is the axis of symmetry? What happens in this case to the value of the function when x is replaced by $-x$?

3. When does the parabola $y = x^n$ have a centre of symmetry? What happens then to the value of the function when replacing x by $-x$?

548. Construct on one drawing the graphs of the functions $y = \sqrt{x}$ and $y = \sqrt[4]{x^2}$. In what respect do the graphs resemble each other and differ? Solve a similar problem for the functions $y = \sqrt{x}$ and $y = \sqrt[3]{x}$.

549. Graph the following functions:

(1) $y = 2\sqrt{x}$; (2) $y = -3\sqrt[3]{x}$; (3) $y = -2\sqrt[3]{x^2}$;

(4) $y = \frac{2}{\sqrt{x}}$; (5) $y = -\frac{3}{\sqrt[3]{x}}$; (6) $y = \frac{2}{\sqrt[3]{x^2}}$.

550. 1. Construct the graph $y = x^2+2$ by adding the graphs $y = x^2$ and $y = 2$.

2. Construct the graphs $y = -x^2+1$, $y = -x^2-1$ by adding and subtracting the graphs $y = -x^2$ and $y = 1$.

3. Construct the graphs $y = x^2+x$, $y = x^2-x$ by adding and subtracting the graphs $y = x^2$ and $y = x$.

551. In a free fall of a body the distance S and the time t are connected by the relation $S = \frac{1}{2}gt^2$, where S is measured in metres, t in seconds, $g \approx 10$ m/s^2.

(1) Construct the graph of this function to a suitable scale.

(2) Determine graphically and by calculation what distance will be covered during 3 seconds?

(3) How many seconds are needed for the body to cover the distance of 20 metres?

552. 1. On one drawing construct the graphs:

$$y = x^2, \quad y = 2x^2, \quad y = 0.5x^2.$$

2. How does the shape of the parabola $y = ax^2$ change along with a change in the parameter a?

3. How does each of the given functions change when the argument x varies from $-\infty$ to 0 and from 0 to ∞?

4. For what value of x does each of the given functions have the least and the greatest value? Compute them.

553. According to the plan of the preceding problem, analyse the behaviour of the following functions:

$$y = -x^2, \quad y = -3x^2, \quad y = -\frac{1}{3}x^2.$$

554. 1. On one drawing construct the graphs of the functions:

$$y = \frac{1}{2}x^2, \quad y = \frac{1}{2}x^2+2, \quad y = \frac{1}{2}x^2-2.$$

2. Determine the values of x for which each of the functions has the least value and find these values.

3. Determine the intervals of increase and decrease.

555. 1. According to the plan of the preceding problem analyse the following functions:

$$y = -x^2, \quad y = -x^2+2, \quad y = -x^2-2.$$

2. Find the roots of these functions.

3. How should the parabola $y = x^2$ be displaced so that its new equation becomes $y = x^2+1, y = x^2-1, y = -x^2+5, y = -x^2-2$.

4. Write the equations of the parabolas obtained by displacements of the parabola $y = x^2$: (a) 3 units upward, (b) 4 units downward, (c) 1.2 units upward, (d) 1.3 units downward, (e) the direction of its branches is reversed.

5. Make a stencil of the parabola $y = x^2$. With the aid of the stencil construct the graphs of the functions (a) $y = x^2-\frac{1}{2}$; (b) $y = x^2+\frac{3}{2}$.

556. 1. On one drawing construct the graphs of the following functions:

$$y = \frac{1}{2}x^2, \quad y = \frac{1}{2}(x-3)^2, \quad y = \frac{1}{2}(x+3)^2.$$

2. On each curve find the point having the ordinate equal to 8, and compare the abscissas of the points on the graphs $y = \frac{1}{2}(x-3)^2$ and $y = \frac{1}{2}(x+3)^2$ with that of the corresponding point on the graph $y = \frac{1}{2}x^2$.

3. What can be said about the shape and location of the parabolas $y = \frac{1}{2}(x-3)^2$ and $y = \frac{1}{2}(x+3)^2$ as compared with the shape and location of the parabola $y = \frac{1}{2}x^2$?

4. Find the coordinates of the vertex of each of the parabolas.

5. Determine for what values of x each of the given functions increases, decreases, assumes the least or the greatest value.

557. Analyse the following functions as in the preceding problem:

(a) $y = -\frac{1}{3}x^2$; (b) $y = -\frac{1}{3}(x-2)^2$;

(c) $y = -\frac{1}{3}(x+2)^2$.

558. 1. Given the function $y = (x-3)^2$. Without drawing its graph, describe its properties.

(a) Describe the shape and location of the curve with respect to the coordinate axes.

(b) Will the function have the greatest or the least value and for what x? Compute it.

(c) For what values of x does the function y decrease, increase, vanish?

(d) At what points does the curve intersect the coordinate axes?

2. Describe the properties of the following functions:

(a) $y = (x+5)^2$; (b) $y = -(x-3)^2$; (c) $y = 2(x-2)^2$.

3. With the aid of the stencil of the parabola $y = x^2$ construct the graphs of the functions:

(a) $y = (x-3.5)^2$; (b) $y = (x+4)^2$.

559. 1. Represent the right-hand member of the function $y = x^2-6x+9$ in the form of a squared binomial; find the roots of the function; conclude on its sign; find the intervals of increase and decrease of the function, as also its greatest or least value.

According to the above plan analyse the following functions:

(a) $y = x^2+4x+4$; (b) $y = -x^2+2x-1$;

(c) $y = 4x^2-4x+1$; (d) $y = -x^2-5x-6.25$.

560. 1. On one drawing construct the graphs of the following functions: (a) $y = (x-2)^2$; (b) $y = (x-2)^2-4$. Describe the mutual location of these parabolas.

2. On one drawing construct the graphs of the following functions: (a) $y = (x-2)^2-4$; (b) $y = x^2$, and describe their location with respect to each other and to the coordinate axes.

3. For the function $y = (x-2)^2-4$ find the intervals of increase and decrease, the least or the greatest value, the intervals of constant sign, the coordinates of the vertex of the parabola $y = (x-2)^2-4$.

561. In the trinomials (1) $y = x^2+6x+12$; (2) $y = -x^2+6x-12$ single out the perfect squares and analyse them as in the preceding problem.

562. 1. Write the equation of each of the parabolas obtained by the displacements of

(a) the parabola $y = x^2$ 4 units leftward and 3 units upward;

(b) the parabola $y = -\frac{1}{2}x^2$ 3 units rightward and 3 units downward.

2. With the aid of the stencil of the parabola $y = x^2$ construct the graphs of the functions:

(a) $y = (x-1)^2+2$, (b) $y = (x+2)^2-3$.

563. 1. Single out the perfect square in the trinomials $y = x^2+px+q$ and $y = -x^2+px+q$.

Write the formulas for the coordinates of the vertices of these parabolas.

2. Determine the coordinates of the vertices of the following parabolas:

(a) $y = x^2-4x-5$; (b) $y = x^2+3x+5$; (c) $y = -x^2-2x+3$;

(d) $y = -x^2+5x-1$.

564. Transform the quadratic trinomial $y = 2x^2-4x-6$ to $y = 2(x-1)^2-8$.

Find the roots of this trinomial, the least or the greatest value, the intervals of increase and decrease, the intervals of variation of x within which $y > 0$, $y < 0$. Find the coordinate of the vertex of the parabola $y = 2x^2-4x-6$.

2. On one drawing construct the graphs of the following functions:

(a) $y = 2x^2$; (b) $y = 2(x-1)^2$; (c) $y = 2(x-1)^2-8$

and describe their location.

565. 1. Analyse the following trinomials as in the preceding problem:

(a) $y = 2x^2+8x+9$; (b) $y = 3x^2+9x$; (c) $y = -2x^2-8x+1$;

(d) $y = -2x^2-4x$.

2. What can be said about the shape of the parabolas $y = ax^2$ and $y = ax^2+bx+c$?

566. 1. Reduce the quadratic trinomial $y = ax^2+bx+c$ to the form $y = a(x+m)^2+q$.

Write the formulas for m and q, write the formulas for the coordinates of the vertex of the parabola $y = ax^2+bx+c$ and the formula for the greatest or the least value of the function $y = ax^2+bx+c$.

2. Using the obtained formulas, find the coordinates of the vertex of the parabola and the greatest or the least value of the functions

(a) $y = 2x^2-3x+4$; (b) $y = -3x^2+9x$;

(c) $y = -2x^2-8x+1$; (d) $y = x^2+2x+3$.

567. Write the equation of each of the parabolas obtained by the displacement of

(1) the parabola $y = 2x^2$ 4 units rightward and 3 units downward;

(2) the parabola $y = -3x^2$ 6 units leftward and 5 units downward.

568. 1. Given rectangles of equal perimeter of 20 cm. Find the sides of the one whose area is the greatest.

2. Break the number 12 into two positive parts so that the sum of their squares has the least value.

CHAPTER VIII

Quadratic Equations and Equations Reduced to Quadratics. Quadratic Inequalities

24. PURE QUADRATICS

569. Solve the following equations (orally):

(1) $x^2 = 25$; (2) $x^2 = 0.49$; (3) $x^2 = 6\frac{1}{4}$;

(4) $x^2+4 = 32$; (5) $x^2+2x = 0$; (6) $2x^2 = x$;

(7) $x^2-0.2x = 0$.

570. Compute the values of the roots to within 0.01:

(a) $x^2-3 = 0$; (b) $x^2-50 = 0$;

(c) $\frac{x^2}{2}-0.75 = 0$.

2. Solve the following equations determining the permissible values of the letters:

(a) $x^2-c^2 = 0$; (b) $a^2x^2-4 = 0$; (c) $ax^2-\frac{1}{a} = 0$;

(d) $ax^2-\frac{b^4}{a} = 0$; (e) $ax^2 = 9$; (f) $4x^2 = a^3$.

3. Are the following equations equivalent?

(a) $x^2+3x-5 = x^2+4$ and $3x-5 = 4$;

(b) $(x-2)x = x$ and $x-2 = 1$;

(c) $\frac{x^2-1}{x-2} = \frac{8}{x-2}$ and $x^2-1 = 8$;

(d) $\frac{x^2+3}{x+3} = \frac{3-3x}{x+3}$ and $x^2+3 = 3-3x$.

571. Solve the following equations:

(1) $4x^2+6x = 9x^2-14x$; (2) $\frac{4}{25}x^2 = 6\frac{1}{4}$;

(3) $2\frac{1}{4} = \frac{2}{9}x^2+2\frac{1}{8}$;

(4) $0.08x^2 = 0.4608$; (5) $5x^2-7.2 = 0$;

(6) $x(12-x) = 3(4x-3)$;

(7) $1.2x^2-2x = 0.9x^2-2(x-0.0135)$;

(8) $10(x+2)-19 = (1+5x)(1-5x)$.

572. 1. Find m if the root of the given equation equals zero:

(a) $3x^2-5x+m^2-4 = 0$; (b) $10x^2+4x-2m = 0$.

2. Write a quadratic equation one of whose roots would be equal to zero.

3. For what values of c does the equation $x^2 = c+2$ have two different real roots?

Solve the following equations:

573. (1) $\frac{4x^2-1}{3}-\frac{3x^2+8}{5} = 1$;

(2) $\frac{5x^2-48}{8}-\frac{33-2x^2}{6} = 3\frac{5}{6}$;

(3) $\frac{2x}{x+1}+\frac{2x}{x-1} = 5\frac{1}{3}$;

(4) $\frac{3x+5}{x-2}-\frac{x-2}{x+2} = -1\frac{1}{2}$;

(5) $\frac{x\sqrt{3}}{2x+\sqrt{3}} = \frac{2x}{2\sqrt{3}+2}$;

(6) $\frac{x\sqrt{3}}{2\sqrt{3}-x} = \frac{2x}{5-x\sqrt{3}}$.

574.

(1) $x^2-4a^2+4a-1=0$; (2) $(ax+b)^2+(a-bx)^2=2(a^2+b^2)$;

(3) $\frac{bx+a}{b}=\frac{ab}{b^2-x}$; (4) $\frac{a+b}{a-b}x^2=a^2-b^2$;

(5) $\frac{ax-b}{x+a}=\frac{x+b}{x-a}$; (6) $\frac{2x}{x-a}+\frac{2x}{x+a}=5\frac{1}{3}$.

25. AFFECTED QUADRATICS

575. 1. Solve the given quadratic equations by factoring their left member:

(a) $x^2-6x+8=0$; (b) $x^2+2x-8=0$.

2. Solve the following equations by singling out the perfect square of the binomial:

(a) $x^2-6x+8=0$; (b) $x^2+8x+15=0$;

(c) $x^2-2x-35=0$; (d) $x^2+11x+30=0$;

(e) $x^2-x-12=0$; (f) $x^2+px+q=0$.

576. Solve the following equations:

(1) $x^2-6x+8=0$; (2) $x^2+5x+6=0$;

(3) $x^2-x-2=0$; (4) $x^2-2x-8=0$;

(5) $x^2-17x-18=0$; (6) $x^2+4x+9=0$;

(7) $x^2-4x+2=0$; (8) $x^2-2x-2=0$;

(9) $x^2+\frac{1}{2}x-\frac{1}{2}=0$.

577. Transform the given equations to the reduced form and then solve them:

(1) $4x^2-8x+3=0$; (2) $9x^2+18x+5=0$;

(3) $3x^2+5x-78=0$; (4) $ax^2+bx+c=0$.

Solve the following equations:

578.

(1) $5x^2+8x+3=0$; (2) $3x^2+5x-2=0$;

(3) $3x^2-11x+6=0$; (4) $3x^2+2x-1=0$;

(5) $9x^2+9x+2=0$; (6) $4x^2+12x+9=0$.

579.

(1) $(x+3)(x-2)+(x+2)^2-3x-10=0$;

(2) $(x-5)^2+(3-x)^2-4(x+5)(3-x)-48=(x+1)^2$;

(3) $(x-1)(x-2)(x-3)-(x^2+3)(x-5)+2x-33=0$.

580.

(1) $\frac{x^2}{6}-\frac{2x}{3}=\frac{3x-10}{4}$; (2) $\frac{x(x+4)}{2}-3=\frac{7x}{4}-\frac{5x-4}{6}$;

(3) $\frac{(x-11)^2}{10}-\frac{(6x-1)^2}{5}=7-\frac{7x-3}{2}$;

(4) $\frac{x-3}{4}+\frac{2x+3}{6}=\frac{x^2-11}{12}$.

581. (1) $8x+11+\frac{7}{x}=\frac{21+65x}{7}$;

(2) $\frac{21}{v+5}=3\frac{2}{7}-\frac{v}{7}$;

(3) $\frac{12}{x}-\frac{7x-6}{6}+5x-26=0$;

(4) $\frac{5(x-1)}{8}=\frac{x}{10}+\frac{10}{x}$.

582.

(1) $\frac{1}{t}+\frac{1}{t^2}=4\left(\frac{1+t^2}{t^2}-\frac{3}{2t}\right)$;

(2) $\frac{x^2}{x-3}-\frac{x+6}{x-3}=1$;

(3) $\frac{6}{y^2-1}-\frac{2}{y-1}=2-\frac{y+4}{y+1}$;

(4) $\frac{3x}{2x-2}+\frac{x}{1-x}-\frac{9}{2x+2}=0$;

(5) $\frac{2}{3-z}-\frac{5}{1-z}+\frac{4}{(z-3)(z-1)}=7$;

(6) $\frac{6}{4x^2-1}-\frac{2}{2x-1}=1-\frac{3}{2x+1}$.

583.

(1) $\frac{y+2}{y+1}=\frac{y-2}{1-y}-\frac{4}{y-1}$;

(2) $7-\frac{48}{9x^2-1}=\frac{6x}{3x-1}-\frac{8}{3x+1}$;

(3) $\frac{8}{x-1}+\frac{x-9}{2x+2}=\frac{19-3x^2}{1-x^2}$;

(4) $\frac{3x-1}{x+2}-\frac{18}{2-x}=\frac{7x^2-28}{x^2-4}+\frac{7}{2+x}$.

584.

(1) $\frac{3x-2}{x-3}-\frac{x-4}{x+3}=\frac{15x-3}{x^2-9}$;

(2) $\frac{8}{6-z}=\frac{z}{10-z}+\frac{32}{z^2-16z+60}$;

(3*) $\frac{z^2+2z+2}{z+1}+\frac{z^2+8z+20}{z+4}-\frac{z^2+4z+6}{z+2}-\frac{z^2+6z+12}{z+3}=0$;

(4*) $\frac{3}{u}+\frac{1}{u-1}+\frac{4}{u-2}+\frac{4}{u-3}+\frac{1}{u-4}+\frac{3}{u-5}=0$.

585. (1) $\frac{1}{2x-2}-\frac{2x-1}{x^2+x+1}+\frac{3}{2x+2}=0$;

(2) $\frac{8y}{y+1}-\frac{6}{y^2+3y+2}-\frac{9}{y+2}=0$;

(3) $\frac{3x}{x^3-1}-\frac{5}{4x^2+4x+4}=\frac{11}{2(1-x)}$;

(4) $\frac{2}{x^2-x+1}-\frac{1}{x+1}=\frac{2x-1}{x^3+1}$.

586. (1) $2x^2+(3-2\sqrt{2})x-3\sqrt{2}=0$;

(2) $\sqrt{3}y^2-8\sqrt{2}y+4\sqrt{3}=0$;

(3) $16z^2-8\sqrt{2}z+1=0$;

(4) $x^2+2\sqrt{2}x+\sqrt{3}=0$.

587. Compute the roots of the equations to within 0.1:

(1) $x^2-0.2x-0.3=0$; (2) $3x^2+7x-1=0$;

(3) $4x^2+6x+1=0$; (4) $2x^2-15x+5=0$.

Solve the following equations:

588. (1) $(x^2-16x)^2-2(x^2-16x)-63=0$;

(2) $(x^2+3x-4)^2+(x^2+3x+2)^2-36=0$;

(3) $\left(\frac{1}{x}+2\right)^2+\frac{1}{x}-10=0$;

(4) $\left(\frac{1-x}{1+x}\right)^2-\frac{1-x}{1+x}-20=0$;

(5) $\left(\frac{x}{x+1}\right)^2+\left(\frac{x+1}{x}\right)^2=\frac{17}{4}$;

(6)* $(x^2-4x+5)^2-(x-1)(x-3)=4$.

589*. (1) $(x^2+x+1)(x^2+x+2)=12$;

(2) $\frac{x^2+2x+7}{x^2+2x+3}=x^2+2x+4$;

(3) $\frac{1}{x^2-2x+2}+\frac{2}{x^2-2x+3}=\frac{6}{x^2-2x+4}$;

(4) $x^2-4x+\frac{10}{x^2-4x+5}=2$.

In Problems 590 through 594 solve the literal equations with the following conditions: (1) the coefficient of the squared unknown is non-zero, (2) none of the denominators of the fractions entering the given equations is equal to zero, (3) the discriminant of the quadratic equation is a nonnegative number.

590.

(1) $x^2-5ax+6a^2=0$; (2) $x^2-2mx+m^2-n^2=0$;

(3) $ay^2-(a+1)y+1=0$; (4) $z^2-2(p+q)z+4pq=0$.

591. (1) $\frac{m}{x-n}-2=\frac{n}{m-x}$;

(2) $t+\frac{b}{a}=\frac{1}{t}+\frac{a}{b}$;

(3) $u-a=\frac{1}{a}-\frac{1}{u}$;

(4) $v+\frac{1}{v}=\frac{m-n}{m+n}+\frac{m+n}{m-n}$.

592. (1) $\frac{a}{x-a}-\frac{x}{x+a}=\frac{1}{3}$;

(2) $\frac{(t-a)^2+(t-b)^2}{(t-a)^2-(t-b)^2}=\frac{a^2+b^2}{a^2-b^2}$;

(3) $\frac{1}{a+b+x}=\frac{1}{a}+\frac{1}{b}+\frac{1}{x}$;

(4) $\frac{m}{x+m}+\frac{n}{x+n}+\frac{p}{x+p}=3$.

593. (1) $\dfrac{x(x+2b)}{a^2+2ab+b^2} = \dfrac{2x}{a+b} + \dfrac{b-a}{a+b}$;

(2) $\left(\dfrac{y+m}{y-m}\right)^2 + \dfrac{7}{2}\left(\dfrac{y+m}{y-m}\right) + 3 = 0$;

(3) $\left(\dfrac{a-x}{x-b}\right)^2 - 8\dfrac{a-x}{x-b} + 15 = 0$.

594. (1)* $\dfrac{m^2+2n}{ny+2n+my+2m} - \dfrac{m}{y+2} - \dfrac{y}{m+n} + \dfrac{y}{y+2} = 0$;

(2)* $\dfrac{z}{az-3a} - \dfrac{a+3}{az-3a-bz+3b} = \dfrac{1}{3-z} - \dfrac{z}{a^2-ab}$;

(3)* $\dfrac{x}{a^2+ab} - \dfrac{x}{ax+3a} + \dfrac{1}{x+3} = \dfrac{a+3}{ax+3a+bx+3b}$.

26. PROPERTIES OF THE ROOTS OF A QUADRATIC EQUATION

595. Find the sum and the product of the roots of the following equations:

(1) $x^2-8x+6 = 0$; (2) $32x^2+28x+3 = 0$;

(3) $3x^2-2ax-a^2 = 0$; (4) $x^2-2mx+m^2-n^2 = 0$.

596. Solve the given equations (orally):

(1) $x^2-3x+2 = 0$; (2) $x^2+6x+8 = 0$;

(3) $x^2-4x-5 = 0$; (4) $x^2-7ax+12a^2 = 0$.

Set up quadratic equations given their roots:

597. (1) 2 and 5; (2) 4 and -3; (3) 0 and 2;

(4) -5 and -4; (5) -2 and $+2$; (6) 3 and 3;

(7) $2+\sqrt{2}$ and $2-\sqrt{2}$; (8) 2 and $\frac{1}{2}$;

(9) -3 and $-\frac{2}{3}$; (10) -5 and $\frac{3}{5}$;

(11) 0.2 and 0.5; (12) $-\frac{5}{6}$ and $\frac{3}{5}$.

598. (1) $\sqrt{2}$ and $-\sqrt{3}$; (2) $2+\sqrt{2}$ and $-\sqrt{2}$; (3) a and a;

(4) $\sqrt{a}$ and $-\sqrt{a}$; (5) $a+b$ and $a-b$; (6) $2a$ and $-a$;

(7) 0 and $3m$; (8) $-\frac{a}{3}$ and $\frac{a}{5}$; (9) $\frac{a}{b}$ and $\frac{b}{a}$;

(10) $\frac{a-b}{a+b}$ and 1; (11) $a \pm \sqrt{b}$; (12) $3a \pm 2b\sqrt{5}$.

599. 1. Can a quadratic equation with rational coefficients have different irrational roots, equal irrational roots?

2. Can a quadratic equation with rational coefficients have one rational, and the other irrational root?

3. Can a quadratic equation with real coefficients have one real and one imaginary root?

4. Set up a quadratic equation with rational coefficients if one root is known:

(a) $2+\sqrt{3}$; (b) $5-\sqrt{5}$; (c) $\frac{1+\sqrt{3}}{2}$;

(d) $\frac{6-\sqrt{2}}{5}$; (e) $\frac{-m+\sqrt{n}}{2}$; (f) $a-b\sqrt{m}$.

600. Without using the formula for the roots of a quadratic equation find its second root if one of the roots is known:

(a) $x^2-x-12=0$, $x_1=4$; (b) $6x^2+5x+1=0$,

$x_1=-\frac{1}{3}$.

601. 1. Without finding the roots x_1 and x_2 of the equation $x^2+9x+14=0$ set up a new quadratic equation, whose roots are

(a) $2x_1$ and $2x_2$; (b) $\frac{x_1}{5}$ and $\frac{x_2}{5}$.

2. Set up a quadratic equation whose roots would be reciprocal to the roots of the equations:

(a) $x^2+33x-108=0$; (b) $x^2+px+q=0$;

(c) $ax^2+bx+c=0$.

602. 1. Set up a quadratic equation such that its roots (a) exceed by three the roots of the equation $x^2+6x+8=0$, (b) are less than the roots of the equation $x^2+4x+4=0$ by two, (c) exceed by p the roots of the equation $x^2+px+q=0$, (d) equal the sum and the product of the roots of the equation $x^2+4x-5=0$, of the equation $2x^2-5x+4=0$.

2. Given the equation $x^2-4x+3=0$. Without solving this equation, set up a new one such that its roots (a) are opposite to the roots of the given equation, (b) are the squared roots of the given equation.

603. Express in terms of p and q (or in terms of a and b):

(1) the sum and the difference of the squared roots of the equations

(a) $x^2+px+q=0$; (b) $ax^2+bx+c=0$;

(2) the sum and the difference of the cubed roots of the equations

(a) $x^2+px+q=0$; (b) $ax^2+bx+c=0$.

604. Without solving the equation $x^2-3x-10=0$, compute: (a) the sum and the product of its roots, (b) the sum of their squares, (c) the difference of their squares, (d) the square of their difference, (e) the sum of their cubes, (f) the difference of their cubes.

605. 1. The equation $x^2+px+3=0$ has the roots x_1 and x_2. For what p is (a) the difference of the roots of the equation equal to 2, (b) the sum of the squared roots of the equation equal to 19, (c) the difference between the squared roots of the equation equal to $12\sqrt{6}$?

2. The equation $x^2-4x+q=0$ has the roots x_1 and x_2. For what q (a) is one root of the equation three times greater than the other, (b) are the roots equal to each other, (c) are the roots reciprocal by magnitude?

3. Find m in the equation $x^2+(m+3)x-16=0$, knowing that the roots of this equation are opposite numbers.

606. 1. Solve the equation $x^2-8x+q=0$ if the sum of the squared roots is equal to 34.

2. In the equation $x^2-8x+q=0$ determine the value of q for which its roots x_1 and x_2 satisfy the equation $3x_1+5x_2=20$.

3. In the equation $4x^2+bx+5=0$ determine the value of b for which its roots satisfy the system of equations $\begin{cases} 3x_1+x_2=-8 \\ x_1+3x_2=-4. \end{cases}$

607. In the equation $(a-2)x^2-(5-a)x-5=0$ determine a if it is known that $x_1-x_2=2\sqrt{6}$.

2. The roots of the equation $x^2+3x+m=0$ are x_1 and x_2.

What values should the parameter m assume so that

(a) $3x_1-x_2=4$; (b) $\frac{x_1}{x_2}=-\frac{2}{5}$; (c) $x_1=x_2$, (d) $\frac{x_1+x_2}{x_1-x_2}=\frac{3}{2}$.

608. 1. Without using the formula for the roots solve the equation $mnz^2-(m^2+n^2)z+mn=0$.

2. Define a so that one of the roots of the equation $x^2-\frac{15}{4}x+a=0$ becomes the square of the other.

609*. Knowing that x_1 and x_2 are the roots of the equation $x^2+px+q=0$, set up an equation such that its roots are:

(1) $1+x_1$ and $1+x_2$; (2) x_1^2 and x_2^2;

(3) $\frac{2}{x_1}-1$ and $\frac{2}{x_2}-1$; (4) $x_1+\frac{1}{x_2}$ and $x_2+\frac{1}{x_1}$;

(5) $\frac{x_1}{x_2}$ and $\frac{x_2}{x_1}$; (6) $\frac{x_1-x_2}{x_1+x_2}$ and $\frac{x_1+x_2}{x_1-x_2}$.

610. 1. Find the roots of the following quadratic binomials and trinomials:

(a) $8x^2-6x$; (b) $4x^2-64$; (c) $3x^2+5$;

(d) $x^2-8x+15$; (e) $4x^2+20x+25$; (f) $2x^2-3x+2$.

2. Given two numbers: (a) -2 and -3, (b) 2 and $-\frac{1}{3}$, (c) 0 and 3, (d) 2 and 5. Write several quadratic trinomials with these numbers as their roots.

611. Factorize the following quadratic trinomials:

(1) $x^2+10x+9$; (2) $x^2+2x-15$; (3) $-x^2+2x+35$;

(4) $-x^2+x+42$; (5) $8x^2+10x+3$; (6) $x^2-25x+114$;

(7) $n+mn(2-n)x-2m^2n^2x^2$;

(8) $2x^2+(3a+4b)x+a^2+3ab+2b^2$.

612. Factorize the following:

(1) $2x^2-ax-a^2$; (2) $x^2-2ax+a^2$;

(3) $2a^2x^2+abx-b^2$; (4) $2m^2-5mny+3n^2y^2$;

(5) $4x^2-2bx+ab-a^2$;

(6) $2a^3b^3+ab(2a-b)x-x^2$;

(7) $2mnx^2+(2m^2+2mn-3n^2)x+2m^2-3mn$.

613. Reduce the following fractions:

(1) $\frac{a^2-3a+2}{a^2+2a-8}$; (2) $\frac{2a^2-8a-90}{3a^2+36a+105}$;

(3) $\frac{a^2+ab+2b^2}{a^2+9ab+14b^2}$; (4) $\frac{3a^2-5ab-8b^2}{2a^2+3ab+b^2}$;

(5) $\frac{2m^2-3mn+n^2}{3mn-m^2-2n^2}$; (6) $\frac{x^2+(\sqrt{2}-1)x-\sqrt{2}}{\sqrt{2}+(1-3\sqrt{2})x-3x^2}$.

614. 1. Show that the trinomial x^2+px+q can be represented in the form $\left(x+\frac{p}{2}\right)^2+q-\frac{p^2}{4}$, and the trinomial ax^2+b+c in the form $a\left[\left(x+\frac{b}{2a}\right)^2+\frac{4ac-b^2}{4a^2}\right]$.

2. For what condition does the trinomial (a) x^2+px+q, (b) ax^2+bx+c represent a perfect square?

3. Find the value of b in the trinomial (a) $10x^2+bx+40$, (b) $3x^2+bx+16$ if it is known that the latter is a perfect square.

27. ANALYSING THE ROOTS OF A QUADRATIC EQUATION

615. Without solving the equations draw the conclusions on their roots: (a) whether they are real, different or equal, or imaginary, (b) what signs have the roots if they are real, (c) if the roots have different signs, then which of them (positive or negative) is greater in absolute value:

(1) $x^2-7x+12=0$; (2) $x^2+3x+4=0$;

(3) $4x^2-6x+9=0$; (4) $3x^2+4x+2=0$;

(5) $3x^2-8x-4=0$; (6) $x^2+8x+20=0$;

(7) $x^2+14x+49=0$; (8) $12x^2+4x-1=0$;

(9) $9x^2-12x+4=0$.

616. For what value of c does the equation

(1) $x^2+3x+c=0$ have equal roots;

(2) $3x^2+x-c=0$ have different real roots;

(3) $2x^2-8x+c+10=0$ have different real roots;

(4) $4x^2+x+2c=0$ have real roots;

(5) $x^2-2x+3c=0$ have no real roots?

617. 1. For what value of b does the equation

(a) $2x^2+bx+2=0$ have equal roots;

(b) $3x^2-bx+1=0$ have equal roots;

(c) $x^2+2bx+3=0$ have different real roots;

(d) $2x^2-bx+5=0$ have different real roots;

(e) $3x^2+bx-3=0$ have real roots;

(f) $5x^2-bx+2=0$ have no real roots?

2. For what k do the equations

(a) $x^2-2(k-4)x+k^2+6k+3=0$;

(b) $(2k+1)x^2+3(k-1)x-k+1=0$

have two equal roots?

618. 1. Determine for what condition the roots of the equation $ax^2+bx+c=0$ where a, b, and c are rational numbers, are (a) rational, (b) irrational?

2. Without solving the equations, indicate which of them have rational roots and which irrational:

(a) $7x^2+9x+2=0$; (b) $x^2-6x-10=0$; (c) $x^2-6x-16=0$; (d) $7x^2+10x+3=0$.

619. 1. For what integral values of a are the roots of the equation $ax^2+(2a-1)x+a-2=0$ rational?

2. For what values of k are the roots of the equation $(k+3)x^2+(3+2k)x+5+k=0$ real numbers?

620. Set up a quadratic equation (a) having positive roots, (b) having roots of different signs, with the negative root greater in absolute value than the positive one.

621. For what condition will the positive root of the quadratic equation $ax^2+bx+c=0$, where $a>0$, be greater (less) in magnitude than the negative root?

622. 1. In what case may one assert, without computing the discriminant, that a quadratic equation has real roots?

2. Write a quadratic equation having real roots.

3. What is the value of the discriminant of the quadratic equation (a) $x^2+px+q=0$; (b) $ax^2+bx+c=0$, whose left member is a perfect square?

4. What dependence must exist between the coefficients of the equation $ax^2+4x+c=0$ so that the sum of its roots is half their difference?

623. 1. Prove that the roots of the equation $x^2+2ax+a^2-b^2-c^2=0$ are real if a, b and c are real numbers.

2. For what p does the equation

(a) $(p+2)x^2+(2p+3)x+p-1=0$;

(b) $(p+1)x^2-(2p+3)x+p+2=0$

have real roots?

3. Prove that the equation $3x^2+2(a+b+c)x+a^2+b^2+c^2=0$ cannot have real roots if a, b, c are not equal to one another.

28. SETTING UP QUADRATIC EQUATIONS

624. The product of two successive natural numbers is equal to 156. Find these numbers.

625. One of the digits of a two-digit number exceeds the other by 2, and the sum of the squares of this number and the number obtained by interchanging its digits equals 4034. Find this number.

626. What base must a number system have so that the number 307 is written as 616?

627. The numerator of a fraction is less than its denominator by 4. The sum of this fraction and its reciprocal is equal to $2\frac{16}{21}$. Find this fraction.

628. By what divisor should the number 206 be divided so that the quotient exceeds the divisor by 5, and the remainder is less than the divisor by 10.

629. Find the two-digit number knowing that the units digit of the sought-for number is less than its tens digit by 4, and that the product of the number by the sum of its digits equals 306.

630. In multiplying two positive numbers, one of which exceeds the other by 40, the pupil made a mistake by reducing the hundreds digit in the product by 5. Dividing then the obtained product by the smaller factor he got 145 as the quotient with 50 as the remainder. Find these numbers.

631. A river ship went with the stream 150 km and came back. Determine the rate of the river flow if the round trip took 16 hours and the speed of the ship in still water equals 20 km/h.

632. A construction site of a rectangular shape should be fenced in. One side of the rectangle is 20 m longer than the other, the fence must be 2 m high. Determine how many cubic metres of boards 4 cm thick are required to build the fence if the area of the site is 2400 m^2 and the cut-off pieces of the boards constitute 4 per cent of their total amount.

633. If the perimeter of a square is reduced by 40 m, then its area becomes equal to $\frac{16}{9}$ of the initial area. Determine the perimeter of the given square.

634. A rectangle, whose one side is 11 m longer than the other, is transformed into an equal rectangle whose larger side becomes equal to 10 m, and the smaller one becomes 2 m longer. Determine the area and the sides of the rectangle.

635. Inside a rectangle with the sides 2 m and 1.5 m another rectangle is placed so that the distance between their sides is equal over the entire perimeter. The area of the frame thus obtained is 2.32 m^2 smaller than the area of the smaller rectangle. Determine the width of the frame and its area.

636. The plan of a house has the shape of a rectangle 21 m long and 10 m wide. Determine the thickness of its walls if the inside area of the house, the partitions included, equals 180 m^2.

637. From a rectangular sheet of tin, by cutting out squares of equal size at each corner and then bending the tin to form sides, a box without a cover is made. Determine the original measurements of the sheet of tin, given that its length is 18 cm greater than its width, and the height of the box is 6 cm and its capacity is 8.64 l.

638. Two bodies are moving along different sides of a right angle in the direction of its vertex. Initially the bodies are 60 and 80 cm, respectively, from the vertex of the angle. Two seconds after the commencement of movement the distance between the bodies equals 25 cm. Determine the speeds of movement of the bodies, given that the second body is moving 7.5 cm/s faster than the first. Can the bodies meet at the vertex of the angle?

639. For the repair of a technical school gymnasium two kinds of paint are purchased, the first to the amount of 36 roubles, the second to the amount of 24 roubles. Six kilograms more of the se-

cond kind of paint are purchased, but it is 70 kop. cheaper per kilogram. How much paint of each kind is purchased?

640. The total milk production at a certain sovkhoz for a one-year period comprises 820 t, whereas at another sovkhoz with identical working conditions the annual milk production from all cows comprises 1050 t, despite the fact that there are 60 fewer cows than in the first sovkhoz. The annual yield of milk per cow in the second sovkhoz exceeds by 1 t the annual yield in the first sovkhoz. Determine the number of livestock in each sovkhoz and calculate the annual milk production per cow in each herd.

641. At a certain kolkhoz, thanks to advanced methods of planting and cultivation, 680 t of potatoes are harvested from one plot of land. Another plot of land, where these advanced methods of planting and cultivation are not applied, yields the same harvest (680 t), but the total land area is larger by 45 ha, since productivity of potatoes using ordinary methods is 9 t less a hectare than when employing advanced methods. Determine the productivity of potatoes for 1 ha for each plot of land.

642. A convoy of automobiles is assigned to transport 60 t of freight from a warehouse to a river port. Because of unfavourable weather conditions it is necessary to reduce the load for each car by 0.5 t from the original estimate. To compensate, four cars are added to fill out the convoy. How many cars are in the original convoy?

643. Make up a problem whose solution will yield the following equation:

$$\text{(a)}\ \frac{90}{x}-\frac{90}{x+5}=1; \quad \text{(b)}\ x(x-2)=80.$$

644. During the 1971 soccer championship the teams in the top league played 240 games for the season in two rounds. How many teams made up the top league?

645. Two gears, which form a pair, are turning such that one gear completes 1 revolution more per minute than the other. The larger gear takes one second longer than the smaller to complete 1/5 revolution. How many revolutions does each gear make in a minute?

646. The distance between cities A and B is 150 km. Two automobiles set out simultaneously from city A for city B. The first car goes 10 km/h faster than the second car and therefore arrives 30 minutes earlier at B. Determine the speed of each car.

647. A tourist walked 9 km at a given speed; subsequently, reducing his speed by 2 km/h, he walked 10 km further. If he had walked the entire distance at the same speed as he had walked the first 9 km, he would have been en route 50 minutes less. Determine the original speed of the tourist.

648. A passenger travels 60 km on a train, spends 40 minutes on business at the terminal station and makes the return trip on a train which travels 3 km/h faster than the first train. In total $3\frac{1}{4}$ h are expended on the trip. Determine the speed of each train.

649. A passenger train departs from city A at 6 : 00 P.M. for city B. At 9 : 00 P.M. an express train, whose average speed exceeds that of the passenger train by 15 km/h, leaves city B for city A. The trains meet each other mid-route. At what time do they meet, given that the distance between the cities is 1080 km?

650. Two tourists on bicycles depart simultaneously from point A for point B. The first tourist has planned his trip such that each day he travels exactly the same number of kilometers. The second tourist travels on the average 16 km farther each day than the first tourist. How many days is each tourist en route if the distance between points A and B is 480 km and the first tourist arrives at point B one day later than the second?

651. The distance between cities A and B by railroad is 200 km. A train goes from A to B, making 4 three-minute stops en route: at distances of 40, 85, 135 and 180 km from A. The train travels the first 60 km on an uphill, the next 100 km on level ground and the remaining 40 km again on an uphill. The average rate of speed of the train on the uphill is 10 km/h less than its speed on level ground. Work out a schedule of the train's movement from A to B, given that the trip without stops takes 4.5 h.

652. Points A and B are located on the bank of a river. A postman delivers mail from A to B in a motorboat, crossing against the current a number of kilometers equal to the value of k in the equation $x^2-(2k+1)x+k^2+5=0$ in which the discriminant is equal to 1. Determine the speed of the river's current, given that the trip there and back takes 1 h 6 min 40 s, and the speed of the boat in still water is 9.6 km/h.

653. In travelling a distance of 175 m the front wheel of a carriage has made 20 more revolutions than the rear wheel, whose circumference is 1 m larger than the circumference of the front wheel. Find the circumference of each wheel.

654. Two teams of technical school students are charged with landscaping the dormitory yard. Working together they can complete the work in 3 days. One of the teams, working separately, can complete the work 2.5 days faster than the second team. How many days are required for each team, working separately, to complete the work?

655. Two pipes conduct water into a swimming pool. When each pipe works separately the first pipe fills the pool 12 h faster than the second pipe. After the first pipe has been left running for 10 h, it is turned off, the second pipe is turned on and it fills the remaining part of the pool in 16 h. How many hours are required for each pipe, operating independently, to fill the empty pool?

656. A swimming pool has three water pipes. The first two, working together, fill the pool in the same amount of time that is required for the third pipe alone to fill the pool. The second pipe, operating alone, fills the pool 5 h faster than the first pipe and 4 h slower than the third. How much time is required for each pipe, operating alone, to fill the pool?

657. A bathtub has two taps. Water flows in through the first tap and flows out through the second. If both taps are turned on the empty tub will be filled in 24 min. How much time is required for the filled tub to empty itself if only the second tap is turned on and if it is known that it takes 2 min longer for the second tap to empty the filled tub than for the first tap to fill the empty tub?

658. Three identical tanks are filled with gas. The first tank is filled through tap A and the second through tap B, which has a greater capacity than tap A. It is further known that the second tank is filled 22 min faster than the first. The third tank is filled in 1 h by two taps which have the same capacity as taps A and B. How much time is required for the first and second tanks to be filled?

659. A worker, working alone, can complete all his work 5 days faster than another worker performing the same work and also working alone. If the first worker works two times faster and the second worker three times more slowly, then working together they can complete the work in 4.5 days. How many days are required for each worker, working independently, to complete the work?

660. A team of construction workers planned to complete a construction project four days faster than another team working on a similar project. In how many days can each team complete

the project if, previously, working together, they managed to complete five such projects in 24 days.

661. Two excavators, working simultaneously, can complete an excavating project in 3 h 45 min. The more powerful excavator, working alone, completes the job 4 h faster than the other. How much time is needed for each excavator, working separately, to complete the excavation project?

662. Two stonemasons are charged with laying a wall. After working together for 5.5 days one of the workers transfers to another job and the other finishes laying the wall in 1.5 days. If the work were to be executed by one stonemason alone, then the second worker would require 3 days longer than the first to complete the job. How many days are required for each of them, working independently, to construct the wall?

663. Two workers are charged with making a certain number of machine parts. It happens that one worker sets to work 1 hour later than the other and at the end of the shift each of them made 360 parts. On the following day they begin to work at the same time and in 3 h they make 630 parts. How much time is required for each worker, working independently, to make 720 parts?

664. Two road construction teams are charged with the construction of a highway between points A and B. For a period of 40 days the teams work separately, in shifts; working thus, one team completes 1/3 of the work and the other 1/6 of the work. On day 41 the teams begin to work together and they complete the remaining section of road in 18 days. Determine how many days are required for each team, working separately, to construct the highway.

665. Two alcohols of varying strength are combined to form 10 buckets of 79° mixture. The first alcohol is 85° strength, and the strength of the second alcohol amounts to a number 66 times greater than the number of buckets of this alcohol in the mixture. How many buckets of each alcohol are needed to form the mixture?

666. In a two-year period the population of a city increased from 200,000 to 220,500 persons. Find the average annual percentage of growth of the city's population.

667. One part of a sum of money totalling 2000 roubles produces an annual interest of 30 roubles, while the remaining part produces 15 roubles in interest. This second part earns 1% more interest than the first. What percent interest does each part of the money earn per annum?

668. After two successive raises at the same rate of increase (in per cent) an employee's salary rises from 80 roubles to 96 roubles 80 kop. What is the percentage rate of increase in salary in each instance?

669. From a vessel holding 40 l and filled with alcohol, a certain quantity of alcohol is poured out and an equivalent amount of water is poured in. From the resulting mixture the same amount is again poured out and an equivalent amount of water is poured in. The resulting alcohol in the vessel has a strength of 25°. How many liters of liquid is poured out each time?

670. The Komsomol committee of a technical school organizes a work day to plant m trees around a classroom building. Since n participants more than expected show up at the work day, each participant plants p fewer trees than it was originally planned. How many participants take part in the work day?

671. A military unit makes a vehicular march moving at a constant speed of v. The length of the column of vehicles on the march is d km. The commander of the unit, at the head of the column, dispatches a motorcycleman to the rear of the column with an order; the motorcycleman transmits the order and immediately sets out back, returning to the commander in t min. Determine the average speed of the motorcycleman.

672. A kolkhoz has two orchards, apple and cherry. Each orchard has n trees, planted in rows. In each row of the cherry orchard there are m trees more than in each row of the apple orchard; however, the number of rows in the cherry orchard is p fewer than in the apple orchard. How many rows of trees are there in each orchard, and how many trees are there in each row?

673. One worker spends m min more than a second worker to lay one square meter of parquet. How many square meters of parquet does each worker lay in k hours, given that during that time the second worker can lay p m^2 of parquet more than the first?

674. Two automobiles, A and B, with constant speeds of v_1 and v_2, move along two streets toward an intersection. At a certain time the cars are at distances from the intersection equal to a_1 and a_2 ($a_1 > a_2$). After what interval of time will the distance of car B from the intersection be 2 times less than the distance between the cars?

29. HIGHER-DEGREE EQUATIONS WHOSE LEFT MEMBERS ARE FACTORIZABLE

Solve the following equations:

675. (1) $(x+2)(x-3)(2x-5)(x+3)^2 = 0$;

(2) $x^4-4x^2 = 0$; (3) $x^4-8x = 0$;

(4) $3x^4+x^3-12x^2-4x = 0$;

(5) $x^3-x^2-4x+4 = 0$;

(6) $x^5-x^4-5x^3+5x^2+4x-4 = 0$.

676. (1) $(a^2x^2-b^2)(a^2-b^2x^2) = 0$;

(2) $x^4+2x^3-x-2 = 0$; (3) $2y^3-7y^2+7y-2 = 0$;

(4) $x^3-(a^2+1)x+a = 0$;

(5) $z^5-a^2z^3+a^3z^2-a^5 = 0$.

677. (1) $(2z+3)^3+(z-1)^3-27z^3-8 = 0$;

(2) $\frac{y^3+3y^23y+1}{y^3+y^2+y+1} = \frac{8}{5}$;

(3) $x^4+4x^3+4x^2-9 = 0$;

(4)* $x^4-6x^3+8x^2-10x-25 = 0$.

678. (1) $x^3+1 = 0$; (2) $8x^3-27 = 0$; (3) $x^4-81 = 0$;

(4) $16x^4-1 = 0$; (5) $x^6-1 = 0$; (6) $x^8-1 = 0$;

(7) $x^4+1 = 0$; (8) $x^6+1 = 0$.

30. BIQUADRATIC EQUATIONS

Solve the following equations:

679. (1) $x^4-13x^2+36 = 0$; (2) $x^4-6x^2+8 = 0$;

(3) $x^4-5x^2-36 = 0$; (4) $36x^4-13x^2+1 = 0$;

(5) $4y^4+7y^2-2 = 0$; (6) $81x^4-45x^2+4 = 0$.

680. (1) $x^4-4x^2-m^2x^2+4m^2 = 0$;

(2) $4x^4+a^2 = x^2+4a^2x^2$;

(3) $x^4-x^2-a^4x^2+a^4 = 0$;

(4) $x^4-a^2x^2 = b^2x^2-a^2b^2$.

681. (1) $x^4-2ax^2+a^2-b^2=0$;

(2) $x^4-13mx^2+36m^2=0$;

(3) $a^2x^4-2ax^2-a^2+1=0$;

(4) $x^4-2(a^2+b^2)x^2+(a^2-b^2)^2=0$.

682. (1) $x^2(x-4)(x+4)=24-2x^2(x^2+5)$;

(2) $\dfrac{3(x^2+1)}{2-x}+2+x=\dfrac{9}{x^2(2-x)}$;

(3) $\dfrac{5x^3-5x^2+1}{x^2}+\dfrac{4(2-x^2)}{x+1}+\dfrac{10}{x(x+1)}=\dfrac{1+10x}{x^2}$;

(4) $\dfrac{x^2+1}{a^2(x+1)}+\dfrac{4(x-1)}{a^2}+\dfrac{a^2}{x^2(x+1)}=\dfrac{3}{x}-\dfrac{3}{x^2}+\dfrac{3}{x+1}$.

683*. Analyse the roots of the following equations without finding them:

(a) $4x^4-8x^2+3=0$; (b) $x^4-5x^2+10=0$;

(c) $x^4-5x^2+4=0$; (d) $x^4-x^2-6=0$;

(e) $x^4+4x^2+5=0$; (f) $x^4-6x^2+9=0$.

684. 1. Prove that the sum of all the roots of the biquadratic equation $x^4+px^2+q=0$ equals zero, and their product is q.

2. Prove that the sum of all the roots of the biquadratic equation $ax^4+bx^2+c=0$ is equal to zero and their product to $\dfrac{c}{a}$.

685. 1. Set up a biquadratic equation given its roots: (a) 3, 4, -3, -4; (b) $2, \sqrt{3}, -2, -\sqrt{3}$; (c) $\pm 3, \pm\sqrt{2}$.

2. Set up a biquadratic equation given its two roots: (a) -1 and 2; (b) $-\sqrt{3}$ and 3; (c) $\sqrt{3}$ and $\sqrt{5}$; (d) $\sqrt{a}$ and 1.

686. 1. Knowing that x_1 and x_2 are the roots of the equation $x^2+px+q=0$, set up an equation whose roots are $x_1, x_2, -x_1, -x_2$.

2. Same for the equation $ax^2+bx+c=0$.

687. Factorize the following:

(1) x^4+4x^2-5; (2) x^4-11x^2+18; (3) x^4-6x^2+9;

(4) x^4-4x^2+3; (5) $4x^4+5x^2+1$; (6) $2x^4-19x^2+9$;

(7) $x^4-(a+b)x^2+ab$; (8) $x^4-(1-ab)x^2+ab$.

688. Reduce the following fractions:

(1) $\frac{x^4-x^2-12}{x^4+8x^2+15}$; (2) $\frac{2a^4+7a^2+6}{3a^4+3a^2-6}$;

(3) $\frac{5a^4+5a^2-3a^2b-3b}{a^4+3a^2+2}$.

31. SECOND-DEGREE INEQUALITIES

689. For what values of x do the following functions have positive (negative) values or vanish:

(1) $y = 2x^2+3x-5$; (2) $y = -2x^2+10x-12$;

(3) $y = 5x^2+x+1$; (4) $y = 4x^2-4x+1$?

Solve the following inequalities:

690. (1) $x^2-5x+4 > 0$; (2) $x^2+4x < 0$;

(3) $2x^2-7x-15 \geqslant 0$; (4) $12x^2-17x-105 < 0$;

(5) $x^2-4x > 0$; (6) $x^2+6x+9 \leqslant 0$.

691. (1) $12x^2-4x+3 < 0$; (2) $3x^2+2x+1 > 0$;

(3) $x^2+13x+36 \leqslant 0$; (4) $x^4-4x^2+4 \leqslant 0$;

(5) $-2+x-3x^2 > 0$; (6) $-5+4x-3x^2 < 0$.

692. (1) $x^2-3x > 10$; (2) $4x^2+9 > 12x$;

(3) $4x-x^2 < 5$; (4) $(x-5)x+4x > 2$;

(5) $(x+5)x \leqslant 2(x^2+2)$; (6) $(x+4)(x+5)-5 \geqslant 5$;

(7) $\frac{1}{3}x^2-3x+6 < 0$; (8) $2(x+2)^2-3.5 \geqslant 2x$;

(9) $2x > \frac{x^2}{2}-5x+5.5$.

693. (1) $\frac{x-3}{x+2} > 0$; (2) $\frac{x-2}{x+2} \geqslant \frac{2x-3}{4x-1}$;

(3) $\frac{x^2-2x+3}{x^2-4x+3} > -3$; (4) $\frac{x^2+6x+15}{x-4} > 0$;

(5) $\frac{x^2-2x-3}{x^2+2x+8} < 0$; (6) $2-\frac{x-2}{x-1} > \frac{x-3}{x-2}$.

694. (1) $\frac{x^2-4x+4}{2x^2-3x-5} > 0$; (2) $\frac{3x^2-x+2}{x^2-2x+3} < 2$;

(3) $x+\frac{2}{x} > -3$; (4) $\frac{5-7x}{x+1} < x$;

(5) $\frac{x-10}{5+x^2} < \frac{1}{2}$; (6) $\frac{x+1}{1-x}-2 < -\frac{1-x}{x}$.

695. (1) $\frac{1}{x+1}+\frac{2}{x+3} > \frac{3}{x+2}$;

(2) $\frac{6}{x-1}-\frac{3}{x+1}-\frac{7}{x+2} < 0$;

(3) $\frac{2x}{2x^2+7x+5} > \frac{x}{x^2+6x+5}$.

696*. (1) $|x^2-2x-3|-2 > 0$;

(2) $(x-3)^2-|x-3|-2 < 0$;

(3) $\frac{x+3}{|x-4|}+2 > 0$; (4) $\frac{|x^2-2x|-3}{x^2+4x-12} < 0$.

697*. (1) $x^4-13x^2+36 < 0$; (2) $x^4-29x^2+100 > 0$;

(3) $20-121x^2-25x^4 < 0$;

(4) $x^4+10x^3+4x^2-5x < 0$.

698. Solve the following systems of inequalities:

(1) $\begin{cases} x^2-2x-3 > 0 \\ x^2-11x+28 \geqslant 0; \end{cases}$ (2) $\begin{cases} x^2-\frac{1}{4} > 0 \\ -2x^2+5x-3 > 0; \end{cases}$

(3) $\begin{cases} 3x^2-4x+1 > 0 \\ 3x^2-5x+2 \leqslant 0; \end{cases}$ (4) $5 < x^2-8x+25 < 18$;

(5) $0 < x^2-3x+2 < 6$;

(6)* $\left(x-\frac{1}{4}\right)^2+4\left(x-\frac{1}{4}\right)+3 < 0$.

699. (1) $\begin{cases} x^2-14x+45 < 0 \\ x^2-11x+30 > 0 \\ 2x-3 > 0; \end{cases}$

(2) $\begin{cases} 3x^2-2x > 3x+2 \\ 2x^2-6 < 7x-x^2 \\ -\frac{2}{3} \leqslant x \leqslant 2.5; \end{cases}$ (3)* $\begin{cases} |x^2-3x|-4 > 0 \\ x^2-3x-18 < 0 \\ |x-2| > 3; \end{cases}$

(4) $\begin{cases} x^2-x-20 < 0 \\ x^2-2x-8 > 0 \\ 2x^2+x-45 < 0. \end{cases}$

700. 1. Find the even values of x satisfying the system of inequalities $\begin{cases} x^2-8x-20 < 0 \\ 5x-1 > 3x-8. \end{cases}$

2. Find the odd values of x satisfying the system of inequalities $\begin{cases} (x-3)^2+2x > 32 \\ x^2 < 400. \end{cases}$

701. 1. Find the integral values of x satisfying the system of inequalities $\begin{cases} x^2-4x-77 < 0 \\ x^2 \geqslant 4. \end{cases}$

2*. Find graphically the pairs of integral values of x and y satisfying the system of inequalities $\begin{cases} y+\frac{1}{2} > |x^2-2x| \\ y \leqslant 2-|x-1|. \end{cases}$

702. For which values of x are the following inequalities valid:

$$\sin\varphi = x^2-6x+9, \quad \text{where} \quad 0 \leqslant \varphi \leqslant 90°$$

$$\tan\varphi = \frac{x^2-3x+2}{x^2-2x+1}, \quad \text{where} \quad 0 < \varphi < 45°?$$

703. Find the domains of definition of the following functions:

(1) $y = \sqrt{5x^2-6x+1}$; (2) $y = \sqrt{-3x^2+6x-3}$;

(3) $y = \sqrt[4]{4+8x-5x^2}$; (4) $y = \dfrac{3}{\sqrt[4]{3x^2-6x+3}}$;

(5) $y = \dfrac{4}{\sqrt[3]{3x-2x^2-2}}$; (6) $y = \dfrac{x+3}{\sqrt[6]{-3x^2-4x-3}}$;

(7) $y = \dfrac{\sqrt{2x^2-x-3}}{\sqrt[4]{2x^2-7x+6}}$; (8) $y = \sqrt{\dfrac{x-1}{x^2-x-2}}$;

(9) $y = \sqrt[3]{\dfrac{2x-1}{x^2+4x+3}}$.

704. For what values of a will the following equations have (a) real and equal roots, (b) real and different roots, (c) no real roots?

(1) $ax^2+(1+3a)x+a = 0$;

(2) $(2-3a)x^2+2ax-a+1 = 0$;

(3) $(a-4)x^2+(1+a)+2a-1 = 0$.

705. For what values of m are the following inequalities satisfied for any real values of x?

(1) $x^2-4x+m > 5$; (2) $x^2-(m+2)x+8m+1 > 0$;

(3) $x^2+4x+(m-2)^2 \geqslant 0$; (4) $\frac{1}{24}x^2+mx-m+1 > 0$;

(5) $mx^2-12x-5 < 0$; (6) $(m+2)x^2+5x-4 < 0$;

(7) $\left|\dfrac{x^2+mx+1}{x^2+1}\right| < 2$; (8) $-3 \leqslant \dfrac{x^2+mx-2}{x^2-x+1} \leqslant 2$.

706*. Solve the given inequalities considering all real values of the parameter a:

(1) $ax^2-x-1 > 0$; (2) $2x^2+ax+3 < 0$;

(3) $x^2-2x+a > 0$; (4) $ax^2+ax-5 < 0$.

707. On the coordinate plane select the points whose coordinates satisfy the given systems of inequalities:

(1) $\begin{cases} y > x^2 \\ y < x; \end{cases}$ (2) $\begin{cases} y > x^2 \\ 3y-x < y; \end{cases}$ (3) $\begin{cases} y > x^2-1 \\ y < 1-x^2; \end{cases}$

(4) $\begin{cases} y > x^2-5x+6 \\ y < 5. \end{cases}$

32. IRRATIONAL EQUATIONS

708. Which of the given equations are irrational:

(1) $\sqrt{3}+2x = 4$; (2) $\sqrt{x}+7 = x$; (3) $\sqrt{x^2}-3 = 15x$;

(4) $\sqrt{x+4}-2x = \sqrt{a}$; (5) $(\sqrt{x}-2\sqrt{x+3})^2 = 4$;

(6) $\sqrt[3]{28-\sqrt{3a}} = 2x$.

709. Determine the domain of permissible real values of the unknowns of the following equations:

(1) $\sqrt{x-3}=4$; (2) $\sqrt{x+1}-\sqrt{x-4}=5$;

(3) $\sqrt{x+1}+\sqrt{4-x}=4$; (4) $\sqrt{2x-6}-\sqrt{1+x}=2$;

(5) $\sqrt[3]{2x}=x$; (6) $\sqrt{20-\sqrt{x^4+30x^2}}=x+1$;

(7) $\sqrt{x^2-4}=\sqrt{x}$; (8) $\sqrt{x^2+2x+22}=2x+11$.

710. 1. Without solving the equations explain why they cannot have roots:

(1) $\sqrt{x-1}+\sqrt{x-2}=0$; (2) $\sqrt{x^2+4}+\sqrt{x-1}=-3$;

(3) $\sqrt{4-x}+\sqrt{x-4}=5$; (4) $\sqrt{x}-\sqrt{-3-x}=1$;

(5) $\sqrt{x-3}-\sqrt{1-x}=\sqrt[3]{x^2+x+2}$; (6) $\sqrt{x}+\sqrt{x-6}=2$.

2. Determine the domain of permissible values of x and prove that the equation $\sqrt{x-1}+\sqrt{1-x^2}=\sqrt[3]{x^2+7}-2$ has the unique root $x=1$.

711. Are the following pairs of equations equivalent?

(1) $3x=x+2$ and $(3x)^2=(x+2)^2$;

(2) $3x=x+2$ and $(3x)^3=(x+2)^3$;

(3) $\sqrt{3x-2}=x-3$ and $3x-2=(x-3)^2$;

(4) $\sqrt{x-2}=4$ and $x-2=16$.

Solve the following equations:

712. (1) $\sqrt{x+5}=3$; (2) $\sqrt{5x+1}=\sqrt{2x+10}$;

(3) $\sqrt{x^2-4}=\sqrt{5}$; (4) $(3x+8)^{\frac{1}{3}}=(x^2-2)^{\frac{1}{3}}$;

(5) $\sqrt{4+x}-\sqrt{x-4}=0$; (6) $(25+\sqrt{x-4})^{\frac{1}{5}}=2$.

713. (1) $\sqrt{2x+1}=1-x$; (2) $\sqrt{x-1}\sqrt{10x+5}=x+3$;

(3) $\sqrt{(2x-1)(3x+1)}=x+1$;

(4) $\sqrt[3]{14x-1}\sqrt[3]{x-1}=x+1$;

(5) $\sqrt[4]{x^2-5}\sqrt[4]{6x^2+10}=4$.

714. (1) $\sqrt{4x-3}=\frac{3x-1}{\sqrt{3x-5}}$; (2) $\frac{2-5x}{\sqrt{2-x}}=3\sqrt{6+x}$;

(3) $\sqrt[3]{3x-4}\sqrt[3]{9x^2-5x+1}=3x-2$;

(4) $\sqrt[4]{x^2+1}\sqrt[4]{9x^2-1}=\sqrt{x^2+3}$.

715. (1) $\sqrt{x^2-11}=2x-7$; (2) $\sqrt[3]{x^3-5x-4}=x-1$;

(3) $2\sqrt[3]{x+3}=\sqrt[3]{x^2+39}$; (4) $\sqrt{11-\sqrt[3]{x^2+7}}=3$;

(5) $\sqrt[3]{24+\sqrt{x^2+5}}=3$;

(6) $\sqrt{x^2-2x-1}+5=\frac{14}{\sqrt{x^2-2x-1}}$.

716. (1) $\sqrt{2+\sqrt{x+5\sqrt{2x^2+3x+2}}}=2$;

(2) $\sqrt{1+y\sqrt{y^2-24}}=y-1$;

(3) $\left(\sqrt{\sqrt{z}+1}-1\right)\sqrt{\sqrt{z}+1}-\sqrt{z}=0$;

(4) $\sqrt{\sqrt{x+3}+x+1}=2\sqrt{2x-1}$;

(5) $\sqrt{\sqrt{x^4-14x^2+5x-1}+32+10x}=x+5$;

(6) $\sqrt{8+\sqrt[3]{2x-\sqrt{4x^2-5x+4}}}=3$;

(7) $\sqrt{1.5-x\sqrt{x^2-5.25}}=2$;

(8) $\sqrt{6-\sqrt[3]{2x+\sqrt{x^2+6x}}}=2$.

717. (1) $\sqrt{y+4}+\sqrt{y-4}=\frac{y+7}{\sqrt{y+4}}$;

(2) $\sqrt{x+10}=\frac{15}{\sqrt{x+10}}=\sqrt{x}$;

(3) $(t+5)^{\frac{1}{2}}-24(t+5)^{-\frac{1}{2}}+(5t-63)^{\frac{1}{2}}=0$;

(4) $\sqrt{x}-\frac{18}{\sqrt{x+12}}+\sqrt{x+12}=0$;

(5) $2\sqrt{4x-3}-\sqrt{5x+1}=\frac{3(x-1)}{\sqrt{4x-3}}$;

(6) $3\sqrt{2x+13}+\sqrt{x+3}=\frac{10(x+5)}{\sqrt{2x+13}}$.

718. (1) $\frac{\sqrt{x}-3}{\sqrt{x}-4}=\frac{\sqrt{x}-5}{\sqrt{x}-8}$; (2) $\frac{3(4-\sqrt{x})}{3\sqrt{x}-8}=\frac{2\sqrt{x}}{0.5(\sqrt{x}+1)}$;

(3) $\frac{3\left(5x^{\frac{1}{2}}-7\right)}{3x^{\frac{1}{2}}-5}=\frac{4.5x^{\frac{1}{2}}}{3-\sqrt{x}}$;

(4) $\frac{2\sqrt{x}+1}{3-\sqrt{x}}=\frac{11-3\sqrt{x}}{5\sqrt{x}-9}$.

719. (1) $\frac{\sqrt{5+x}+\sqrt{5-x}}{\sqrt{5+x}-\sqrt{5-x}}=\frac{5}{x}$;

(2) $\frac{y+\sqrt{24-y}}{y-\sqrt{24-y}}=\frac{\sqrt{2}+1}{\sqrt{2}-1}$;

(3) $\frac{x+\sqrt{x^2-1}}{x-\sqrt{x^2-1}}+\frac{x-\sqrt{x^2-1}}{x+\sqrt{x^2-1}}=98$;

(4) $\frac{\sqrt{17+x}+\sqrt{17-x}}{\sqrt{17+x}-\sqrt{17-x}}=\frac{x}{2}$.

720. (1) $\frac{(\sqrt{z+7}-\sqrt{z+3})^2}{\sqrt{z+7}+\sqrt{z+3}}=2$;

(2) $\frac{2}{1-\sqrt{1-x^2}}+\frac{2}{1+\sqrt{1-x^2}}=9$;

(3) $\frac{x+\sqrt{x^2-4}}{x-\sqrt{x^2-4}}-\frac{x-\sqrt{x^2-4}}{x+\sqrt{x^2-4}}=3\sqrt{x^2-4}$;

(4) $\frac{\sqrt{x}}{\sqrt{3-0.25x}+\sqrt{0.25x}}=\frac{\sqrt{x}}{\sqrt{3-0.25x}-\sqrt{0.25x}}+4=0$.

721. (1) $\frac{8}{\sqrt{x}}-\frac{x}{\sqrt{x}+\sqrt{4-x}}=\frac{x}{\sqrt{x}-\sqrt{4-x}}$;

(2) $\frac{\sqrt{4-x}+\sqrt{x-2}}{\sqrt{4-x}-\sqrt{x-2}}=\frac{1}{3-x}$;

(3) $\sqrt{2+x}+\sqrt{3+2x}=2\sqrt[4]{(2+x)(3+2x)}$;

(4) $\left(1+\sqrt{\sqrt{x}-2}\right)\sqrt{\sqrt{x}-2}=\sqrt{x}$.

722. (1) $\sqrt{2x+5}-\sqrt{10x+5}=2$;

(2) $\sqrt{4x+8}-2=\sqrt{3x-2}$;

(3) $\sqrt{x+8}-\sqrt{7x+9}+1=0$;

(4) $\sqrt{7-x}+\sqrt{x-5}=\sqrt{2}$;

(5) $\sqrt{10+y}=\sqrt{20}-\sqrt{10-y}$;

(6) $\sqrt{2z+4}-\sqrt{7-z}=3$.

723. (1) $\sqrt{41-3x}-\sqrt{9-3x}=2\sqrt{5+x}$;

(2) $\sqrt{y-2}+\sqrt{2y+3}=\sqrt{3y+31}$;

(3) $2\sqrt{z-2}-\sqrt{2z+3}=\sqrt{z-10}$;

(4) $\sqrt{13u-30}-2\sqrt{u-3}=3\sqrt{u-2}$.

724. (1) $\dfrac{a}{\sqrt{x}+1}=\sqrt{x}-1$; (2) $\dfrac{a-1}{\sqrt{x}+1}=1$;

(3) $\dfrac{m}{\sqrt{x}+2}=\sqrt{x}-2$;

(4) $\sqrt{m+x}-\sqrt{m-x}=\sqrt{2m}$;

(5) $\sqrt{m-x}+\sqrt{x-n}=\sqrt{m-n}$.

725. (1)* $\sqrt{4(x+m)+n}-\sqrt{4(m-x)+n}=4\sqrt{m}$;

(2) $\sqrt{5y-3a+4b}+\sqrt{5y-3a+4b}=2\sqrt{y+a}$;

(3) $\dfrac{\sqrt{2z+k}+\sqrt{2(z-k)}}{\sqrt{2z+k}-\sqrt{2(z-k)}}=\dfrac{4z-k}{3k}$.

726. 1. Prove the identities:

(a) $(a+b)^3=a^3+b^3+3ab(a+b)$;

(b) $(a-b)^3=a^3-b^3-3ab(a-b)$.

2. Using the above formulas solve the following equations:

(1) $\sqrt[3]{y+1}+\sqrt[3]{y+2}+\sqrt[3]{y+3}=0$;

(2) $(\sqrt{x}+3)^{\frac{1}{3}}+(13-\sqrt{x})^{\frac{1}{3}}=4$;

(3) $(10x+3)^{\frac{1}{3}}-(10x-5)^{\frac{1}{3}}=2$;

(4) $\sqrt[3]{2x-1}+\sqrt[3]{x-1}=1$;

(5) $(\sqrt{x}-4)^{\frac{1}{3}}+(\sqrt{x}+4)^{\frac{1}{3}}=4^{\frac{1}{3}}$;

(6) $\sqrt[3]{2x}+\sqrt[3]{x-4}=2$.

Solve the following equations:

727. (1) $x+2\sqrt{x}-8=0$; (2) $2\sqrt[3]{z^2}-5\sqrt[3]{z}-3=0$;

(3) $2x^2+\sqrt{2x^2-8}=20$;

(4) $(2x-3)^{\frac{1}{2}}+6=5(2x-3)^{\frac{1}{4}}$;

(5) $x^2-3x-\sqrt{x^2-3x+5}=1$;

(6) $3x^2+15x+2\sqrt{x^2+5x+4}=4$.

728. (1) $\sqrt{3x^2-5x+18}-\sqrt{-3x^2-5x+3}=3$;

(2)* $x^2+3-\sqrt{2x^2-3x+2}=\frac{3}{2}(x+1)$;

(3)* $(x-2)^2-4(x-2)+\sqrt{x}(4-\sqrt{x})=0$;

(4)* $(x-4)^2+4x-27=2\sqrt{x^2-4x+4}$.

729. (1) $\frac{x+1}{x}-\sqrt{\frac{x+1}{x}}-2=0$;

(2) $\left(\frac{x+9}{x}\right)^{\frac{1}{2}}+\left(\frac{x+9}{x}\right)^{-\frac{1}{2}}=2.05$;

(3) $\sqrt[3]{\frac{5x+4}{x-1}}+\sqrt[3]{\frac{x-1}{5x+4}}=2\frac{1}{2}$;

(4)* $3\frac{\sqrt{x+2}+\sqrt{x+7}}{\sqrt{x+7}}=10\frac{\sqrt{x+7}+\sqrt{x+2}}{\sqrt{x+2}}$.

730*. (1) $\sqrt{x^2-2x+1}=x^2-3$;

(2) $\sqrt{(x-2)^2}+\sqrt{(x+1)^2}=\sqrt{(x+2)^2}$;

(3) $\sqrt{x^2-4x+4}-\sqrt{x^2-2x+1}=\sqrt{x^2-6x+9}$.

731*. (1) $\sqrt{4x^2+9x+5}-\sqrt{x^2-1}=\sqrt{2x^2+x-1}$;

(2) $\dfrac{1}{\sqrt{3x-2}}+\dfrac{1}{\sqrt{x+2}}=\dfrac{4}{\sqrt{3x^2+4x-4}}$;

(3) $\dfrac{x-8}{\sqrt{3x-4}}=\sqrt{2x-5}-\sqrt{x+3}$;

(4) $\sqrt{2x+4}-\sqrt{8-4x}=\dfrac{12x-8}{\sqrt{6x+20}}$.

732. (1) $\sqrt[4]{\dfrac{1-y}{1+y}}\sqrt{\dfrac{1-m}{1+m}}+\sqrt[4]{\dfrac{1+y}{1-y}}=2\sqrt[4]{\dfrac{1-m}{1+m}}$;

(2)* $\sqrt[7]{1+\dfrac{x}{p}}+\sqrt[7]{\dfrac{p+x}{q}}=\sqrt[7]{\dfrac{x}{pq}}$;

(3) $\sqrt[3]{x+n}-\sqrt[3]{x-n}=\sqrt[3]{m}$;

(4) $\sqrt[3]{a+\sqrt{x}}+\sqrt[3]{a-\sqrt{x}}=\sqrt[3]{b}$.

Solve the following inequalities:

733*. (1) $\sqrt{x^2-55x+250}<x-14$;

(2) $2\sqrt{x(x+7)}<29-2x$;

(3) $\sqrt{x^2-3x+2}>2-x$;

(4) $\sqrt{2x+1}-\sqrt{x+8}>3$;

(5) $\sqrt{x+1}+\sqrt{x+2}<3$;

(6) $\sqrt[3]{x^3-3x^2+5x-6}>x-2$.

33. SYSTEMS OF SECOND-DEGREE EQUATIONS

734. 1. Solve the equation $x^2+y^2=9$ with respect to y and using the given table of values of x find the corresponding values of y (to within 0.1).

x	-3	-2	-1.5	-1	-0.5	0	1	1.5	2	3
y										

2. Construct the points whose coordinates are equal to the found solutions.

3. Join the constructed points with a smooth curve and prove that the locus of the points whose coordinates satisfy the equation $x^2+y^2 = 9$ is a circle with the origin as centre and radius equal to 3.

4. Prove that the circle of radius r, whose centre is at the origin, has the equation $x^2+y^2 = r^2$.

5. Prove that each of the following equations is that of a circle:

(a) $2x^2+2y^2 = 8$; (b) $\frac{x^2}{3}+\frac{y^2}{3} = 27$; (c) $0.1y^2 = 10-0.1x^2$.

735. Solve the equation $xy = 24$ with respect to y and plot the graph of the function. Judging from the equation and from the graph, consider the following properties of this function:

(1) x is an arbitrary real number, except for zero;

(2) the behaviour of the function y as $x \to 0$ and $x \to +\infty$;

(3) the presence of two axes of symmetry $y = x$ and $y = -x$.

736. 1. Solve the equation $x^2-y^2 = 9$ with respect to y and show that (a) x cannot attain the values less than 3 in absolute value, (b) each value of x is accompanied by two values of y which are equal in absolute value but have opposite signs, (c) y vanishes for $x = \pm 3$.

2. Solve the equation $x^2-y^2 = 9$ for the following values of x, computing y to within 0.1:

x	-7	-6	-5	-4	-3	3	3.5	4	5	6	7	8
y												

3. Construct the found points and connect them with a smooth curve, taking into account the results of the analysis of the equation.

4. Show that the coordinate axes serve as the axes of symmetry of the curve.

5. Construct the straight lines $y = x$, and $y = -x$ and, using the drawing, show that the branches of the curve approach these straight lines (its asymptotes) indefinitely.

737. Find graphically the coordinates of the points of intersection of the

(1) straight line $2x-y+3 = 0$ and the parabola $y = x^2-5$;

(2) straight line $x-y = 1$ and the parabola $x^2-y = 7$;

(3) parabola $y = -x^2+3$ and the hyperbola $xy+2 = 0$.
Verify the answers by solving the systems of equations.

738. Solve analytically and graphically the following systems of equations:

(1) $\begin{cases} xy = 12 \\ 3x-2y = 12; \end{cases}$ (2) $\begin{cases} x^2-y = 8 \\ y+5 = 1; \end{cases}$ (3) $\begin{cases} x^2+y^2 = 25 \\ x-7y = 25. \end{cases}$

739*. 1. For what real values of x and y does a polynomial takes on its least value? Find the least values of the given polynomials:

(a) $2x^2-2xy+5y^2+5$; (b) $5x^2+8xy+5y^2-2x+2y$.

2. For what real values of x and y does a polynomial yield its greatest value? Find the greatest values of the given polynomials:

(a) $-5x^2-2xy-2y^2+14x+10y-1$;

(b) $-5x^2-5y^2+8x-6y-1$.

Solve the following systems of equations:

740. (1) $\begin{cases} x^2-y = 1 \\ x-y = -5; \end{cases}$ (2) $\begin{cases} x^2-xy+y^2 = 39 \\ x-y = 7; \end{cases}$

(3) $\begin{cases} x^2+y^2 = 25 \\ 3x-y = 15; \end{cases}$ (4) $\begin{cases} 6x^2+y^2-5y = 0 \\ 3x+y = 0; \end{cases}$

(5) $\begin{cases} xy = 24 \\ x+2y = 14; \end{cases}$ (6) $\begin{cases} x^2+xy = 10 \\ 2x+y = -7. \end{cases}$

741. (1) $\begin{cases} 5x^2-3y^2+2xy-4x-33 = 0 \\ x+y = 3; \end{cases}$

(2) $\begin{cases} 5xy+3x^2-4y^2 = 38 \\ x-y = 2; \end{cases}$

(3) $\begin{cases} 3x^2+2xy+5y^2 = 6 \\ 5x+6y = 1; \end{cases}$

(4) $\begin{cases} 2x^2-3xy-y^2+2x-2y+1 = 0 \\ 2x-3y = -1. \end{cases}$

742. (1) $\begin{cases} x+y=-a \\ xy=-2a^2; \end{cases}$ (2) $\begin{cases} y-x=2b \\ xy=a^2-b^2; \end{cases}$ (3) $\begin{cases} x^2+y^2=5a^2 \\ x-y=a; \end{cases}$

(4) $\begin{cases} x^2+xy+y^2=\frac{7}{4}a^2 \\ 4x-2y=-7a; \end{cases}$ (5) $\begin{cases} x+y=2a \\ \frac{1}{2}(x^2-2a^2+y^2)=1. \end{cases}$

743. (1) $\begin{cases} x^2-y^2=3a^2 \\ 2x+3y=a; \end{cases}$ (2) $\begin{cases} \frac{x+y}{x}=a \\ \frac{xy}{a^2-1}-1=a; \end{cases}$

(3) $\begin{cases} \frac{m}{x+m}+\frac{y}{m+1}=\frac{3}{2} \\ y-x=1; \end{cases}$ (4) $\begin{cases} \frac{x+y}{4a}+\frac{b}{x-y}=1 \\ x+y=2a. \end{cases}$

744. (1) $\begin{cases} \frac{4}{x-1}-\frac{5}{y+1}=1 \\ \frac{4}{x+5}=\frac{2}{y}; \end{cases}$ (2) $\begin{cases} \frac{3}{x+5}+\frac{5}{y-3}=2 \\ \frac{1}{x-1}=\frac{1}{5-y}; \end{cases}$

(3) $\begin{cases} \frac{1-x+x^2}{1-y+y^2}=7 \\ \frac{x-1}{y+1}=1; \end{cases}$ (4) $\begin{cases} \frac{3x+5}{y+3}+\frac{x}{y}=4 \\ x-y=4. \end{cases}$

745. (1) $\begin{cases} \frac{1}{3x}+\frac{1}{2y}=4 \\ 3x+2y=1; \end{cases}$ (2) $\begin{cases} \frac{2x+y}{2y-x}+\frac{y}{x}+\frac{5}{4}=0 \\ 2x+4y=0; \end{cases}$

(3) $\begin{cases} \frac{3x+7y}{2x-y}+\frac{4x-2y}{3x+y}=10\frac{1}{2} \\ 2x-3y=1; \end{cases}$

(4) $\begin{cases} \frac{xy}{(x+y)(5x+y)}=\frac{1}{12} \\ y-3x=2. \end{cases}$

746. (1) $\begin{cases} x^2+y^2+x+y=2 \\ 2x^2-y^2+2x-y=4; \end{cases}$

(2) $\begin{cases} 2x^2-5xy+3x-2y=2 \\ 5xy-2x^2+7x-8y=-22; \end{cases}$

(3) $\begin{cases}(x-y)(9-x)=10\\(x-y)(12-y)=20;\end{cases}$

(4) $\begin{cases}2x^2-3xy+5y-5=0\\(x-2)(y-3)=0.\end{cases}$

747. (1) $\begin{cases}x^2-3y^2-5x-y+14=0\\(x+3)(y-2)=y^2+y-6;\end{cases}$

(2) $\begin{cases}(x+y)^2-5(x+y)+4=0\\(x-y)^2-(x-y)-2=0;\end{cases}$

(3) $\begin{cases}x+xy+y=5\\x^2+xy+y^2=7;\end{cases}$ (4) $\begin{cases}xy-x-y=29\\x^2+y^2-(x+y)=72.\end{cases}$

748. (1) $\begin{cases}2x^2+xy-2x-y=5\\2x^2-3xy-2x+3y=1;\end{cases}$

(2) $\begin{cases}2xy+6x-y^2-3y=14\\2x^2+4x-xy-2y=35;\end{cases}$

(3) $\begin{cases}x^2-5y^2-3x-y+24=0\\(x-3)(y-2)=y^2-3y+2;\end{cases}$

(4) $\begin{cases}x^2-3xy+y^2=61\\xy=-12.\end{cases}$

749. (1) $\begin{cases}x^2+y^2+3xy-4(x+y)+3=0\\xy+2(x+y)-5=0;\end{cases}$

(2) $\begin{cases}x^2+xy+y^2=75\\x^2-xy+y^2=25;\end{cases}$ (3) $\begin{cases}x-3y+2xy=3\\y-x-xy=-3;\end{cases}$

(4) $\begin{cases}x^2-y^2+2x-3y=5\\2y^2-x^2+4y-2x=-5.\end{cases}$

750. Using the properties of the roots of a quadratic equation, solve the following systems:

(1) $\begin{cases}x+y=5\\xy=4;\end{cases}$ (2) $\begin{cases}2x+3y=7\\xy=2;\end{cases}$ (3) $\begin{cases}x+5y=-2\\5xy=-8;\end{cases}$

(4) $\begin{cases}x-y=7\\xy=18;\end{cases}$ (5) $\begin{cases}2y-x=2\\2xy=3;\end{cases}$ (6) $\begin{cases}5x-3y=5\\5xy=-2.\end{cases}$

751. (1) $\begin{cases} x^2-xy+2y^2=0 \\ 2x^2+3xy-5y^2=0; \end{cases}$ (2) $\begin{cases} 2x^2+xy-y^2=0 \\ 2x^2-3xy-2y^2=0; \end{cases}$

(3) $\begin{cases} 2x^2+xy-y^2=0 \\ 2x^2-3xy+y^2=0; \end{cases}$ (4) $\begin{cases} 2x^2-xy+y^2=0 \\ 2x^2+xy+y^2=0. \end{cases}$

752. (1) $\begin{cases} x^2-y^2=0 \\ x^2+y^2=4; \end{cases}$ (2) $\begin{cases} x^2+y^2-10y+20=0 \\ 2x^2-xy-y^2=0; \end{cases}$

(3) $\begin{cases} 2x^2+xy-y^2=0 \\ x^2+y^2-10y+21=0; \end{cases}$

(4) $\begin{cases} 12x^2-7xy-12y^2=0 \\ x^2+y^2-2x-14y=0. \end{cases}$

753. (1) $\begin{cases} x^2-xy+3y^2=3 \\ 2x^2+5xy-7y^2=0; \end{cases}$ (2) $\begin{cases} 2x^2-3xy+3y^2=80 \\ x^2+xy-2y^2=-56; \end{cases}$

(3) $\begin{cases} 3x^2-3xy+2y^2=8 \\ 2x^2-3xy+3y^2=8; \end{cases}$ (4) $\begin{cases} 2x^2-xy+y^2=28 \\ x^2+3xy-3y^2=28. \end{cases}$

754. (1) $\begin{cases} 2x^2+3xy+2y^2=4 \\ 4x^2-4xy-y^2=8; \end{cases}$ (2) $\begin{cases} x^2-xy+y^2=3 \\ 2x^2-xy-y^2=5; \end{cases}$

(3) $\begin{cases} x^2+xy+y^2=75 \\ x^2-xy+y^2=25; \end{cases}$ (4) $\begin{cases} x^2-5xy+2y^2=8 \\ x^2+y^2=2. \end{cases}$

755. (1) $\begin{cases} 4x^2-4xy+y^2+2x-y=0 \\ 4x^2+4xy+y^2+2x+y=0; \end{cases}$

(2)* $\begin{cases} x^2+y^2-2x=a^2+b^2-2a \\ 2x-y=2a-b; \end{cases}$

(3) $\begin{cases} x^2-6xy+9y^2-3x+9y=18 \\ (3x-y)^2-8(3x-y)=20; \end{cases}$

(4) $\begin{cases} (x+y)^2-4n(x+y)=4a^2-8an \\ (x-y)^2-(a+1)(x-y)=4n^2-2an-2n. \end{cases}$

Introducing an auxiliary unknown, solve the following systems

756. (1) $\begin{cases} \dfrac{x}{y}+\dfrac{y}{x}=2\dfrac{1}{12} \\ 4x^2+9y^2=5; \end{cases}$ (2) $\begin{cases} \dfrac{x+y}{x-y}-\dfrac{x-y}{x+y}=4.8 \\ \sqrt{x^2-2y^2}=1 \end{cases}$

(3)* $\begin{cases} \left(x+\frac{1}{y}\right)^2+\frac{y^2}{(1+xy)^2}=17\times 2^{-2} \\ y^{-2}-3x^2=-2; \end{cases}$

(4) $\begin{cases} x^2-2y+3\sqrt{x^2-2y}-4=0 \\ x^2+4y-19=0. \end{cases}$

757. (1) $\begin{cases} \sqrt{\frac{x}{y}}-\sqrt{\frac{y}{x}}=\frac{5}{6} \\ x-y=5; \end{cases}$

(2) $\begin{cases} \sqrt{\frac{16x+3y}{4x}}-\sqrt{\frac{4x}{16x+3y}}=\frac{15}{4} \\ x^2-xy+y^2=241; \end{cases}$

(3) $\begin{cases} \sqrt{\frac{2x-y}{2y-x}}+\sqrt{\frac{2y-x}{2x-y}}=2 \\ \sqrt{x}+\sqrt{y}=4; \end{cases}$

(4)* $\begin{cases} \frac{x-5y}{5y+x}-2\sqrt{\frac{(x-5y)(x+1)}{xy+5y^2}}+\frac{x+1}{y}=1 \\ y-x=(x+y)^0. \end{cases}$

758. (1) $\begin{cases} \left(\frac{\sqrt{x}+1}{\sqrt{y}+1}\right)^{\frac{1}{2}}+\left(\frac{\sqrt{x}+1}{\sqrt{y}+1}\right)^{-\frac{1}{2}}=2 \\ xy=1; \end{cases}$

(2) $\begin{cases} \sqrt{\frac{x^2-a^2}{y^2-a^2}}+\sqrt{\frac{y^2-a^2}{x^2-a^2}}=\frac{4a^2+1}{2a} \\ x^2+4a^2y^2-12a^4+a^2=0; \end{cases}$

(3) $\begin{cases} y^2+\sqrt{3y^2-2x+3}=\frac{2}{3}x+5 \\ 3x-2y=5; \end{cases}$

(4) $\begin{cases} \frac{x+\sqrt{x^2-y^2}}{x-\sqrt{x^2-y^2}}+\frac{x-\sqrt{x^2-y^2}}{x+\sqrt{x^2-y^2}}=\frac{17}{4} \\ x(x+y)+\sqrt{x^2+xy+4}=52. \end{cases}$

Solve the following systems:

759. (1)* $\begin{cases} x^2+4xy-2y^2=5(x+y) \\ 5x^2-xy-y^2=7(x+y); \end{cases}$

(2) $\begin{cases} x+xy+xy^2 = 7 \\ xy^2+xy^3+xy^4 = 28; \end{cases}$ (3)* $\begin{cases} x^3 : y+xy = 25 \\ y^3 : x+xy = 16; \end{cases}$

(4)* $\begin{cases} x^2+y^2-m(x+y)+2n(m-2n) = 0 \\ xy-2n(x+y)+4n^2 = 0. \end{cases}$

760. (1) $\begin{cases} x^2+y^2 = m^2 \\ \dfrac{x}{y} = \dfrac{a}{b}; \end{cases}$ (2) $\begin{cases} \dfrac{y^2}{x^2} = \dfrac{m^2}{n^2} \\ a-y = b-x; \end{cases}$

(3) $\begin{cases} \dfrac{\sqrt{x}+\sqrt{y}}{\sqrt{x}-\sqrt{y}} = \dfrac{a}{b} \\ xy = (a^2-b^2)^2; \end{cases}$ (4) $\begin{cases} \dfrac{x}{y}-xy = a \\ \dfrac{y}{x}-xy = \dfrac{1}{a}. \end{cases}$

761. (1) $\begin{cases} x^3+y^3 = 35 \\ x^2-xy+y^2 = 7; \end{cases}$ (2) $\begin{cases} x^3-y^3 = 98 \\ x^2+xy+y^2 = 49; \end{cases}$

(3) $\begin{cases} x^3-y^3 = 56 \\ x-y = 2; \end{cases}$ (4) $\begin{cases} x^3+y^3 = 91 \\ x+y = 7. \end{cases}$

762. (1) $\begin{cases} xy = 2 \\ yz = 6 \\ xz = 3; \end{cases}$ (2) $\begin{cases} xy = -6 \\ yz = -2 \\ xz = 3; \end{cases}$ (3) $\begin{cases} xy = 2 \\ xz = 3 \\ x^2+z^2 = 10; \end{cases}$

(4) $\begin{cases} x+y = 1 \\ y+z = 5 \\ xz = 6; \end{cases}$ (5) $\begin{cases} x-y = 5 \\ y+z = -2 \\ x^2+z^2 = 13; \end{cases}$ (6) $\begin{cases} x-y = 5 \\ y-z = -2 \\ x^2-z^2 = 3. \end{cases}$

763. (1) $\begin{cases} y-x = 3 \\ y-z = 4 \\ x^2+y^2+z^2 = 30; \end{cases}$ (2) $\begin{cases} x+y = -3 \\ x+z = -2 \\ xy+xz+yz = 2; \end{cases}$

(3) $\begin{cases} xy+yz = 5 \\ yz+xz = 9 \\ xy+xz = 8. \end{cases}$

764*. (1) $\begin{cases} x^2+y^2+z^2 = 29 \\ xy+yz+xz = -10 \\ x+y = -1; \end{cases}$ (2) $\begin{cases} x^2+y^2+z^2 = 6 \\ xy+yz = -1 \\ x+y+z = 0; \end{cases}$

(3) $\begin{cases} x+y+z=a \\ xy+xz+yz=0 \\ xyz=0; \end{cases}$ (4) $\begin{cases} x^{-1}+y^{-1}+z^{-1}=9 \\ xy=12^{-1} \\ yz=8^{-2}. \end{cases}$

765. 1. Prove that the straight line $3x+4y=25$ is tangent to the circle $x^2+y^2=25$ and find the point of tangency.

2. Set up the equation of the tangent to the circle $x^2+y^2=2$, parallel to the bisector of the first quadrant.

766*. 1. Find out for what a does the system $\begin{cases} x^2+y^2=z \\ x+y+z=a \end{cases}$ have a unique solution and find this solution.

2. Find the real solutions of the system $\begin{cases} x+y+z=1 \\ 2xy-z^2=1. \end{cases}$

34. SETTING UP SYSTEMS OF EQUATIONS

767. The difference between two numbers is equal to 3, and the difference between their squares to 33. Find these numbers.

768. The sum of two squared numbers is equal to 34, and the difference between them to 16. Find these numbers.

769. The sum of two squared numbers equals 221. If each of the numbers is increased by unity, then the sum of their squares will be equal to 265. Find the numbers.

770. The difference of two numbers constitutes $\frac{1}{6}$ of their sum, and the difference between the squares of these numbers is equal to 24. Find the numbers.

771. Find the two-digit number which is 7 times greater than the sum of its digits, and 10.5 times greater than the product of its digits.

772. The sum of the numerator and the denominator of a fraction is equal to 16. If 2 is added to the numerator and subtracted from the denominator, then the reciprocal fraction is obtained. Find this fraction.

773. Find the two-digit number which, when divided by the sum of the digits, yields 5 as the quotient and 9 as the remainder, and, when divided by the product of the digits, 1 as the quotient and 18 as the remainder.

774. If a two-digit number is divided by the product of its digits, then 2 is obtained as the quotient with the remainder 5. If the digits of this number are interchanged, and the obtained number is

divided by the sum of its digits, then the quotient 7 is obtained with the remainder 3. Find the number.

775. The arithmetic mean of two numbers is equal to 15, and the geometric mean to 12. Find these numbers.

776. Find the two numbers whose sum, product and the difference of the squares are equal to one another.

777. The area of a rectangle is equal to 108 cm^2 and the diagonal to 15 cm. Find the sides of the rectangle.

778. A rectangular plot whose area amounts to 1600 m^2 is fenced in. The fence is 200 m long. Find the dimensions of this plot.

779. The hypotenuse of a right triangle is equal to 82 cm, and its area to 720 cm^2. Find the legs of the triangle.

780. The perimeter of a right triangle is equal to 72 cm, and its area to 216 cm^2. Find the sides of the triangle.

781. A rectangular plot whose area is 600 m^2 is paved with concrete slabs. The total of 20 rows were laid, 30 slabs per row. If the size of the slab is changed, i.e. its length is reduced by 1 m, and the width is increased by 0.5 m, then the same area can be covered with the same amount of slabs. Determine the dimensions of the slab.

782. Two trains were heading towards each other from two stations A and B 400 km away from each other. If the train from station A had left one hour earlier than the other, then they would have met at the midpoint of their journey. But if they left at the same time, then the distance between them four hours later would be equal to 40 km. Determine the speeds of the trains.

783. Two cyclists set off toward each other at the same time, and meet two hours later. Without stopping, they continue on their way at the same speed, and the first one arrives at point B 1 hour and 10 minutes earlier than the second arrives at point A. Determine the speed of each cyclist, if the distance between the points is 56 km.

784. Two people set off toward each other on foot at the same time from two places 27 km apart, and meet three hours later. The first arrives at the place from which the second had set out 1 hour and 21 minutes later than the second arrives at the place from which the first had set out. How long did it take each one to travel the whole distance?

785. A cyclist leaving point A has to arrive at point B by a set time. If he travels at a speed of 15 km/h, he will arrive 1 hour and 20 minutes earlier than the set time. But if he travels at a speed

of 10 km/h, he will have covered only 5/6 of the distance from A to B by the set time. Find the distance from A to B and the time in which the cyclist has to cover this distance.

786. The distance between towns A and B is equal to 270 km. Two trains set off toward each other at the same time from each city and travel for three hours before they meet. One train takes $1\frac{7}{20}$ hours more than the other to make the whole journey. Determine the speed of each train.

787. At 7 A.M. two loaded dump trucks left town in the same direction—one at an average speed of 40 km/h, the other, of 48 km/h. 3/4 h later a passenger car left the same town in the same direction, and overtook the second dump truck $1\frac{1}{2}$ hours later than it did the first. Find the passenger car's average speed, and find out at what time it overtook the second dump truck.

788. A freight train, a passenger train and an express train travel from one city to another. The freight train's speed is a km/h less than the passenger train's speed, and so it takes b hours longer than the passenger train to make the entire journey. The express train's speed is a km/h greater than the passenger train's speed, and so it takes c hours less than the passenger train does to make the entire journey. Find each train's speed and the time it takes each to make the entire journey.

789. Two summer sports camps are located on a river bank a few kilometres from each other. A sportsman rows from one camp to the other and back, and notices that it took him 39 minutes longer than if he had been rowing in still water. The next day he repeats his trip with another sportsman in the same boat, and comes to the conclusion that the two of them together, rowing in still water, would, in one hour, have rowed one and a half the distance he would have alone. This time they took eight minutes longer than they would have had there been no current. Find the distance between the camps and the speed at which the sportsman would have rowed in still water, if the speed of the river's current is equal to 4 km/h.

790. 1200 tons of winter wheat were gathered in a kolkhoz. If the yield had been 25% greater, the same amount of wheat could have been harvested within an area 80 hectares smaller than the one actually sown. How many hectares were sown and what was the harvest yielded by one hectare?

791. Three workers can manufacture a certain quantity of parts in three hours. In how many hours can each of them working on his own do the job if the first worker, working at an improved automatic machine, produces as many parts in one hour as the second and third combined manufacture in an hour on their machines, and if the second produces as many parts in two hours as the third worker does in three?

792. The rationalizers in a factory section devised and brought into production a new kind of part lighter in weight, lower in cost and superior in quality to parts of the old kind. Determine the weight of a part of the old and new kinds, if 6 parts of the new kind are 0.5 kg lighter than 5 parts of the old kind, and if the factory now makes one more part of the new kind from 22 kg of metal than it made parts of the old kind from 25 kg of metal.

793. A mathematics competition was held at a technical college. The technical college's trade union committee spent 39.5 roubles to buy a few books as first and second prizes for the winners in the competition; the ratio of the total cost of the first prizes to the total cost of the second prizes was 72 : 7. The number of roubles paid for each first prize is one fourth of the number of all the first and second prizes. The number of roubles paid for each second prize is one fourth of the difference between the numbers of the first and second prizes. How many first and second prizes were handed out to the winners in the competition?

794. Two student brigades were created to prepare a sports field for meets. The brigades could have done all of the various jobs in the equipping of the field in 3.6 hours. But it so happened that at first one brigade worked 1/3 of the time required by the second to do the whole job, and then the second brigade worked 1/3 of the time which the first would have spent in doing the whole job. It then turned out that they had done 13/18 of the whole job. How much time would each brigade working separately have required to do the whole job?

795. The resultant of two forces at right angles to each other is equal to 13 N. If one of them is increased by 4 N and the other is left unchanged, the resultant will increase by 2 N. Find the magnitudes of the component forces.

796. The resultant of two forces at right angles to each other is equal to 50 N. If the greater force is increased by 8 N and the smaller by 16 N, their resultant will remain unchanged. Find the component forces.

797. Two forces applied to one and the same point are at right angles to each other. The ratio of the two forces is equal to 2 : 5; the resultant is equal to 29 N. Determine the forces.

798. A body performs a uniformly accelerated motion (without an initial speed) and, after travelling 270 m, has acquired a speed of 9 m/s. At what acceleration and for how long did the body move?

CHAPTER IX

Review Problems

799. For what condition are the following equalities correct?

(1) $\sqrt{4a^2b} = 2a\sqrt{b}$; (2) $\sqrt{(x-3)^3} = (x-3)\sqrt{x-3}$;

(3) $\sqrt[3]{(x-3)^4} = (x-3)\sqrt[3]{x-3}$;

(4) $\sqrt{(a+1)(a-3)} = \sqrt{a+1}\,\sqrt{a-3}$;

(5) $\sqrt{(x-3)^4} = (x-3)^2$; (6) $\sqrt{(x+2)^6} = (x+2)^3$.

800. 1. May one assert that (a) if $x^4 = y^4$, then $x = y$; (b) if $x^3 = y^3$, then $x = y$?

2. What signs have the differences:

(a) $\sqrt[3]{11} - \sqrt{5}$; (b) $\sqrt{13} - (1+\sqrt{6})$?

3. Prove the following identities:

(a) $\dfrac{a - 2\sqrt{ab^2} + b^2}{\sqrt{a} - b} = \sqrt{a} - b, \quad b \geqslant 0$;

(b) $\dfrac{x\sqrt{x} - y\sqrt{y}}{\sqrt{x} - \sqrt{y}} = x + y + \sqrt{xy}$.

801. 1. For what values of the argument x do the given functions assume the least (the greatest) values? Find them.

(1) $y = \sqrt{4-x^2}$; (2) $y = \sqrt[3]{4+x^2}$; (3) $y = \dfrac{2}{\sqrt{1-x^2}}$;

(4) $y = \dfrac{12}{\sqrt[3]{8-x^2}}$.

802. Simplify and evaluate:

(1) $\left[\dfrac{b(ab^{-2})\left(a^{-\frac{2}{3}}\right)^{-1}}{a\sqrt{a}}\right]^6$ for $a = 2^{\frac{1}{2}}$, $b = 2^{-\frac{1}{3}}$;

(2) $\dfrac{x\sqrt{x}-y\sqrt{y}}{(x^2-xy)^{\frac{2}{3}}} : \dfrac{\sqrt[3]{x-y}}{\sqrt[3]{x^2}\left(x^{\frac{3}{2}}+y^{\frac{3}{2}}\right)}$ for $x = 1.2$, $y = 0.6$.

803. 1. Represent the following numbers in the form of a sum, difference, product and quotient of two irrational numbers:

$$4, \quad -3, \quad 7, \quad -7.$$

2. Find the reciprocal numbers:

$$3-2\sqrt{2}, \quad 7+4\sqrt{3}, \quad 5-\sqrt{3}, \quad \frac{\sqrt{3}}{3}.$$

804. Solve the following equations:

(1) $\dfrac{2}{3+2x} - \dfrac{2}{3-2x} - \dfrac{4x^2-21}{4x^2-9} = 0$;

(2) $x(x+3)-a(3-a) = 2(ax-1)$;

(3) $\dfrac{1}{2x+3} - \dfrac{1}{x^2-4} - \dfrac{4}{2x^2+7x+6} + \dfrac{7}{2x^3+3x^2-8x-12} = 0$;

(4) $(x^3-10)^4-5(x^3-10)^2+4 = 0$;

(5) $(x-2)^6-26(x-2)^3 = 27$.

805. (1) $\sqrt{2x^2-9} = 2x+3$; (2) $\sqrt{3x^2-2x+1}+2x = 1$;

(3) $\sqrt{2x^2-3x+2}+4 = x$;

(4) $\sqrt{2x^2+3x+2}+\sqrt{2x^2-3x+2} = \sqrt{7x^2+8}$.

806. (1) $2x^3-7x^2+7x-2 = 0$; (2) $x^4-6x^3+9x^2-9 = 0$;

(3)* $(x+3)^4+(x-1)^4 = 82$;

(4) $(x^2+x+1)(x^2+x+3)-15=0$;

(5)* $(2x-3)(4x-3)(x+1)(4x+1)=9$;

(6)* $(x-4)(x-2)(x-1)(x+1)+8=0$.

807. 1. Prove that the given equations have no real roots:

(a) $x^4+4x^3+6x^2+2x+1=0$; (b) $x^2+3x+2\sqrt{x-2}=0$.

2. Prove that the given systems of equations have no real solutions:

(a) $\begin{cases} x^2+y^2+x+y+1=0 \\ x^2+y^2-2x-2y+3=0; \end{cases}$

(b) $\begin{cases} x^2+y^2=9 \\ x^2+y^2-2xy+2x-2y+2=0. \end{cases}$

808. Without solving the equations, carry out a complete analysis of their roots:

(a) $x^2-4x+2=0$; (b) $3x^2-3x+1=0$;

(c) $2x^2+4x+1=0$; (d) $x^2+2x-3=0$;

(e) $5x^2-4x-10=0$; (f) $2x^2+5x+4=0$.

809. 1. Set up a quadratic equation given its roots:

(a) $x_1=2$, $x_2=-8$; (b) $x_{1,2}=\pm\frac{1}{2}$; (c) $x_1=2$, $x_2=\frac{1}{2}$;

(d) $x_{1,2}=2\pm\sqrt{3}$; (e) $x_{1,2}=\frac{a\pm\sqrt{b}}{2}$;

(f) $x_{1,2}=\frac{5}{3\pm\sqrt{2}}$.

2. Factorize:

(a) x^2-5x+4; (b) $3x^2-x-4$; (c) x^4-13x^2+36;

(d) $abx^2+(a^2-b^2)x-ab$.

810. Determine the value of the parameter m in the equation $2x^2+(2m-1)x-2m-1=0$ so that its roots x_1 and x_2 satisfy the relation $2x_1+2x_2=5$.

811. 1. Without solving the equation $3x^2-17x-14=0$, evaluate the expression

$\frac{2x_1^2+3x_1x_2+2x_2^2}{4x_1x_2^2+4x_1^2x_2}$, where x_1 and x_2 are the roots of the equation.

2. Without solving the equations, find the sums and the differences of their cubed roots:

(a) $4x^2-\sqrt{97}x+6=0$; (b) $8x^2+4\sqrt{39}x-41=0$.

812. 1. Given the equation $3x^2-5x+2=0$, whose roots are x_1 and x_2. Set up a new quadratic equation whose roots would be $\frac{x_1}{x_2}$ and $\frac{x_2}{x_1}$.

2. What must be p and q so that the roots of the equation $x^2+px+q=0$ are also p and q?

813. 1. For what k does the equation $(k-1)x^2+2(k+1)x+k-2=0$ have equal roots?

2. For what a does the equation $(a+5)x^2+(2a-3)x+a-10=0$ have real roots of the same sign?

814. Solve the following systems of equations:

(1) $\begin{cases} x^2+y^2=80 \\ x+y=12; \end{cases}$ (2) $\begin{cases} x^2+y^2=41 \\ xy=20; \end{cases}$ (3) $\begin{cases} x+xy^2=20 \\ x+xy=8. \end{cases}$

815. Solve the given inequalities:

(1) $3x^2+4x-15>0$; (2) $\frac{x-5}{x+2}>0$;

(3) $\frac{2x^2-3x+7}{x^2+1}>3$; (4) $\frac{x^2-2x-1}{1-2x}\geq\frac{2}{3}$;

(5) $|x-2|>3$; (6) $|x^2-2x|>3$.

816. Solve the following inequalities and systems of inequalities:

(1) $\begin{cases} \frac{x}{2}-\frac{3-x}{4}>\frac{x}{2} \\ x<5-2(1-x); \end{cases}$ (2) $\begin{cases} 3x-\frac{x-1}{2}<4x-5 \\ x+\frac{2}{7}>4-\frac{x}{5}; \end{cases}$

(3) $|2x+5|<3$; (4)* $|x^2-2x|-x>0$;

(5) $|2x^2-8x-5|<5$; (6) $|2x^2+8x+3|>3$.

817. On one and the same drawing construct the graphs of the following functions to the same scale:

(a) $y=3x$; (b) $y=3x+3$; (c) $y=3x-3$; (d) $y=-3x$.

818. Given the functions

(1) $y = \frac{1}{2}x^2$; (2) $y = -\frac{1}{2}x^2$; (3) $y = \frac{1}{2}x^2+2$;

(4) $y = \frac{1}{2}(x-1)^2$; (5) $y = \frac{1}{2}(x+1)^2$;

(6) $y = \frac{1}{2}x^2+x$; (7) $y = x^2+4x+4$;

(8) $y = x^2+4x+3$.

Determine for what values of x each of the given functions (a) increases, (b) decreases, (c) vanishes, (d) attains the least or the greatest value; find it.

819. Plot the graphs and find the least and the greatest values of the following functions:

(1) $y = \left|\frac{1}{2}x-1\right|-\left|\frac{1}{2}x+1\right|+1$;

(2) $y = |x+1|-|x-1|+|x-2|-1$;

(3) $y = |x^2-2x+8|$; (4) $y = |x^2-4x-5|-2$.

820. 1. A cyclist left Voroshilovgrad for Kadievka. The graph of his journey is given in Fig. 7. Using this graph, comment the journey. (When did the cyclist leave the city of Voroshilovgrad, when did he come back, when did he arrive in Kadievka, how long did he stay there, what happened during his return journey, when did he cycle most rapidly, and so on?)

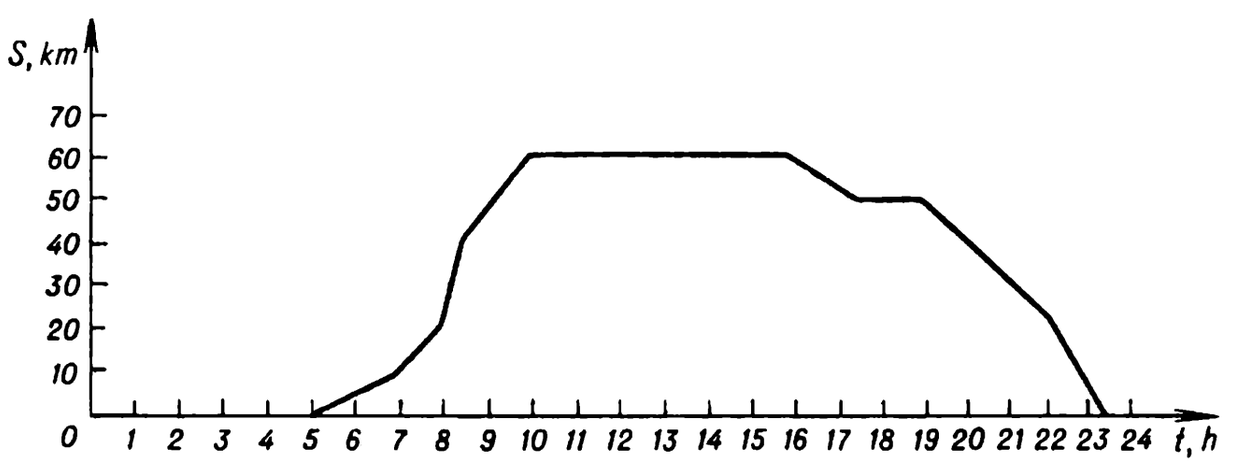


FIG. 7

2. For what values of m does the system

$$\begin{cases} mx+(1+m)y = 4m \\ 2x+3y = 1 \end{cases}$$

have $x = 4$? What is then y equal to? Construct for this case the graphs of the equations.

3. Simplify the expression

$$\frac{2b^{\frac{1}{2}}}{a^{\frac{1}{2}}+b^{\frac{1}{2}}}+(a-b)^{-1}\left[\frac{a\sqrt{a}+b\sqrt{b}}{a^{0.5}+b^{0.5}}-\sqrt{ab}\right].$$

4. Solve the system of inequalities

$$\begin{cases} \frac{x}{4}-\frac{7}{8} > \frac{3}{4}x-\frac{5}{2} \\ \frac{x+1}{4} < 2-\frac{1-2x}{3}. \end{cases}$$

821. 1. Carry out the operations and compute the result to within 0.1:

$$\sqrt{\frac{2+\sqrt{3}}{\sqrt{3}-1}+\frac{2+2\sqrt{3}}{\sqrt{3}-3}}\ \sqrt{(1-\sqrt{3})^2}.$$

2. Solve the equation

$$\sqrt{(x-4)^2}+\sqrt{(1-x)^2} = 5.$$

3. Solve the inequality

$$(3x^2-14x-17)(2x-1) > 0.$$

822. 1. A body situated at a certain altitude is thrown vertically upwards. It is known to have reached the maximum height in 1 second and to have fallen to the earth in 6 seconds after its start. Construct the graph of the altitude reached by the body from the starting instant till the time it hits the ground, and determine the altitude from which the body was thrown, its initial velocity and the maximum height reached.

2. Perform the operations:

$$\left(\sqrt{\sqrt{a^2-2a+1}+\frac{a}{\sqrt{1-a}}}\right):\left(\frac{1}{\sqrt{a+1}}-\frac{\sqrt{a+1}}{a-1}\right).$$

3. Solve the equation

$$\sqrt{1+x+x^2}+\sqrt{1-x+x^2} = 4.$$

823. 1. In an isosceles triangle change the sides preserving its area constant. Express the length l of the lateral side of the triangle as a function of the base and indicate the domain of definition of the function.

2. The straight line $2x+y-14=0$ intersects the ellipse $\frac{x^2}{25}+\frac{y^2}{100}=1$. Find the length of the line segment contained inside the ellipse.

3. Carry out the indicated operations:

$$\frac{0.25^{-1}a^2b^{-7}}{9x^{4n-3}}\sqrt[6]{\frac{0.000064a^{-12}x^{24n-6}}{27^{-2}b^{-42}}}.$$

824. 1. Carry out the operations and compute the result to within 0.1:

$$\sqrt{(2-\sqrt{5})^2}\sqrt{\frac{4+\sqrt{5}}{\sqrt{5}-2}+\frac{4\sqrt{5}+1}{2\sqrt{5}-5}}.$$

2. Compute:

$$\sqrt[6]{4-2\sqrt{3}}\sqrt[3]{1+\sqrt{3}}.$$

3. Solve the system of equations

$$\begin{cases} xy+x+y=11 \\ x^2y+xy^2=30. \end{cases}$$

825. 1. Simplify:

$$\frac{m+n}{\sqrt{m}+\sqrt{n}}:\left(\frac{m+n}{\sqrt{mn}}+\frac{n}{m-\sqrt{mn}}-\frac{m}{\sqrt{mn}+n}\right).$$

2. Find the coordinates of the points at which the graphs of the functions $y=x^2-2$ and $y=-2x-3$ intersect and plot these graphs.

3. Prove that

$$\sqrt[3]{5\sqrt{2}+7}+\sqrt[3]{5\sqrt{2}-7}=2.$$

826. 1. Due to repair of separate sections of the highway the bus reduced its speed by an average of 5 km/h and spent half an hour longer than usually to cover the distance of 320 km. Find the original speed of the bus (the answer should be given accurate to 1 km/h).

2. Solve the equation $\sqrt{(x-3)^2}+\sqrt{(1-x)^2}=2$ for $-1<x\leqslant 3$.

3. Construct the graph of the function

$$y = \begin{cases} \dfrac{x^3+x}{|x|} & \text{if} \quad |x| \leqslant 1 \\ 2 & \text{if} \quad |x| > 1. \end{cases}$$

827. 1. Two workers finish together a certain job in 24 hours. If one of them accomplished half the amount of work and then the other the remaining half, it would have taken them 50 hours to carry out the job. How much time would it take each worker to complete the job separately?

2. Solve the equation

$$\sqrt{x+15}+\sqrt{x-24}-\sqrt{x-13}=\sqrt{x}.$$

3. Prove that the expression

$$\left[(9+a^2-6a)^{0.5}(a^{0.5}-3^{0.5})^{-1}-\frac{a^{1.5}-3^{1.5}}{a-3}\right]:(a\sqrt{3}-3\sqrt{a})$$

is rational with respect to a for $a > 3$.

4. Find the domain of definition of the function

$$y = \sqrt[3]{1+x}+\sqrt{|x+2|-7}.$$

828. 1. Prove that the expression

$$\left[\frac{(a^2+16b^2-8ab)^{0.5}}{a-(ab)^{0.5}-6b}+\frac{a-9b}{a-6(ab)^{0.5}+9b}\right]b^{-0.5}\times(\sqrt{a}+3\sqrt{b})^{-1}$$

is rational with respect to a and b for $0 < a < b$.

2. Solve the inequality

$$|x+3 > 2x-1|.$$

3. Solve the equation $\dfrac{2x+a}{2x+5a}-\dfrac{4x}{5a-2x}=\dfrac{10ax+75a^2}{4x^2-25a^2}$.

829. 1. The distance between the points A and C is equal to 80 km. A skier starts from the point A in the direction of C and moves at a speed of 10 km/h. In 15 minutes another skier is to start from the point C and, moving at a constant speed, to meet the first one halfway from A to C. But since the second skier started 55 minutes later than it had been scheduled, they met at another point (B). What is the distance between A and B?

2. Simplify:

$$(n^{-0.5}+0.25m-m^{0.5}n^{-0.25})^{0.5}-2m^{0.5}n^{-0.25}(0.25n^{0.25}-m^{-0.5}).$$

3. Graph the function

$$y = \frac{2x}{\sqrt{1-\left(\frac{1-x^2}{1+x^2}\right)^2}}.$$

830. 1. Find the product

$$(1+\sqrt{a})(1+\sqrt[4]{a})(1+\sqrt[8]{a})(1+\sqrt[16]{a})(1-\sqrt[16]{a}) \quad \text{for} \quad a = 2.$$

2. Graph the function

$$y = \left(\sqrt{x+2\sqrt{x-1}}+\sqrt{x-2\sqrt{x-1}}\right)^2.$$

3. To get from the point A into the point B it is necessary to go by train the portion of this distance from the point A to the point C, and the remaining 15 km (from C to B) must be covered by bus. The travel from A to B takes 1 hour and 15 minutes, the bus starting right at the moment of the arrival of the train. The time spent by the train and by the bus on stops may be neglected. By mistake a passenger got off the train one station earlier, walked the remaining way to C (5 km) at a speed of 4 km/h, and then waited for the bus another 20 minutes. Finally, he reached the point B an hour and a half later than it had been scheduled. Find the distance from A to C if it is known that the speed of the train is twice that of the bus.

831. 1. Three pumps, installed at the river bank to water the adjacent fields, were switched on simultaneously. The first and the third pumps were switched off at the same time, while the second one after two hours of operation. As a result, the first pump pumped out $9m^3$ of water, and the second and the third pumps together —28 m^3. What amount of water is pumped out by each pump per hour, if it is known that the capacity of the third pump is by $3m^3$ higher as compared with the first pump, and that the total capacity of the three pumps is $14m^3$ of water per hour?

2. Graph the function

$$y = \frac{\sqrt{x(x-2)^2}}{x-2}.$$

3. Simplify the following expression for $a > 1$

$$\left\{(1-a^2)\left[\frac{1-a^{1.5}}{1-a^{0.5}}+a^{0.5}\right]^{-1}\left[\frac{1+a^{1.5}}{1+a^{0.5}}-a^{0.5}\right]^{-1}+1\right\}:\sqrt{(1-2a+a^2)^{-1}}.$$

4. Solve the inequality

$$|x+2|-|1-x| > 3x-2.$$

832. 1. If a steamer and a cutter go with the stream, then the steamer covers the distance between the landing stages A and B 1.5 times faster than the cutter, and the latter falls 8 km behind the steamer each hour. If they go against the stream, then the steamer covers the distance between B and A twice as fast as the cutter does. Find the speeds of the steamer and the cutter in still water.

2. Determine the greatest value of the function

$$y = \frac{1}{x^2+x+1}.$$

3. Simplify the expression

$$\left[(a^2x^{0.25}+a^{0.5}x)(ax^{0.25}+a^{0.5}x^{0.5})^{-1}-\sqrt{a^2x+2ax^{0.5}}\right]^{-4}a^2x$$

for $a > 0$ and $x > 0$.

833. 1. Two bodies move uniformly in a circle in one and the same direction. The first body covers the circle by two seconds faster than the second one and catches up with the latter each twelve seconds. What time is it necessary for each moving body to complete the circle?

2. Carry out the indicated operations

$$\left(u\sqrt{\frac{6v}{u}}+4v\sqrt{\frac{u}{2v}}\right)^2-\frac{40}{u^3v^2}\sqrt{0.12u^8v^6}.$$

3. Describe the properties of the function $y = x^{-1}$ on the interval $-2.5 \leqslant x \leqslant 2.5$ and plot its graph.

834. 1. A motor-boat started from the point A for the point B upstream. Suddenly, something became wrong with the motor, and while it was being repaired during 20 minutes the boat was drifting downstream. Determine the time by which the boat was late to reach the point B due to the motor fault if it is known that the way from A to B is usually covered by the boat 1.5 times slower than that from B to A.

2. Simplify:

$$\left(\frac{a\sqrt[3]{a}-2a\sqrt[3]{b}+\sqrt[3]{a^2b^2}}{\sqrt[3]{a^2}-\sqrt[3]{ab}}+\frac{\sqrt[3]{a^2b}-\sqrt[3]{ab^2}}{\sqrt[3]{a}-\sqrt[3]{b}}\right):\sqrt[3]{a}.$$

3. Prove the inequality

$$\frac{k^2+k+1}{k^2+1} \leqslant \frac{3}{2}.$$

835. 1. Simplify the expression

$$\frac{(1-a^{-2}b^2)\,a^2}{(\sqrt{a}-\sqrt{b})^2+2\sqrt{ab}}.$$

2. Solve the inequality

$$x^4-5x^2+4>0.$$

3. A mixture is composed from two liquids whose densities are 2 g/cm^3 and 3 g/cm^3. How many grams of each liquid was taken and what is the density of the mixture, if 7 cm^3 of the mixture constitute 1/5 of the total weight of the first liquid entering the mixture, and 70 cm^3 of the mixture weigh as much as the total amount of the second liquid entering the same mixture? It is assumed that the volume of the mixture is equal to the sum of the volumes of its components.

CHAPTER X

Vectors

35. THE VECTOR. ADDITION AND SUBTRACTION OF VECTORS

836. Which of the listed quantities are scalars and which are vectors: mass, force, length, area, volume, velocity, acceleration, temperature, pressure?

837. Compute the length of the directed line segment AB if:

(1) $A(2, 0)$, $B(5, 0)$; (2) $A(4, 0)$, $B(1, 0)$; (3) $A(-1, 0)$, $B(2, 0)$; (4) $A(-2, 0)$, $B(-5, 0)$; (5) $A(0, 0)$, $B(0, -3)$; (6) $A(0, -2)$, $B(0, 3)$.

838. 1. Let respectively $A(x_1, y_1)$ and $B(x_2, y_2)$ be the initial and the terminal points of vector $\overrightarrow{AB}$. Express $|\overrightarrow{AB}|$ in terms of the coordinates of the points A and B. Using the obtained formula, compute the length of the vector $\overrightarrow{AB}$ if (a) $A(0, 0)$, $B(-3, 4)$; (b) $A(1, 2)$, $B(7, 10)$; (c) $A(-2, 1)$, $B(2, 4)$.

2. Construct the vector $\overrightarrow{AB}$ and determine its length and direction with respect to the x-axis if (a) $A(0, 0)$, $B(\sqrt{3}, 1)$; (b) $A(-1, 2)$, $B(-5, 2)$; (c) $A(0, 2)$, $B(0, -3)$.

839. 1. Prove that for any two vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ the commutative law is obeyed.

2. Prove that for any three vectors $\boldsymbol{a}$, $\boldsymbol{b}$, $\boldsymbol{c}$ the equality $(\boldsymbol{a}+\boldsymbol{b})+\boldsymbol{c} = \boldsymbol{a}+(\boldsymbol{b}+\boldsymbol{c})$ is correct (the associative law).

840. 1. Given the vector $\boldsymbol{a}$. Construct the vector

(a) $2\boldsymbol{a}$; (b) $\frac{1}{2}\boldsymbol{a}$; (c) $-\frac{2}{3}\boldsymbol{a}$; (d) $0\times\boldsymbol{a}$; (e) $-0.3\boldsymbol{a}$.

2. Given the vectors $\boldsymbol{a}$ and $\boldsymbol{b}$. Construct the vectors

(a) $\boldsymbol{a}+\boldsymbol{b}$; (b) $\boldsymbol{b}-\boldsymbol{a}$; (c) $-\boldsymbol{a}-\boldsymbol{b}$; (d) $-\frac{1}{3}\boldsymbol{b}$; (e) $3\boldsymbol{a}+\frac{1}{2}\boldsymbol{b}$.

841. 1. Given the points $A(-2, 1)$, $B(1, 2)$, $C(3, -2)$. Construct the vectors $\overrightarrow{AB}$, $\overrightarrow{BC}$ and $\overrightarrow{AB}+\overrightarrow{BC}$.

2. Given the points $A(-3, -2)$, $B(1, 2)$, $C(3, 5)$, $D(10, 8)$. Construct the vectors $\overrightarrow{AB}+\overrightarrow{CD}$, $\overrightarrow{AB}-\overrightarrow{DC}$ and $\overrightarrow{AB}-\overrightarrow{CD}$.

842. 1. The vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ are mutually perpendicular, $a = 3$ and $b = 5$. Determine $|\boldsymbol{a}+\boldsymbol{b}|$ and $|\boldsymbol{a}-\boldsymbol{b}|$.

2. Compute $|\boldsymbol{a}-\boldsymbol{b}|$, if $|\boldsymbol{a}| = 13$, $|\boldsymbol{b}| = 19$, $|\boldsymbol{a}+\boldsymbol{b}| = 24$.

3. Compute $|\boldsymbol{a}+\boldsymbol{b}|$, if $|\boldsymbol{a}| = 11$, $|\boldsymbol{b}| = 23$, $|\boldsymbol{a}-\boldsymbol{b}| = 30$.

843. 1. Prove that $|\boldsymbol{a}+\boldsymbol{b}| \leqslant |\boldsymbol{a}|+|\boldsymbol{b}|$. In what case does the sign of equality occur?

2. What condition must the vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ satisfy so that

(a) $|\boldsymbol{a}+\boldsymbol{b}| = |\boldsymbol{a}-\boldsymbol{b}|$; (b) $|\boldsymbol{a}+\boldsymbol{b}| > |\boldsymbol{a}-\boldsymbol{b}|$;

(c) $|\boldsymbol{a}+\boldsymbol{b}| < |\boldsymbol{a}-\boldsymbol{b}|$; (d) $|\boldsymbol{a}+\boldsymbol{b}| = |\boldsymbol{a}|-|\boldsymbol{b}|$;

(e) $|\boldsymbol{a}-\boldsymbol{b}| = |\boldsymbol{a}|+|\boldsymbol{b}|$; (f) $|\boldsymbol{a}+\boldsymbol{b}| = 0$?

844. On the sides OA and OB of the rectangle $OACB$ unit vectors $\boldsymbol{i}$ and $\boldsymbol{j}$ are laid off. Express the vectors $\overrightarrow{OA}$, $\overrightarrow{OB}$, $\overrightarrow{OC}$, $\overrightarrow{AC}$, $\overrightarrow{BO}$, $\overrightarrow{BA}$ in terms of $\boldsymbol{i}$ and $\boldsymbol{j}$ if $OA = 5$ and $OB = 7$.

845. Given in the parallelogram $OACB$ are the vectors $\overrightarrow{OA} = \boldsymbol{a}$ and $\overrightarrow{OB} = \boldsymbol{b}$. Find the vectors $\overrightarrow{OC}$, $\overrightarrow{AB}$, $\overrightarrow{MO}$, $\overrightarrow{MA}$, $\overrightarrow{MB}$, $\overrightarrow{MC}$, where M is the point of intersection of the diagonals.

846. Let O be the point at which the medians of the triangle ABC intersect. Prove that $\overrightarrow{OA}+\overrightarrow{OB}+\overrightarrow{OC} = 0$.

847. In an isosceles trapezoid $ABCD$ the angle BAD is equal to $60°$; $AB = BC = CD = 4$, M and N are the midpoints of the sides BC and CD. Express the vectors $\overrightarrow{CD}$, $\overrightarrow{AM}$, $\overrightarrow{AN}$, $\overrightarrow{MN}$ in terms of the unit vectors $\boldsymbol{m}$ and $\boldsymbol{n}$ of the directions AD and AB.

848. The point B divides the circular arc $AC = 90°$ in the ratio $1 : 2$; O is the centre of the circle. Find the vector $\overrightarrow{OC}$ if $\overrightarrow{OA} = \boldsymbol{a}$ and $\overrightarrow{OB} = \boldsymbol{b}$.

849. Given the vectors $\boldsymbol{a}$ and $\boldsymbol{b}$, the angle between them is 120°. Construct the vector $\boldsymbol{c} = 4\boldsymbol{a} - 3\boldsymbol{b}$ and determine its length if $a = 3$ and $b = 4$.

850. Decompose the vector $\boldsymbol{a}$ into two vectors, given:

(1) the directions of the component vectors;

(2) the direction and the length of one of the component vectors;

(3) the direction of one component vector and the length of the other;

(4) the lengths of both component vectors.

851. 1. The resultant of two mutually perpendicular forces equals 10 N. One of the component forces is equal to 8 N. Find the other component force.

2. Given the vectors $\overrightarrow{OA} = \boldsymbol{a}$ and $\overrightarrow{OB} = \boldsymbol{b}$; OC is the median of the triangle AOB. Decompose analytically and geometrically the vector $\overrightarrow{OC}$ into the vectors $\boldsymbol{a}$ and $\boldsymbol{b}$, and the vector $\boldsymbol{a}$ into the vectors $\boldsymbol{b}$ and $\overrightarrow{OC}$.

852. Three coplanar forces, 20 N each, act on a body. Find graphically the resultant of these forces, if two of them form an angle of 90°, and the third force with each of the first two angles of 135°.

853. A barrel whose mass is 100 kg is kept on a plane inclined at an angle of 30° to the horizontal. Find the restoring force and the force with which the barrel presses on the plane (the friction force may be neglected).

36. THE PROJECTION OF A VECTOR ON AN AXIS. COORDINATES OF A VECTOR ON A PLANE

854. 1. Let $A(x_1, y_1)$ and $B(x_2, y_2)$ be respectively the initial and the terminal points of the vector $\overrightarrow{AB}$. Express its projections a_x and a_y on the x- and y-axes in terms of the coordinates of the points A and B.

2. Using the obtained formulas, compute the projections of the vector $\overrightarrow{AB}$ on the axes of coordinates if

(a) $A(0, 0)$, $B(-4, 3)$; (b) $A(2, -3)$, $B(2, 5)$;

(c) $A(2, 0)$, $B(0, 3)$.

855. 1. Compute the projection of vector $\boldsymbol{a}$ on the x-axis, if the angle between the positive direction of the x-axis and the direction

of the vector is equal to α:

(a) $\boldsymbol{a}$, α; (b) $|\boldsymbol{a}| = 6$, $\alpha = 120°$; (c) $|\boldsymbol{a}| = 1$, $\alpha = 45°$;

(d) $|\boldsymbol{a}| = 5$, $\alpha = 180°$; (e) $|\boldsymbol{a}| = 3$, $\alpha = 90°$;

(f) $|\boldsymbol{a}| = 1$, $\alpha = 130°28'$.

2. Compute the length of the vector $\boldsymbol{a}$, if its projection on the x-axis is equal to a_x and the angle between the vector $\boldsymbol{a}$ and the x-axis is equal to α:

(a) a_x, α; (b) $a_x = -5$, $\alpha = 120°$; (c) $a_x = 10$, $\alpha = 75°$;

(d) $a_x = -3$, $\alpha = 180°$; (e) $a_x = 0$, $\alpha = 90°$.

856. Determine the point B which is the terminal point of the vector $\boldsymbol{a}$, if its initial point coincides with point A. (Given in braces are the projections of the vector on the coordinate axes.)

(1) $\boldsymbol{a}\{3, 1\}$, $A(2, 0)$; (2) $\boldsymbol{a}\{-3, 2\}$, $A(1, 2)$;

(3) $\boldsymbol{a}\{-4, -5\}$, $A(-2, 6)$.

857. Prove that equal vectors have equal projections on a given axis.

858. Prove that if vectors have the same direction, then their projections on one and the same axis, which is not perpendicular to them, are proportional to the lengths of the vectors.

859. Given arbitrary vectors $\overrightarrow{AB}$ and $\overrightarrow{AC}$. Through the point A draw an axis l so that (1) $\text{pr}_{\overrightarrow{l}}\,\overrightarrow{AB} = \text{pr}_{\overrightarrow{l}}\,\overrightarrow{AC}$; (2) $\text{pr}_{\overrightarrow{l}}\,\overrightarrow{AB} = -\text{pr}_{\overrightarrow{l}}\,\overrightarrow{AC}$.

860. Given an equilateral triangle ABC; D is the midpoint of AC. Find the projections of the vectors $\overrightarrow{AB}$, $\overrightarrow{BC}$, $\overrightarrow{AC}$ on $\overrightarrow{BD}$, if the side of the triangle is equal to 10.

861. Given two vectors $\boldsymbol{a}\{3, -2\}$ and $\boldsymbol{b}\{-2, 1\}$. Determine the projections of the following vectors on the coordinate axes:

(a) $\boldsymbol{a}+\boldsymbol{b}$; (b) $\boldsymbol{a}-\boldsymbol{b}$; (c) $2\boldsymbol{a}$; (d) $-\frac{1}{4}\boldsymbol{b}$; (e) $3\boldsymbol{a}+2\boldsymbol{b}$.

862. 1. Let vector $\boldsymbol{a}\{a_x, a_y\}$ be specified by its projections. Express $|\boldsymbol{a}|$ in terms of a_x and a_y.

2. Using the obtained formula, find the lengths of the vectors

(a) $\boldsymbol{a}\{1, \sqrt{3}\}$; (b) $\boldsymbol{b}\{5, 12\}$; (c) $\boldsymbol{a}\{-5, -12\}$; (d) $\boldsymbol{b}\{2, 2\}$.

863. 1. Prove that any vector $\boldsymbol{a}$ may be represented in the form $\boldsymbol{a} = a_x\boldsymbol{i}+a_y\boldsymbol{j}$, where a_x and a_y are the projections of the vector $\boldsymbol{a}$ on the coordinate axes, $\boldsymbol{i}$ and $\boldsymbol{j}$ are unit vectors of the coordinate axes.

2. Express the vector $\overrightarrow{AB}$ in terms of $\boldsymbol{i}$ and $\boldsymbol{j}$ if:

(a) $A(2, 0)$, $B(1, 0)$; (b) $A(-1, -2)$, $B(-1, 8)$;

(c) $A(2, 2)$, $B(-2, 2)$; (d) $A(1, -1)$, $B(-2, 3)$;

(e) $A(0, 0)$, $B(-3, 4)$; (f) $A(0, 2)$ $B(-3, 0)$.

3. Find the unit vector of the vector $\boldsymbol{a} = 3\boldsymbol{i}+4\boldsymbol{j}$.

864. 1. Given the vectors $\boldsymbol{a} = 3\boldsymbol{i}+\frac{1}{2}\boldsymbol{j}$ and $\boldsymbol{b} = -\boldsymbol{i}+5\boldsymbol{j}$. Find the projections of the vectors $-\boldsymbol{a}+\boldsymbol{b}$, $-\boldsymbol{a}-\boldsymbol{b}$, $\frac{1}{3}\boldsymbol{a}+2\boldsymbol{j}$, $-\frac{3}{4}\boldsymbol{a}-\frac{1}{5}\boldsymbol{j}$.

2. Given on a plane are the points $A(0, 2)$, $B(4, 2)$, $C(4, -2)$. The forces $\overrightarrow{OA}$, $\overrightarrow{OB}$, $\overrightarrow{OC}$ are applied at the origin. Construct their resultant $\overrightarrow{OM}$, find its projections on the coordinate axes and the magnitude.

3. A point is acted upon by three forces: $\boldsymbol{F}_1 = 2\boldsymbol{i}+\boldsymbol{j}$, $\boldsymbol{F}_2 = 4\boldsymbol{i}-3\boldsymbol{j}$, $\boldsymbol{F}_3 = -5\boldsymbol{i}+4\boldsymbol{j}$. Find the magnitude and the direction of the resultant force.

865. 1. Construct the point M and determine the length and the direction of its radius vector:

(a) $M(3, 4)$; (b) $M(0, -3)$; (c) $M(-6, 8)$.

2. Construct the vector $\overrightarrow{OM}$ and determine its length and direction:

(a) $\overrightarrow{OM} = 3\boldsymbol{i}+4\boldsymbol{j}$; (b) $\overrightarrow{OM} = -3\boldsymbol{j}$; (c) $\overrightarrow{OM} = 8\boldsymbol{i}-6\boldsymbol{j}$;

(d) $\overrightarrow{OM} = -2\boldsymbol{i}-2\boldsymbol{j}$; (e) $\overrightarrow{OM} = -\boldsymbol{i}+\sqrt{3}\boldsymbol{j}$.

866. Given the points $A(0, -4)$ and $B(3, 4)$. Required:

(1) Construct radius vectors OA and OB of these points and find their projections.

(2) Determine the projections of the vectors $\overrightarrow{OM} = \overrightarrow{OA}+\overrightarrow{OB}$, $\overrightarrow{ON} = \overrightarrow{OB}-\overrightarrow{OA}$.

(3) Write down the vectors $\overrightarrow{OA}$, $\overrightarrow{OB}$ and $\overrightarrow{ON}$ with the aid of the unit vectors $\boldsymbol{i}$ and $\boldsymbol{j}$ of the coordinate axes.

867. Find the radius vector of the midpoint M of the line segment AB decomposed along the unit vectors $\boldsymbol{i}$ and $\boldsymbol{j}$ of the coordi-

nate axes if:

(1) $A(-8, 3)$, $B(2, 5)$; (2) $A(-6, 1)$, $B(4, -1)$;

(3) $A(-3, 4)$, $B(5, 5)$; (4) $A(-4, 2)$, $B(4, -2)$.

868. Construct a parallelogram on the vectors $\vec{OA} = \boldsymbol{i}+\boldsymbol{j}$; $\vec{OB} = \boldsymbol{i}-3\boldsymbol{j}$ and determine its diagonals.

869. The unit vectors $\boldsymbol{a}$, $\boldsymbol{b}$, $\boldsymbol{c}$ form with the axis l angles of 60°, 120° and 180°, respectively. Find the projections of the vectors $\boldsymbol{a}+\boldsymbol{b}+\boldsymbol{c}$ and $3\boldsymbol{a}+2\boldsymbol{b}+\boldsymbol{c}$ on the axis l.

870. Find the projection of the sum of vectors $\boldsymbol{a}$, $\boldsymbol{b}$, $\boldsymbol{c}$, $\boldsymbol{d}$ on the axis l if $a = 5$, $b = 6$, $c = 8$, $d = 12$, and the angles between these vectors and the axis l are equal to 0°, 120°, 180°, 360°, respectively. Make a drawing and explain the geometrical meaning of the obtained result.

37. SCALAR PRODUCT OF TWO VECTORS

871. 1. Prove that the scalar product possesses the commutative property, i. e. $\boldsymbol{ab} = \boldsymbol{ba}$.

2. Prove that $\boldsymbol{ab} = \boldsymbol{a}\,\mathrm{pr}_{\boldsymbol{a}}\,\boldsymbol{b}$.

3. Prove that the scalar product possesses the distributive property, i.e. $(\boldsymbol{a}+\boldsymbol{b})\boldsymbol{c} = \boldsymbol{ac}+\boldsymbol{bc}$ (use the formula of item 2 of this problem).

872. Prove that diagonals of a rhomb are mutually perpendicular.

873. 1. Find the scalar product of the unit vectors:

$$\boldsymbol{ii},\quad \boldsymbol{ij},\quad \boldsymbol{jj}.$$

2. Prove that the scalar product of vectors specified by their projections is equal to the sum of products of like projections, i.e. if $\boldsymbol{a}\{a_x a_y\}$; $\boldsymbol{b}\{b_x, b_y\}$, then $\boldsymbol{ab} = a_x b_x + a_y b_y$.

3. Find the scalar product of two vectors:

(a) $\boldsymbol{a} = 2\boldsymbol{i}+3\boldsymbol{j}$ and $\boldsymbol{b} = -\boldsymbol{i}+\boldsymbol{j}$; (b) $\boldsymbol{a} = 2\boldsymbol{i}-3\boldsymbol{j}$ and $\boldsymbol{b} = 3\boldsymbol{i}+2\boldsymbol{j}$.

874. 1. Compute the work performed by the force $\boldsymbol{F}\{6, -2\}$ when the point of its application moves rectilinearly from $A(3, 4)$ to $B(-1, 3)$.

2. Given three forces $\boldsymbol{F}_1\{3, -4\}$, $\boldsymbol{F}_2\{2, 3\}$, $\boldsymbol{F}_3\{-3, -2\}$ applied to one and the same point. Compute the work performed by their

resultant if its point of application moves rectilinearly from $A(5, 3)$ to $B(4, -1)$.

875. 1. Prove that the angle between two vectors $\boldsymbol{a}\{a_x, a_y\}$, $\boldsymbol{b}\{b_x, b_y\}$ is computed by the formula

$$\cos\varphi = \frac{a_x b_x + a_y b_y}{\sqrt{a_x^2 + a_y^2}\sqrt{b_x^2 + b_y^2}}.$$

2. Determine the angle between the vectors:

(a) $\boldsymbol{a}\{2, 1\}$, $\boldsymbol{b}\{1, -4\}$; (b) $\boldsymbol{a}\{0, 2\}$, $\boldsymbol{b}\{-3, 4\}$;

(c) $\boldsymbol{a}\{-3, 4\}$, $\boldsymbol{b}\{5, 12\}$; (d) $\boldsymbol{a}\{2, -3\}$, $\boldsymbol{b}\{-3, 2\}$.

876. Given a triangle with the vertices $A(-2, 3)$, $B(-2, -1)$, $C(-1, 0)$. Find (1) its interior angles, (2) the angle between the median drawn from the vertex A and the side AC.

877. 1. Given $|\boldsymbol{a}| = 3$, $|\boldsymbol{b}| = 5$. For what value of m will the vectors $\boldsymbol{a}+m\boldsymbol{b}$ and $\boldsymbol{a}-m\boldsymbol{b}$, be perpendicular to each other?

2. What condition must be satisfied by vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ so that the vector $\boldsymbol{a}+\boldsymbol{b}$ is perpendicular to the vector $\boldsymbol{a}-\boldsymbol{b}$?

878. 1. Prove that (a) $a_x b_x + a_y b_y = 0$ is the condition of perpendicularity, (b) $\frac{a_x}{b_x} = \frac{a_y}{b_y}$ is the condition of parallelism of the vectors $\boldsymbol{a}\{a_x, a_y\}$, $\boldsymbol{b}\{b_x, b_y\}$.

2. Given the vertices of a rectangle $A(6, -1)$, $B(5, 1)$, $C(1, 2)$, $D(2, -4)$. Prove that its diagonals AC and BD are mutually perpendicular.

3. Prove that the triangle with the vertices $A(1, 0)$, $B(1, 3)$, $C(4, 3)$ is an isosceles right triangle.

CHAPTER XI

Trigonometric Functions of an Arbitrary Angle

38. MEASURING ANGLES AND ARCS

879. 1. Compute (in degrees and radians) the angle described by the small hand during (a) 1 hour, (b) 1 hour and 30 minutes, (c) 4 hours, (d) 12 hours, (e) 24 hours, (f) 15 minutes.

2. Compute (in degrees and radians) the angle described by the large hand during (a) 15 minutes, (b) 1 hour, (c) 20 minutes.

3. Compute (in degrees and radians) the angle formed by the clock hands when they show 4 : 00; 6 : 00; 12 : 00; 11 : 30; 5 : 30.

880. 1. Construct the angle of 340°. Write the general formula for the angles whose terminal side coincides with the terminal side of the given angle. Solve this problem for the following angles 75°, 145°, 220°, −65°, −136°, −249°.

2. Given the formula of a set of angles $A = 20° + 360°\ k$. Construct the positive and negative angles belonging to this set. The absolute value of each of them should not exceed 360°.

881. 1. Write down a general formula for all the arcs of one and the same circle whose initial point is A and end-point is B, if one of these arcs is equal to 140°.

2. What is the negative angle whose terminal side coincides with that of the angle α, if $\alpha = 76°$, $\alpha = 128°$, $\alpha = 176°$, $\alpha = 354°$. The angles are measured from one and the same point.

882. Given the following angles: 540°, 1120°, −690°, −90°,

1800°, −1760°. Represent them as the sum $360°k+\alpha$ where k is a whole number and α is a nonnegative angle less than 360°.

883. 1. Write down the values of the angles (in degrees and radians) shown in Figs. 8 and 9.

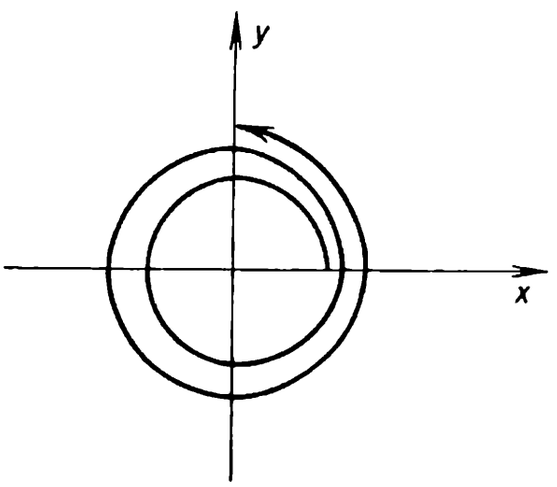


FIG. 8

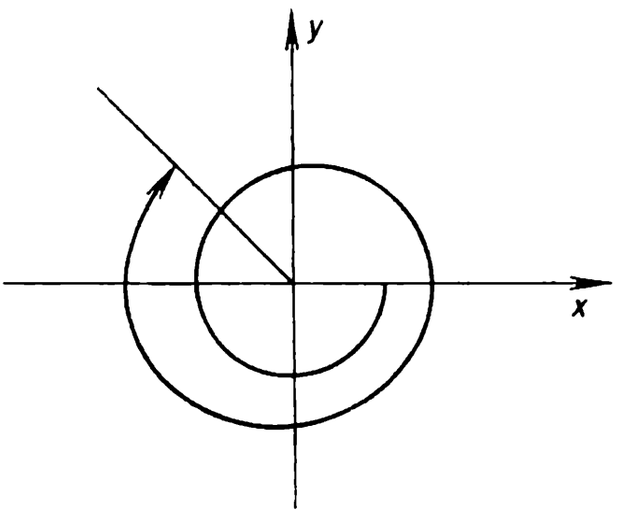


FIG. 9

2. Draw the terminal sides of the angles equal to 30°, −45°, 135°, −225°, −90°, 450°, −810°, 2070°.

884. 1. In what quadrant is the terminal side of the angle situated if

$$\alpha = 400°,\ \alpha = 460°,\ \alpha = 481°,\ \alpha = 541°,\ \alpha = 632°,\ \alpha = 2184°,$$
$$\alpha = 12652°,\ \alpha = -362°,\ \alpha = -534°,\ \alpha = -1748°,$$
$$\alpha = 90°(2k+1)+20° \text{ for } k = 1, -1, 2, -2, 3, -3?$$

2. The angle $0 < \alpha < 360°$ terminates in (a) the second quadrant, (b) the third quadrant, (c) the fourth quadrant. In what quadrant does the angle $\frac{\alpha}{2}$ terminate?

885. A cyclist rides at a speed such that the wheel of the bicycle performs 240 revolutions per minute. What angle is described by the spoke during 15 seconds? during 2 seconds? during 0.5 second?

886. A gear and a pinion forming a toothed gearing have 24 and 30 teeth, respectively. Through what angle must the smaller wheel turn so that the larger wheel turns through 720°? Through what angle will the larger wheel turn, if the smaller one turns through 5 teeth?

887. A circle is divided into 6 equal parts (see Fig. 10). (1) Write the general form of the angles, if their terminal sides occupy the

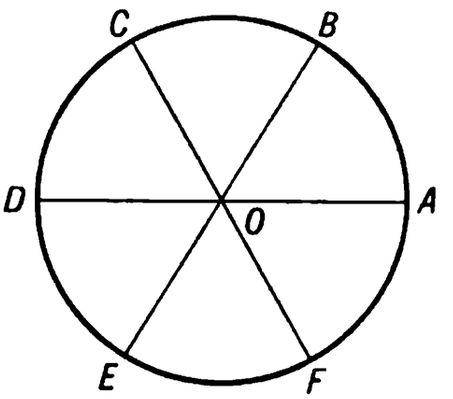


FIG. 10

following positions: (a) OC, (b) OE and if the initial side of the angles coincides with OA.

(2) Find $\smile ABC + \smile CDE + \smile ED + \smile FED + \smile DEFA$.

888. 1. Find the radian measure of the following angles in terms of the angle π:

$60°$, $135°$, $23°30'$, $-75°$, $108°$, $225°$, $-300°$, $1°$, $\alpha°$, $7200°$.

2. Using the Table convert the following angles to radian measure:

$7°30'$, $16°34'$, $-43°53'$, $83°58'$, $126°$, $-250°20'$, $352°10'$.

3. Express in degrees and minutes the magnitudes of the arcs given in radians

$$\frac{1}{5}\pi,\ \frac{2}{3}\pi,\ \frac{\pi}{8},\ \frac{\pi}{10},\ -\frac{3}{5}\pi,\ 1\frac{1}{4}\pi,\ -\frac{7}{12}\pi.$$

4. Using the Table find the degree measure of the following angles: 0.3491, 1.5708, −1.2800, 0.0098, 0.5000, −2.5405, 6.283.

889. 1. Two angles of a triangle are equal to $\frac{8}{15}\pi$ and $\frac{\pi}{5}$ radians. Find the value of each angle of the triangle in degrees.

2. The angles of a triangle are as 4 : 5 : 6. Determine the radian measure of these angles.

890. 1. The angle at the base of an isosceles triangle is equal to $37°15'$. Find the vertex angle and express it in radians.

2. One angle of an isosceles trapezoid equals $48°$. Express the remaining angles of the trapezoid in radians.

891. Determine the radian measure of the interior angle of a regular triangle, quadrilateral, pentagon, n-gon.

892. 1. Using the Tables, determine the radian measure of the angles of a quadrilateral if they are as 4 : 5 : 9 : 6.

2. A circle is divided by four points in the ratio 6 : 3 : 4 : 5. Express (in radians) each of the obtained arcs and the angles of the quadrilateral with the division points as vertices.

893. 1. Find the radian measure of the complementary to a right angle for the following acute angles: $\frac{\pi}{3}$, $\frac{\pi}{10}$, $\frac{7\pi}{20}$, 0.7, 0.7421.

2. For the given (in radians) angles find their adjacent angles in degrees: $\frac{3}{5}\pi$, $\frac{9\pi}{20}$, $\frac{2\pi}{9}$, 1.6021.

894. 1. The radius of a circle is equal to 5 cm. Express in degrees the arcs of the circle if their lengths are 12 cm, 18 cm, 31 cm.

2. The radius of a circle equals 25 cm. Determine the length of the arc subtending a central angle of (a) 1.25 radians, (b) 144°.

3. The arc of a rounded section of the railway track contains 20° and is 500 m long. Find the radius of the rounding.

895. 1. Compute using the Tables:

(a) sin 0.473, cos 1.1736, tan 0.9367, cot 1.2130;

(b) sin (cos 75°15′), cos (sin 0.1348), tan (sin 84°17′).

2. Find x in radians if:

(a) $\sin x = 0.7381$, $\cos x = 0.3154$, $\tan x = 15.75$;

(b) $\cot x = 7.1389$, $\sin(\tan x) = 0.372$, $\cos(\tan x) = 0.5462$.

Compute with the aid of a slide rule:

896. (1) sin 22°40′, sin 43°30′, sin 54°25′, sin 81°15′;

(2) sin 2°20′, sin 4°28′, tan 1°30′, tan 3°48′.

897. (1) tan 20°, tan 20°30′, tan 20°37′, tan 30°, tan 33°15′;

(2) tan 44°, tan 50°12′, tan 54°15′, tan 60°, tan 68°18′.

898. (1) cos 33°48′, cos 4°25′, cos 60°, cos 72°14′, cos 86°40′;

(2) cos 152°30′, cos 94°, cot 3°, cot 5°16′, cot 14°26′;

(3) cot 68°21′, cot 77°20′, cot 86°39′, cot 95°48′.

899. Find the angle x if:

(1) $\sin x = 0.43$, $\sin x = 0.061$, $\sin x = 0.0785$;

(2) $\cos x = 0.8960$, $\cos x = 0.032$, $\cos x = 0.264$;

(3) $\sin x = 0.785$, $\tan x = 0.0612$, $\tan x = 3.84$;

(4) $\tan x = 1.324$, $\cot x = 0.284$, $\cot x = 1.35$;

(5) $\cot x = 2.98$, $\cot x = 0.056$, $\cot x = 0.935$.

900. Compute:

(1) $4.2 \times \tan 12^\circ 30'$; (2) $760 \times \cos 80^\circ 6'$;

(3) $300 \times \sin 63^\circ 25'$; (4) $0.246 \times \cot 43^\circ 17'$.

901. Compute:

(1) $\dfrac{240}{\tan 32^\circ 57'}$; (2) $\dfrac{30.25}{\sin 38^\circ 20'}$; (3) $\dfrac{\sin 23^\circ 20'}{\tan 14^\circ 15'}$;

(4) $\dfrac{106 \tan 4^\circ 40'}{\tan 38^\circ 20'}$; (5) $\dfrac{\sin 2^\circ \sin 40^\circ}{\tan 10^\circ \tan 20^\circ}$.

902. The magnitude of a circular arc is $\frac{4\pi}{9}$. At what angle is the chord subtending this arc seen from the points on the arc?

2. At the end-point of a chord whose length is equal to the radius a tangent line is drawn to the circle. Determine the angle between the chord and the tangent in radians and degrees.

903. 1. Find the perimeter p of a circular sector if the radius is equal to R, and the corresponding central angle to α.

2. Compute the perimeter of a circular sector with the central angle of 50° and the radius equal to 1 m.

904. 1. Find the area of a circular sector whose radius is equal to 10 cm and the central angle to 72°.

2. Find the radius of a circular sector if its area equals 27 square units, and the central angle contains 35°.

3. Find the radian measure of the arc of a sector whose area is equal to 25.6 cm^2, and the radius of the arc to 4 cm.

905. 1. A body performs uniform motion in a circle making 4200 revolutions per hour. Find its angular velocity in radians per second.

2. A point taken on a circle rotates at a speed of 900 metres per minute. Find its angular velocity if the radius of the circle is equal to 2 dm.

906. 1. The shaft of a machine makes 25 revolutions per second. Find the angular velocity (in radians per second) and the linear speed of the point of the shaft 10 cm distant from the axis of rotation.

2. The angular velocity of a generator armature is 100π radians per second. How many revolutions per minute does the armature make?

907. 1. What angular velocity has a disk rotating at 240 revolutions per minute?

2. The angular velocity of a shaft is 62.8 1/s. How many revolutions per minute does the shaft make?

908. 1. A wheel rotates at an angular velocity $\omega = \frac{\pi}{4}$ rad/s. How many seconds does it take the wheel to complete one revolution?

2. What linear velocity has the point on a rotating disk 27 cm distant from the axis of revolution if the angular velocity of the disk is equal to $\frac{\pi}{3}$ rad/s. How long is the arc described by this point during 30 minutes?

909. The card of a marine compass is graduated into 32 equal parts which are called compass points. Express the compass point in radians and degrees.

910. Artillerymen are used to measure angles in mils. This unit is equal to $\frac{1}{6000}$ of a complete circle.

1. Express (a) the mil in minutes, the minute in mils, the degree in mils, (b) the angles of 36′, 3°, 12°, 48° in mils.

2. A gun is laying fire on a moving target. A layer has received a second order: "To the right 0-20 (20 mils)." Determine approximately the angular displacement of the target in degrees.

911. 1. Find the arc length of one mil putting $\pi \approx 3$ and denoting by R the radius.

2. Derive the formula for approximate calculation of the arc length l if its radius is R, and its corresponding central angle contains n mils. Find R from the obtained formula.

3. A tree 12 m high is seen by an observer at an angle of 15 mils. Find the distance between the observer and the tree.

39. TRIGONOMETRIC FUNCTIONS OF AN ARBITRARY ANGLE

912. Show that the definition of the sine, cosine, tangent and cotangent of the acute angle familiar to us from geometry are particular cases of the general definitions of the trigonometric functions.

913. For what values of the angle α does the equality $\sin\alpha = \cos\alpha$ take place? Derive the general formula of these angles.

914. The vector $\boldsymbol{a}$ forms an angle α with the x-axis. Find the values of $\sin\alpha$, $\cos\alpha$ and $\tan\alpha$ if

(1) $a_x = 8$, $a_y = 6$; (2) $a_x = 8$, $a_y = -15$;

(3) $a_x = 0$, $a_y = -\frac{3}{4}$; (4) $a_x = -3$, $a_y = 0$;

(5) $a_x = -2$, $a_y = 1$; (6) $a_x = -0.11$, $a_y = -0.6$.

915. Using the unit circle, find by direct measurement $\sin\alpha$, $\cos\alpha$, $\tan\alpha$, $\cot\alpha$ if the angle α is equal to 30°, 135°, 210°, -240°, -420°.

916. 1. In what quadrants do $\sin x$ and $\cos x$, $\sin x$ and $\tan x$, $\cos x$ and $\cot x$ have the same signs?

2. In what quadrant is the terminal side of the angle α situated if (a) $\sin a > 0$ and $\cos a < 0$, (b) $\tan\alpha < 0$ and $\cos\alpha > 0$?

917. 1. Determine the signs of:

(a) $\cos 179^\circ$, $\sin(-272^\circ)$, $\sin 345^\circ$, $\cos(-279^\circ)$;

(b) $\tan 135^\circ$, $\cot 200^\circ$, $\cot\frac{3}{5}\pi$, $\cos\frac{2\pi}{3}$, $\tan\left(-\frac{3\pi}{8}\right)$.

2. Determine the signs of the trigonometric functions if $0 < \alpha < 90^\circ$.

(a) $\sin(270^\circ - \alpha)$, $\tan(90^\circ + \alpha)$;

(b) $\sin(\alpha - 180^\circ)$, $\cos(2\alpha - 360^\circ)$.

918. 1. Determine the signs of the given products:

(a) $\sin 85^\circ \cos 280^\circ$; (b) $\sin 100^\circ \cos 100^\circ$;

(c) $\cos 130^\circ \tan 300^\circ$; (d) $\tan 200^\circ \cos 350^\circ$;

(e) $\tan 175^\circ \cot 185^\circ$; (f) $\tan(-18^\circ)\cos(-200^\circ)$.

2. If $0 < \alpha < 90^\circ$, determine the sign of the expression

$$\sin(180^\circ - \alpha) + \tan(90^\circ - \alpha) - \cos(270^\circ - \alpha).$$

3. Determine the sign of the differences:

(a) $\sin 20^\circ - \sin 21^\circ$; (b) $\cos 243^\circ - \cos 250^\circ$;

(c) $\tan 200^\circ - \tan 190^\circ$; (d) $\cot 73^\circ - \cot 293^\circ$;

(e) $\sin 1.2 - \tan 1.2$; (f) $\cos 2 - \cot 2$.

919. Which is greater

(1) $\sin 40^\circ$ or $\sin 40^\circ \sin 60^\circ$; (2) $\sin 55^\circ$ or $(\sin 55^\circ)^2$;

(3) $\cos 173^\circ$ or $\cos 173^\circ \cos 20^\circ$; (4) $\sin 20^\circ$ or $\tan 20^\circ$;

(5) $\sin 340^\circ$ or $\tan 340^\circ$; (6) $\cos 70^\circ$ or $\cot 70^\circ$;

(7) $\cos 140^\circ$ or $\cot 140^\circ$?

920. Indicate the intervals of variation of x within which the given functions attain (a) positive values, (b) negative values:

(1) $y = \sin x \cos x$; (2) $y = \sin x \tan x$;

(3) $y = \tan x + \cot x$; (4) $y = \dfrac{\cos x}{\cot x}$;

(5) $y = \sin\left(x + \dfrac{\pi}{3}\right)$; (6) $y = 1 - \sin x$.

921. Find the intervals within which the following inequalities hold true:

(1) $\cos x < 0$; (2) $\cos 2x > 0$; (3) $\cos \dfrac{x}{2} > 0$;

(4) $\sin 2x < 0$; (5) $\cos\left(x - \dfrac{\pi}{3}\right) > 0$; (6) $\tan 5x < 0$;

(7) $\cos(-2x) < 0$; (8) $\sin(-2x) > 0$; (9) $\sin(3x - 1) > 0$.

922. Let α, β, γ be the angles of a triangle. What sign do the following sums have?

$$\sin\alpha + \sin\beta + \sin\gamma; \quad \cos\alpha + \cos\beta + \cos\gamma;$$

$$\tan\alpha + \tan\beta + \tan\gamma;$$

$$\cos\frac{\alpha}{2} + \cos\frac{\beta}{2} + \cos\frac{\gamma}{2}; \quad \tan\frac{\alpha}{2} + \cot\frac{\beta}{2} + \cot\frac{\gamma}{2}.$$

923. 1. Prove the inequality $\sin x + \cos x > 1$ if $0^\circ < x < 90^\circ$.

2. Can the value of $\tan\alpha$ be less than that of $\sin\alpha$ in absolute value?

3. Can the value of $\cot\alpha$ be less than that of $\cos\alpha$ in its absolute value?

Justify the answers to questions 2 and 3 analytically and geometrically.

924. 1. What is the difference between the domains of definition of the given functions:

(a) $\sin x$ and $\tan x$; (b) $\cos x$ and $\cot x$?

2. Indicate the domains of variation of the following functions:

(a) $1+\sin x$; (b) $1-\cos x$; (c) $3+2\sin x$;

(d) $4-3\cos x$; (e) $1-|\sin x|$; (f) $1+|\cos x|$;

(g) $1+|\tan x|$; (h) $\frac{3\sin x-2}{4}$.

925. 1. On the interval $[0,2\pi]$ indicate the subintervals within which the functions $\sin x$, $\cos x$, $\tan x$, $\cot x$, (a) increase, (b) decrease, (c) attain positive values, (d) attain negative values.

2. On the interval $[0,2\pi]$ indicate the subintervals within the functions $\sin x$ and $\cos x$ (a) increase, (b) decrease simultaneously.

926. In the interval $0 < x < 90°$ which is greater (a) 3 or $3\sin x$, (b) $\sin x$ or $3\sin x$, (c) $\sin x$ or $\sin x\cos x$, (d) $\sin x$ or $\sin x\tan x$.

927. Establish whether the following equalities are possible:

(1) $\sin x = \frac{m}{m-1}$, $m > 1$; (2) $\sin x = \frac{\pi}{4}$;

(3) $\cos x = \frac{\pi}{2}$; (4) $\sec x = \sqrt{0.16}$;

(5) $\sec x = \frac{\pi}{2}$; (6) $\sin\alpha = \sqrt{3}-2$;

(7) $\cos\alpha = -\sqrt[6]{6}$; (8) $\sin\alpha = -\sqrt[3]{1.1}$;

(9) $\cos\alpha = \sqrt{2}-1$; (10) $\sin\alpha = \sqrt[3]{29}-\sqrt{11}$.

928. Which of the trigonometric functions can attain the following values:

(1) $\frac{1}{3}$, -0.75, -2, -1; (2) $\sqrt{0.1}$, $\frac{\pi}{3}$, $m+\frac{1}{m}$;

(3) $\frac{2\sqrt{ab}}{a+b}$, $a > 0$, $b > 0$; (4) $\frac{a+b}{2\sqrt{ab}}$, $a > 0$, $b > 0$.

929. Using the trigonometric circle solve the following inequalities:

(1) $\sin x > \frac{\sqrt{3}}{2}$; (2) $\sin x < \sin 30°$;

(3) $\cos 150° > \cos x$; (4) $|\cos x| > \frac{1}{2}$;

(5) $\tan 2x \geqslant 1$; (6) $|\sin 2x| \geqslant \frac{\sqrt{3}}{2}$;

(7) $2\sin^2 x - \sin x > 0$;

(8) $4\cos^2 x - 2(\sqrt{3}-1)\cos x - \sqrt{3} < 0$.

930. Find the domain of definition of the given function:

(1) $y = \frac{1}{\cos x}$; (2) $y = \frac{\sin x}{\cos x}$; (3) $y = \frac{2}{1-\cos x}$;

(4) $y = \sqrt{\sin x}$; (5) $y = \sqrt[3]{\cos x}$; (6) $y = \sqrt{\sin 2x}$;

(7) $y = \sqrt{-\cos x}$; (8) $y = \sqrt{\sqrt{3}-2\cos x}$;

(9) $y = \frac{5}{\sqrt{3}-\tan x}$; (10) $y = \frac{\cos x}{\sqrt{1-\sin x}}$;

(11) $y = \frac{2}{\sin x+\cos x}$.

931. Prove that

(1) $|3\cos x+2\sin x| < 5$; (2) $|2\sin x-10\cos x| < 12$.

932. Find the least and the greatest values of the following functions:

(1) $y = 2+3\sin x$; (2) $y = 1-2\cos x$;

(3) $y = 5+\sin^2 x$; (4) $y = \frac{1}{1-\sin x}$;

(5) $y = \frac{1}{1+\sin x}$; (6) $y = \frac{2}{3+\tan x}$;

(7) $y = 4\sin^2 x+5\cos^2 x$; (8) $y = \sin^2 x+2\sin x+1$.

933. Which of the given functions are bounded and which are unbounded?

(1) $2\sin x-5$; (2) $1-\tan x$; (3) $3+\cot x$;

(4) $\cos\left(2x-\frac{\pi}{3}\right)$; (5) $x+2\sin x$; (6) $|x\cos x|$;

(7) $\cos x+\tan x$; (8) $\frac{1}{\tan x-1}$; (9) $\frac{4}{\tan^2 x+2}$?

934. 1. For what values of x does the function $y = \sin(\sin x)$ take on the greatest value and for what values of x the least value. Using the Table find these values of the function.

2. Solve an analogous problem with respect to the function $y = 1 - \cos(\sin x)$.

935. Compute:

(1) $2 \sin 270^\circ - \cos 90^\circ + 3 \tan 180^\circ$;

(2) $3 \cos 0^\circ - 2 \sin 270^\circ + 6 \tan 360^\circ$;

(3) $3 \cos 0^\circ - 3 \sin 90^\circ + 4 \cot 90^\circ$;

(4) $5 \sin 270^\circ + 3 \cot 270^\circ + 2 \cos 180^\circ$.

936. Compute: (1) $2 \sin \frac{\pi}{3} + 2 \cos \frac{\pi}{6} + \tan \frac{\pi}{6}$;

(2) $2 \cos \pi + 3 \cos \frac{3\pi}{2} + 6 \tan \frac{\pi}{3}$;

(3) $a \sin \pi + b \cos \pi + c \tan \pi$;

(4) $m \cos \frac{\pi}{2} + n \cos \pi + p \sin \frac{3\pi}{2} + q \tan 2\pi$.

937. (1) $4 \sin 810^\circ + 3 \cos 720^\circ - 3 \sin 630^\circ + 5 \cos 900^\circ$;

(2) $5 \tan 540^\circ + 2 \cos 1170^\circ + 4 \sin 990^\circ - 3 \cos 540^\circ$;

(3) $100 \cot^2 990^\circ + 25 \tan^2 540^\circ - 3 \cos^2 990^\circ$;

(4) $3 \sin \frac{25\pi}{6} - 3 \tan \frac{13\pi}{4} + 2 \cos \frac{19\pi}{3}$.

938. Evaluate the given expression

(1) $2 \sin(\alpha - 45^\circ) + \sin(\alpha + 45^\circ) + 5 \cos 2\alpha + 2 \cos(135^\circ - \alpha)$ for $\alpha = 45^\circ$;

(2) $2 \sin \alpha + \cos 2\alpha + \frac{1}{2} \tan(\alpha + 15^\circ) + 2 \cos 6\alpha$ for $\alpha = 30^\circ$;

(3) $2 \sin(\pi - \alpha) + 6 \cos\left(\alpha - \frac{\pi}{3}\right) + \tan\left(\alpha - \frac{2\pi}{3}\right)$ for $\alpha = \frac{5\pi}{6}$;

(4) $\tan(\alpha - 4\pi) \cot(\alpha - 4\pi) - \sin\left(\frac{14\pi}{3} - \alpha\right)$ for $\alpha = \frac{25\pi}{6}$.

939. Simplify the following expressions:

(1) $m^2 \sin^2 \frac{\pi}{2} - n^2 \sin \frac{3\pi}{2} + 2mn \sec \pi$;

(2) $\dfrac{a^2 \cos 0 - b^2 \sin \frac{\pi}{2}}{a \tan^2 \frac{\pi}{4} + b \cos \pi - 2ab \cot \frac{3\pi}{2}}$;

(3) $\dfrac{a^3 \cot^2 \dfrac{3\pi}{4} + b^3 \cos^2 \pi}{a^2 \sin^2 \dfrac{\pi}{2} + ab \sin \dfrac{3\pi}{2} - b^2 \cos \pi}$;

(4) $4a^2 \sin^2 \dfrac{\pi}{4} - 3\left(a \tan \dfrac{\pi}{4}\right)^2 + \left(2a \cos \dfrac{\pi}{4}\right)^2$.

940. 1. Construct the angle x, if $\sin x = \frac{1}{3}$. How many solutions has this problem? Measure the constructed angles with the aid of a protractor and write the formula for the set of angles corresponding to the terminal side of each angle.

2. Solve a similar problem if

$$\sin x = -\frac{1}{4}, \quad \cos x = \frac{1}{2}, \quad \cos x = -\frac{2}{3},$$

$$\tan x = 1.5, \quad \tan x = -2, \quad \cot x = 2, \quad \cot x = 2.3.$$

941. Construct a positive angle $x(0^\circ < x < 360^\circ)$ if:

(1) $\sin x = -\frac{1}{3}$ and $\cos x > 0$;

(2) $\cos x = -\frac{3}{4}$ and $\tan x > 0$;

(3) $\cot x = -0.75$ and $\sin x < 0$;

(4) $\csc x = -1.5$ and $\cos x < 0$.

942. Using the Table find the acute angle x and write the formula for the set of angles corresponding to the position of the terminal side of the found angle if:

(1) $\sin x = 0.7346$; (2) $\cos x = 0.3748$;

(3) $\cot x = 1.7343$; (4) $\sin (x+18^\circ) = 0.3441$;

(5) $\sin (3x+18^\circ) = 0.3441$;

(6) $\cos \dfrac{x+22^\circ}{3} = 0.1744$;

(7) $\tan \dfrac{2x+30^\circ}{4} = 2.1748$; (8) $\cot \dfrac{3x-60^\circ}{2} = 1.1748$.

943. Find the period of the given function:

(1) $y = \sin x + \cos x$; (2) $y = \cos 2x$;

(3) $y = \sin x + \tan x$; (4) $y = \frac{\sin x}{3}$;

(5) $y = \tan 3x$; (6) $y = \sin \frac{x}{a}$;

(7) $y = |\cos x|$; (8) $y = |\tan x|$; (9) $y = |\cot x|$.

944. Find the period of the given function:

(1) $y = 2 \sin x$; (2) $y = \sin 2x$; (3) $y = \cos \frac{x}{3}$;

(4) $y = \sin 2x + \cos 4x$; (5) $y = |\sin 2x|$;

(6) $y = |\sin x| + 2$.

40. RELATIONS BETWEEN TRIGONOMETRIC FUNCTIONS OF ONE AND THE SAME ANGLE

945. Can the following equalities exist for one and the same value of the argument?

(1) $\sin x = 0.6$, $\cos x = 0.8$; (2) $\sin x = -\frac{1}{2}$, $\cos x = \frac{2}{3}$;

(3) $\sin x = \frac{5}{13}$, $\cos x = -\frac{12}{13}$, $\tan x = -\frac{5}{12}$;

(4) $\sin x = \frac{6\sqrt{2}}{11}$, $\cos x = -\frac{7}{11}$, $\tan x = -\frac{6\sqrt{2}}{7}$;

(5) $\sin x = \frac{7}{9}$, $\cos x = -\frac{4\sqrt{2}}{9}$, $\cot x = \frac{4\sqrt{2}}{7}$.

946. 1. Find the trigonometric functions of a positive acute angle α if:

(a) $\sin \alpha = \frac{3}{5}$; (b) $\cos \alpha = \frac{40}{41}$; (c) $\tan \alpha = 0.75$;

(d) $\cot \alpha = \frac{\sqrt{7}}{3}$; (e) $\sec \alpha = 3$; (f) $\csc \alpha = 1.25$.

2. Find the trigonometric functions of a negative acute angle α if:

(a) $\sin \alpha = -\frac{13}{14}$; (b) $\cos \alpha = \frac{2}{3}$; (c) $\tan \alpha = -\frac{9}{40}$.

947. 1. Find the trigonometric functions of the angle A of a triangle if:

(a) $\sin A = 0.28$; (b) $\cos A = -0.5$; (c) $\tan A = \frac{\sqrt{7}}{3}$.

2. In a right triangle ABC ($C = 90^\circ$) the side $BC = 15$ m and the altitude $CD = 12$ m. Determine the trigonometric functions of the acute angle A.

948. Evaluate the remaining trigonometric functions of the argument α if:

(1) $\sin\alpha = \frac{15}{17}$ and $\frac{\pi}{2} < \alpha < \pi$;

(2) $\cos\alpha = -\frac{4}{5}$ and $\pi < \alpha < \frac{3\pi}{2}$;

(3) $\tan\alpha = 1\frac{1}{3}$ and $\pi < \alpha < \frac{3\pi}{2}$;

(4) $\cot\alpha = -\frac{\sqrt{3}}{3}$ and $\frac{3\pi}{2} < \alpha < 2\pi$;

(5) $\sin\alpha = 0.2$ and $\tan\alpha < 0$;

(6) $\tan\alpha = -3$ and $\cos\alpha < 0$.

949. Evaluate the remaining trigonometric functions of the argument x if:

(1) $\sin x = 0.28$ and $0 < x < 90^\circ$;

(2) $\cos x = -\frac{\sqrt{3}}{2}$ and $90^\circ < x < 180^\circ$,

(3) $\tan x = 1.124$ and $180^\circ < x < 270^\circ$;

(4) $\cot x = -1.5$ and $270^\circ < x < 360^\circ$.

950. Express each of the trigonometric functions of the argument α in terms of the remaining trigonometric functions of this argument.

951. 1. Knowing that $\sin\alpha = \frac{a}{b}$ where $0 < a < b$ and $90^\circ < \alpha < 180^\circ$, find the values of the remaining trigonometric functions of the angle α.

2. Knowing that $\cos\alpha < 0$ and $\tan\alpha = \frac{a^2}{b^2}$, evaluate the remaining trigonometric functions of the angle α. What values can the parameters a and b attain?

952. 1. Evaluate the expression $\frac{\cot\alpha + \tan\alpha}{\cot\alpha - \tan\alpha}$ if $\sin\alpha = \frac{3}{5}$ and $0 < \alpha < 90^\circ$.

2. Evaluate the expression $\frac{\sin\alpha + \cos\alpha}{\cos\alpha - \sin\alpha}$ if $\tan\alpha = -2$.

3. Evaluate the expression $\frac{\sin^2\alpha-\cos^2\alpha}{\sin^2\alpha-2\cos^2\alpha}$ if $\cot\alpha=3$.

953. Simplify and then evaluate the following expressions:

(1) $\tan\alpha-\tan\alpha\sin^2\alpha$ if $\cos\alpha=-\frac{4}{5}$ and $\pi<\alpha<\frac{3\pi}{2}$;

(2) $\sin\alpha+\cos\alpha\tan\alpha$ if $\sec\alpha=2$ and $\frac{3\pi}{2}<\alpha<2\pi$;

(3) $\tan^2\alpha-\frac{1}{\sin\alpha\cos\alpha}+\cot^2\alpha$ if $\tan\alpha+\cot\alpha=3$.

954. Express in terms of $\tan\alpha$ (of $\cot\alpha$):

(1) $\frac{1}{\sin^2\alpha-\sin\alpha\cos\alpha+\cos^2\alpha}$;

(2) $\frac{1}{1-\cos^2\alpha}$; (3) $\frac{4\sin\alpha-2\cos\alpha}{5\cos\alpha+3\sin\alpha}$.

955. 1. Compute (a) $\tan^2\alpha+\cot^2\alpha$, (b) $\tan^3\alpha+\cot^3\alpha$ if $\tan\alpha+\cot\alpha=5$.

2. Compute (a) $\tan^2\alpha+\cot^2\alpha$, (b) $\tan^3\alpha-\cot^3\alpha$ if $\tan\alpha-\cot\alpha=m$.

3. Compute (a) $\sin\alpha\cos\alpha$, (b) $\sin\alpha-\cos\alpha$, (c) $\sin^4\alpha+\cos^4\alpha$ if $\sin\alpha+\cos\alpha=p$.

956. Find (1) $\tan\alpha$ if $\cos\alpha=0.6$; (2) $\sin\alpha$ if $\cot\alpha=\frac{1}{8}$; (3) $\cot\alpha$ if $\sin\alpha=\frac{1}{4}$; (4) $\cos\beta$ if $\tan\beta=1\frac{1}{3}$; (5) $\sec\beta$ if $\sin\beta=1$; (6) $\csc\varphi$ if $\cos\varphi=0$.

Simplify the following expressions:

957. (1) $1-\cos^2\alpha$; (2) $\tan\alpha\cos\alpha$; (3) $\sin^2\alpha-1$;

(4) $\frac{\sin^2\alpha}{\cos\alpha-1}$; (5) $\frac{\tan\alpha+\tan\beta}{\cot\alpha+\cot\beta}$;

(6) $\frac{\tan\alpha}{1-\tan^2\alpha}\times\frac{\cot^2\alpha-1}{\cot\alpha}$;

(7) $1+\cos^2\alpha-\sin^2\alpha$; (8) $\sin\alpha\cot\alpha+\cos\alpha$.

958. (1) $(1-\sin\alpha)(1+\sin\alpha)$; (2) $3-\sin^2\alpha-\cos^2\alpha$;

(3) $\frac{\sec\alpha-\tan\alpha}{\sec\alpha}$; (4) $(\sin\alpha+\cos\alpha)^2+(\sin\alpha-\cos\alpha)^2$;

(5) $\frac{1}{1+\tan^2 2\alpha}+\frac{1}{1+\cot^2 2\alpha}$;

(6) $\cot\alpha-\frac{\cos\alpha}{1+\sin\alpha}$.

959. (1) $\sin^4\alpha-\cos^4\alpha-\sin^2\alpha+\cos^2\alpha$;

(2) $\sin^4\alpha+2\sin^2\alpha\cos^2\alpha+\cos^4\alpha$;

(3) $\frac{2\sin^2\alpha-1}{1-2\cos^2\alpha}$;

(4) $\sqrt{\frac{2}{1+\cos\alpha}+\frac{2}{1-\cos\alpha}}$;

(5) $1-\frac{\sin^2\alpha}{1+\cot\alpha}-\frac{\cos^2\alpha}{1+\tan\alpha}$.

960. (1) $\frac{1}{\cos^2\alpha}-\tan^2\alpha-\sin^2\alpha$;

(2) $\cos^2\beta+\cot^2\beta-\frac{1}{\sin^2\beta}$;

(3) $\frac{1}{\sin^2 x}-\cot^2 x(1+\sin^2 x)$;

(4) $\frac{1}{\cos^2\alpha}-\tan^2\alpha\,(\cos^2\alpha+1)$.

961. (1) $1-\cos^2\alpha+\tan^2\alpha\cos^2\alpha$;

(2) $(1+\cot^2\alpha)\sin^2\alpha$;

(3) $\cos^4\alpha+\sin^4\alpha+2\cos^2\alpha\sin^2\alpha-\cos^2\alpha$;

(4) $\frac{\cos\alpha\cos\beta}{\sin\alpha\sin\beta}\cot\alpha\tan\beta+1$;

(5) $\frac{\sin^2\alpha-1}{\cos^2\alpha-1}+\tan\alpha\cot\alpha$;

(6) $\frac{\sin^2\alpha-\tan^2\alpha}{\cos^2\alpha-\cot^2\alpha}$.

962. (1) $(\tan\alpha+\cot\alpha)^2-(\tan\alpha-\cot\alpha)^2$;

(2) $1-\sin\alpha\tan\alpha\cos\alpha$; (3) $\frac{\cos^2\alpha}{1-\cos^2\alpha}\tan^2\alpha$;

(4) $\frac{\sin^2\alpha-\cos^2\alpha}{\sin\alpha\cos\alpha}$;

(5) $\dfrac{1}{\cos\alpha\,(\sin^2\alpha+\cos^2\alpha+\tan^2\alpha)}$;

(6) $\dfrac{1}{\cos^2\alpha}-\dfrac{1}{\cot^2\alpha}$.

963. (1) $\dfrac{1}{\tan\alpha+\cot\alpha}$; (2) $\left(1-\dfrac{1}{\cos^2\alpha}\right)\left(1-\dfrac{1}{\sin^2\alpha}\right)$;

(3) $\dfrac{\sin\alpha\cot\alpha}{\cos^2\alpha}-\tan\alpha\sin\alpha$;

(4) $\dfrac{1+\cos\alpha}{\sin^2\alpha}:\left[1+\left(\dfrac{1+\cos\alpha}{\sin\alpha}\right)^2\right]$.

964. (1) $\dfrac{\sin\alpha}{1+\cos\alpha}-\dfrac{\sin\alpha}{\cos\alpha-1}$;

(2) $\dfrac{\cos\beta}{1-\sin\beta}+\dfrac{\cos\beta}{1+\sin\beta}$;

(3) $(\tan\alpha+1)^2+(\tan\alpha-1)^2$;

(4) $(\cot\beta+1)^2+(\cot\beta-1)^2$;

(5) $\cot\alpha+\dfrac{\sin\alpha}{1+\cos\alpha}$;

(6) $\tan\beta+\dfrac{\cos\beta}{1+\sin\beta}$.

965. (1) $\sqrt{1-\cos^2 2\alpha}$; (2) $\sqrt{1-\sin^2\dfrac{\alpha}{2}}$;

(3) $\sqrt{\dfrac{1+\sin\alpha}{1-\sin\alpha}}+\sqrt{\dfrac{1-\sin\alpha}{1+\sin\alpha}}$;

(4) $\sqrt{\dfrac{1+\cos\alpha}{1-\cos\alpha}}+\sqrt{\dfrac{1-\cos\alpha}{1+\cos\alpha}}$;

(5) $\left(\sqrt{\dfrac{1+\sin\alpha}{1-\sin\alpha}}-\sqrt{\dfrac{1-\sin\alpha}{1+\sin\alpha}}\right)\times$

$\times\left(\sqrt{\dfrac{1+\cos\alpha}{1-\cos\alpha}}-\sqrt{\dfrac{1-\cos\alpha}{1+\cos\alpha}}\right)$.

966. For what values of x do the following equalities hold true?

(1) $\cos x=-\sqrt{1-\sin^2 x}$; (2) $\tan x=-\dfrac{\sin x}{\sqrt{1-\sin^2 x}}$;

(3)* $\sqrt{\dfrac{1-\cos x}{1+\cos x}}-\sqrt{\dfrac{1+\cos x}{1-\cos x}}=2\cot x$;

(4) $\sin x \sqrt{\frac{1}{1+\cos x}+\frac{1}{1-\cos x}} = \sqrt{2}$.

967. Simplify the following expressions:

(1) $\sqrt{\frac{1-\sin\alpha}{1+\sin\alpha}}-\sqrt{\frac{1+\sin\alpha}{1-\sin\alpha}}$ if $\frac{\pi}{2}<\alpha<\pi$;

(2) $\frac{1}{\sin\alpha-\sqrt{\cot^2\alpha-\cos^2\alpha}}$ if $\pi<\alpha<2\pi$;

(3) $\sqrt{1-\tan^2\alpha\cos^2\alpha}+1$, verify the obtained equality for $\alpha=\frac{\pi}{3}$ and for $\alpha=\pi$.

968. 1. Verify the validity of the equality $\frac{\cos\alpha}{1+\sin\alpha}=2-\tan\alpha$ for $\alpha=60°$ and $\alpha=-60°$.
Will this equality be a trigonometric identity?

2. Prove the equality $\frac{\tan^2\alpha-\tan^2\beta}{\tan^2\alpha\tan^2\beta}=\frac{\sin^2\alpha-\sin^2\beta}{\sin^2\alpha\sin^2\beta}$ and indicate the permissible values for the arguments α and β.

Prove the following identities:

969. (1) $\sin^4\alpha-\cos^4\alpha=\sin^2\alpha-\cos^2\alpha$;

(2) $(1+\tan^2\alpha)\cos^2\alpha=1$;

(3) $\frac{\cos^2\alpha-1}{\sin^2\alpha-1}=\tan^2\alpha$;

(4) $\frac{5\cos\alpha-4}{3-5\sin\alpha}-\frac{3+5\sin\alpha}{4+5\cos\alpha}=0$.

970. (1) $(1-\cos^2\alpha)(1+\tan^2\alpha)=\tan^2\alpha$;

(2) $\frac{1}{\sin\alpha}-\sin\alpha=\cos\alpha\cot\alpha$;

(3) $\frac{1}{\sin^2\alpha}-\frac{1}{\tan^2\alpha}=1$;

(4) $\frac{1-\sin\alpha}{\cos\alpha}=\frac{\cos\alpha}{1+\sin\alpha}$.

971. (1) $\frac{\sin\alpha}{\cos\alpha\tan^2\alpha}=\cot\alpha$;

(2) $\cot^2\alpha-\cos^2\alpha=\cos^2\alpha\cot^2\alpha$;

(3) $\frac{1-2\sin^2\alpha}{2\cos^2\alpha-1}=1$;

(4) $(1+\cot\alpha)^2+(1-\cot\alpha)^2=\dfrac{2}{\sin^2\alpha}$;

(5) $\dfrac{1}{\sin^2\alpha-\cos^2\alpha}=\dfrac{\tan^2\alpha+1}{\tan^2\alpha-1}$;

(6) $\dfrac{\sin^3\alpha-\cos^3\alpha}{1+\sin\alpha\cos\alpha}=\sin\alpha-\cos\alpha.$

972. (1) $\dfrac{\sin\alpha}{1+\cot\alpha}+\dfrac{\cos\alpha}{1+\tan\alpha}=\dfrac{1}{\sin\alpha+\cos\alpha}$;

(2) $\dfrac{\sin\alpha-\cos\alpha}{\sin\alpha+\cos\alpha}=\dfrac{\tan\alpha-1}{\tan\alpha+1}$;

(3) $\dfrac{\sin^2\alpha-\cos^2\alpha}{\sin\alpha\cos\alpha}=\dfrac{\tan^2\alpha-1}{\tan\alpha}$;

(4) $\dfrac{\sin\alpha}{1+\cos\alpha}=\dfrac{1-\cos\alpha}{\sin\alpha}$;

(5) $\cot^2\alpha-\tan^2\alpha=\csc^2\alpha-\sec^2\alpha$;

(6) $\dfrac{\cos\alpha+\tan\alpha}{\cot\alpha+\sec\alpha}=\sin\alpha.$

973. (1) $\dfrac{\cos\alpha+\cot\alpha}{\cot\alpha}=1+\sin\alpha$;

(2) $\dfrac{\sin\alpha+\tan\alpha}{\tan\alpha}=1+\cos\alpha$;

(3) $\dfrac{\tan\alpha}{\tan\alpha+\cot\alpha}=\sin^2\alpha$;

(4) $\dfrac{\cot\alpha}{\tan\alpha+\cot\alpha}=\cos^2\alpha$;

(5) $\dfrac{1+\tan\alpha}{1+\cot\alpha}=\tan\alpha$;

(6) $\dfrac{\cot\beta+1}{\tan\beta+1}=\cot\beta.$

974. (1) $\sin^4\alpha+\sin^2\alpha\cos^2\alpha+\cos^2\alpha=1$;

(2) $\sin^2\alpha+\sin^2\alpha\cos^2\alpha+\cos^4\alpha=1$;

(3) $\dfrac{(\sin 2\alpha+\cos 2\alpha)^2-1}{\cot 2\alpha-\sin 2\alpha\cos 2\alpha}=2\tan^2 2\alpha$;

(4) $\dfrac{\left(\sin\dfrac{\alpha}{2}+\cos\dfrac{\alpha}{2}\right)^2-1}{\tan\dfrac{\alpha}{2}-\sin\dfrac{\alpha}{2}\cos\dfrac{\alpha}{2}}=2\cot^2\dfrac{\alpha}{2}.$

975. (1) $\dfrac{\sin^2\alpha}{\sec^2\alpha-1}+\dfrac{\cos^2\alpha}{\csc^2\alpha-1}=1$;

(2) $\tan\alpha+\cot\alpha=\sec\alpha\csc\alpha$;

(3) $\csc\alpha-\cos\alpha\cot\alpha=\sin\alpha$;

(4) $\cos^2\alpha+\cos^2\alpha\cot^2\alpha=\cot^2\alpha$;

(5) $(\tan^2\alpha-\sin^2\alpha)\dfrac{\cot^2\alpha}{\sin^2\alpha}=1$;

(6) $\left(\dfrac{1}{\cos\alpha}-\cos\alpha\right)\left(\dfrac{1}{\sin\alpha}-\sin\alpha\right)=\sin\alpha\cos\alpha$.

976. (1) $1+\sin\alpha+\cos\alpha+\tan\alpha=(1+\cos\alpha)(1+\tan\alpha)$;

(2) $(1+\sin\alpha)(1+\cos\alpha)=(\sin\alpha+\tan\alpha)(\cos\alpha+\cot\alpha)$;

(3) $\sin^6\alpha+\cos^6\alpha=1-3\sin^2\alpha\cos^2\alpha$;

(4) $\dfrac{\cos^3\alpha+\sin^3\alpha}{1-\sin\alpha\cos\alpha}=\cos\alpha+\sin\alpha$.

977. (1) $\sin^3\alpha(1+\cot\alpha)+\cos^3\alpha(1+\tan\alpha)=\sin\alpha+\cos\alpha$;

(2) $\dfrac{2}{\tan\alpha+\cot\alpha}+\tan\alpha\cot\alpha=(\sin\alpha+\cos\alpha)^2$;

(3) $\sqrt{\dfrac{1-\cos\alpha}{1+\cos\alpha}}=\dfrac{|\sin\alpha|}{1+\cos\alpha}$;

(4) $\sqrt{\dfrac{1+\cos\alpha}{1-\cos\alpha}}=\dfrac{1+\cos\alpha}{|\sin\alpha|}$.

978. Solve the following equations:

(1) $\cos^2 x=1+\sin^2 x$; (2) $3\sin x+2=2\cos^2 x$;

(3) $4\sin^2 x+2\cos^2 x=3$; (4) $\sin x=\cos x$;

(5) $\cos x=2\tan x$; (6) $\cot x-\tan x=\dfrac{3}{2}$.

979. Eliminate t from the following systems of equalities:

(1) $\begin{cases}x=5\cos t\\ y=5\sin t;\end{cases}$ (2) $\begin{cases}x=3\cos t\\ y=5\sin t;\end{cases}$

(3) $\begin{cases}\sin t+\cos t=x\\ \sin t\cos t=y;\end{cases}$ (4) $\begin{cases}\tan^2 t+\cot^2 t=x\\ \tan^4 t+\cot^4 t=y.\end{cases}$

980. Prove the following inequalities:

(1) $\sin\alpha\cos\alpha \leqslant \frac{1}{2}$; (2) $\cos(\sin x) > 0$;

(3)* $0 < \sin^2 x+\cos^4 x \leqslant 1$; (4)* $|\sin\alpha+\cos\alpha| \leqslant \sqrt{2}$;

(5) $\tan x+\cot x \geqslant 2$ if $0 < x < \frac{\pi}{2}$;

(6) $2\sin^2 x-3\sin x+3 > 0$;

(7)* $\sin^4 x-7\sin^2 x+6 \geqslant 0$; (8) $\frac{\sin x+1}{\sin x-1} < 0$.

981. Solve the following inequalities:

(1)* $(\sin x+1)\sin x(2\sin x-1) > 0$;

(2)* $(\cos x+1)\cos x(2\cos x-1) \leqslant 0$;

(3) $\cos^2 x-3\cos x > 0$; (4)* $2\sin^2 x+\sin x < 0$

(5)* $\frac{\sin x}{\sin x-1} < 0$; (6) $\frac{3}{2\sin x+1} < 0$;

(7) $4\sin^4 x-5\sin^2 x+1 \leqslant 0$;

(8) $\sin^2 x-\cos^2 x+\sin x > 0$.

41. REDUCTION FORMULAS

982. Replace by the functions of complementary angles:

(1) $\sin 65°$; (2) $\cos 0.4\pi$; (3) $\tan(45°+\alpha)$;

(4) $\cot\left(60°-\frac{\alpha}{2}\right)$; (5) $\sin(45°-\alpha)$;

(6) $\cos(90°-3\alpha)$.

983. Given: $\cos\alpha+\cos\beta = 1.5$ and $\alpha+\beta = 90°$. Find:

(1) $\sin^2(90°-\alpha)+\sin^2(90°-\beta)$;

(2) $\sin(90°-\alpha)\sin(90°-\beta)$.

984. Given: $\sin(45°+\alpha)+\sin(45°-\alpha) = a$. Find:

(1) $\cos^2(45°+\alpha)+\cos^2(45°-\alpha)$;

(2) $\cos(45°+\alpha)\cos(45°-\alpha)$.

985. Given: $\tan\left(\frac{\pi}{4}-\alpha\right)+\tan\left(\frac{\pi}{4}+\alpha\right) = 3$. Find:

(1) $\tan^2\left(\frac{\pi}{4}-\alpha\right)+\tan^2\left(\frac{\pi}{4}+\alpha\right)$;

(2) $\tan\left(\frac{\pi}{4}-\alpha\right)\tan\left(\frac{\pi}{4}+\alpha\right)$.

986. Compute the following using the reduction formulas:

(1) cos 120°, sin 150°, cot 135°, tan 150°;

(2) sin 210°, cos 225°, tan 240°, cot 225°;

(3) tan 300°, sin 315°, cos 330°, cot 330°.

987. Compute the following using the reduction formulas and the tables of trigonometric functions:

(1) sin (−117°), cos (−136°17′), tan (−172°15′);

(2) cot (−194°47′), sin (−205°34′), cos (−250°41′);

(3) tan (−342°26′), cot (−292°27′), sin (−352°15′).

988. Reduce the following trigonometric functions to those of positive angles smaller than 45°:

(1) sin 54°, cos 81°, tan (−37°), cot (−65°);

(2) sin 146°, cos 132°, tan 174°, cot 164°;

(3) sin 265°, cos 208°, tan 350°, cot 356°;

(4) sin (−343°), cos (−454°), tan (−312°), cot (−275°);

(5) sin (−1364°), cos (−10,742°), tan (−5600°), cot (−3000°).

989. Reduce the following to the like trigonometric functions of positive acute angles:

(1) sin 200°, cos 154°, tan 357°, cot 289°;

(2) sin (−501°), cos (−304°), tan (−1894°), cot (−1342°);

(3) tan 0.8π, sin 2.9π, cos (-6.4π), cot 5.7π;

(4) sin 5, cos 4, tan (−5), cot 2.

990. Reduce the following to the trigonometric functions of positive acute angles:

(1) sin 154°, cos 242°, tan (−253°), cot (−346°);

(2) sin 1242°, cos 930°, tan 2402°, cot 3546°.

991. Reduce the following trigonometric functions to the functions of the least positive argument:

(1) $\sin 17.6\pi$, $\cos 13.5\pi$, $\tan 8.3\pi$, $\cot 5\frac{1}{3}\pi$;

(2) $\sin(-24.3\pi)$, $\cos(-7.12\pi)$, $\tan 12.5\pi$, $\cot(-18.9\pi)$;

(3) $\sin 5.3$, $\cos 7.8$, $\tan 3.4$, $\cot 12.5$.

992. Write several positive and negative angles such that (1) the sine of each of them is equal to $\sin 47°$; (2) the cosine of each of them is equal to $\sin 55°$; (3) the tangent of each of them equals $\cot 88°$.

993. Find the angle α if:

(1) $\sin\alpha = 0.1741$, $90° < \alpha < 180°$;

(2) $\cos\alpha = 0.5431$, $270° < \alpha < 360°$;

(3) $\tan\alpha = 2.347$, $180° < \alpha < 270°$;

(4) $\tan\alpha = -5.478$, $90° < \alpha < 180°$;

(5) $\sin\alpha = -0.4754$, $180° < \alpha < 270°$;

(6) $\cos\alpha = -0.4812$, $180° < \alpha < 270°$;

(7) $\cot\alpha = -3.748$, $270° < \alpha < 360°$;

(8) $\cot\alpha = 4.534$, $180° < \alpha < 270°$.

994. Find:

(1) $\sin(\alpha - 270°)$, $\tan(360° - \alpha)$ if $\sin\alpha = -\frac{2}{3}$, $270° < \alpha < 360°$;

(2) $\cos(180° + \alpha)$, $\cot(90° + \alpha)$ if $\tan\alpha = -\frac{3}{4}$, $90° < \alpha < 180°$;

(3) $\cos(\alpha - 270°)$, $\tan(\alpha - 180°)$ if $\cot\alpha = 2$, $180° < \alpha < 270°$.

995. Find:

(1) $\cos(270° + \alpha)$ if $\sin(180° - \alpha) = 0.3$ and $90° < \alpha < 180°$;

(2) $\sin(90° + \alpha)$ if $\tan(270° + \alpha) = -4$ and $180° < \alpha < 270°$;

(3) $\tan(180° - \alpha)$ if $\cos(180° + \alpha) = 0.6$ and $180° < \alpha < 270°$.

996. Find:

(1) $\sin(180° + \alpha)$ if $\cos(270° - \alpha) = -0.8$ and $0° < \alpha < 90°$;

(2) $\cos(630° + \alpha)$ if $\cot(540° + \alpha) = -5$ and $90° < \alpha < 180°$;

(3) $\cot(900° + \alpha)$ if $\cot(90° - \alpha) = -8$ and $90° < \alpha < 180°$.

997. Simplify (orally):

(1) $\sin 54^\circ+\sin(-54^\circ)+\cos 37^\circ-\cos(-37^\circ)$;

(2) $\tan 48^\circ+\tan(-48^\circ)-3\cos(-25^\circ)+3\cos 25^\circ$;

(3) $2\cos\left(-\frac{\pi}{8}\right)-2\cos\frac{\pi}{2}+3\sin^2\left(-\frac{\pi}{2}\right)$.

Compute the following:

998. (1) $\sin 210^\circ$, $\sin 900^\circ$, $\cos 330^\circ$, $\cos(-855^\circ)$;

(2) $\cos(-240^\circ)$, $\tan 300^\circ$, $\cot 3.5\pi$, $\cos 2\frac{2}{3}\pi$.

999. (1) $\tan 910^\circ-\sin(-1090^\circ)+\cos(-1450^\circ)$;

(2) $\sin(-1120^\circ)+\cos^2(-940^\circ)+\tan 1750^\circ$;

(3) $(\sin 10^\circ+\sin 340^\circ+\sin 30^\circ)-(\cos 80^\circ+\sin 200^\circ-$
$-\cos 60^\circ)$.

1000. (1) $\sin^2 930^\circ-\cos^2(-675^\circ)+\tan^2 855^\circ$;

(2) $\cos(-690^\circ)+\sin^2(-900^\circ)+\cot 495^\circ$;

(3) $\tan 1^\circ \tan 2^\circ \tan 3^\circ \ldots \tan 88^\circ \tan 89^\circ$;

(4) $\cot 15^\circ \cot 16^\circ \ldots \cot 74^\circ \cot 75^\circ$.

1001. (1) $\sin 5^\circ+\sin 10^\circ+\sin 15^\circ+\ldots+$
$+\sin 345^\circ+\sin 350^\circ+\sin 355^\circ$;

(2) $\cos 20^\circ+\cos 40^\circ+\cos 60^\circ+\ldots+$
$+\cos 160^\circ+\cos 180^\circ$;

(3) $\tan 20^\circ+\tan 40^\circ+\tan 60^\circ+\ldots+$
$+\tan 160^\circ+\tan 180^\circ$;

(4) $\cot 195^\circ+\cot 210^\circ+\cot 225^\circ+\ldots+$
$+\cot 330^\circ+\cot 345^\circ$;

(5) $\sin(\alpha+\pi)+\sin(\alpha+2\pi)+\sin(\alpha+3\pi)+\ldots+$
$+\sin(\alpha+2k\pi)$, $k=1, 2, 3$.

1002. (1) $\sin\left(-\frac{14\pi}{3}\right)+\cot^2\frac{29\pi}{4}-\tan^2\frac{3\pi}{4}+1$;

(2) $2\cos\left(-\frac{23\pi}{6}\right)+\cot\frac{23\pi}{4}-\tan^2\frac{16\pi}{3}-1$;

(3) $2\cos^2\frac{17\pi}{4}+\cot^2\frac{33\pi}{4}\tan\frac{3\pi}{4}$;

(4) $\tan^2\left(-\frac{43\pi}{4}\right)-\sin^2\frac{127\pi}{6}\cos^2\left(-\frac{64\pi}{3}\right)$.

1003.

(1) $\frac{\cos(-288^\circ)\cot 72^\circ}{\tan(-162^\circ)\sin 108^\circ}-\tan 18^\circ$;

(2) $\frac{\sin(-234^\circ)-\cos 216^\circ}{\sin 144^\circ-\cos 126^\circ}\times\tan 36^\circ$;

(3) $\frac{(\cot 44^\circ+\tan 226^\circ)\cos 406^\circ}{\cos 316^\circ}-\cot 72^\circ\cot 18^\circ$;

(4) $\frac{\cos^2 197^\circ+\cos^2 287^\circ}{1-\sin^2 217^\circ}-\frac{\sin^2 323^\circ}{\cos^2 37^\circ}$.

Simplify the following expressions:

1004.

(1) $\sin\alpha+\sin(90^\circ+\alpha)+\sin(180^\circ+\alpha)+$
$+\sin(270^\circ+\alpha)+\sin(360^\circ+\alpha)$;

(2) $\cos(\alpha+40^\circ)+\cos(\alpha+130^\circ)+\cos(\alpha+220^\circ)+\cos(\alpha+310^\circ)$;

(3) $\cos(90^\circ+\alpha)\cos(180^\circ-\alpha)[\tan(180^\circ+\alpha)+\tan(270^\circ-\alpha)]$;

(4) $\sin 25^\circ\cos 65^\circ+\sin^2 115^\circ\cos^2 245^\circ+\sin^2 295^\circ\cos^2 335^\circ$;

1005. (1) $\sin(180^\circ+\alpha)-\cos(90^\circ+\alpha)+$
$+\cot(360^\circ-\alpha)+\tan(270^\circ+\alpha)$;

(2) $\cos(\pi-\alpha)+\sin\left(\frac{3\pi}{2}+\alpha\right)-\tan\left(\frac{\pi}{2}+\alpha\right)\cot\left(\frac{3\pi}{2}-\alpha\right)$;

(3) $\cot(\alpha-360^\circ)\cos(\alpha-270^\circ)+\cos(\alpha+360^\circ)-$
$-2\sin(\alpha-180^\circ)$;

(4) $\frac{\cos^3(\alpha-270^\circ)\sin(360^\circ-\alpha)}{\cot^3(\alpha-90^\circ)\sin^3(270^\circ+\alpha)}$.

1006. (1) $\frac{\cos 750^\circ+\sin 420^\circ}{\sin(-330^\circ)-\cos(-390^\circ)}-\frac{1+\cos 1800^\circ\tan(-420^\circ)}{\tan 420^\circ}$;

(2) $\tan(-432^\circ)\tan 18^\circ+\cos(-302^\circ)\cos 508^\circ-$
$-\cos 32^\circ\cos 122^\circ$;

(3) $\sin(-385^\circ)\sin 1555^\circ - \sin(-295^\circ)\sin 4165^\circ - \cos(-1050^\circ)$;

(4) $\sin(-330^\circ)\cos 1230^\circ + \cos(-510^\circ)\sin 1950^\circ + \tan(-480^\circ)\tan 1290^\circ$.

1007. (1) $\dfrac{1}{\tan 368^\circ} + \dfrac{2\sin 2550^\circ \cos(-188^\circ)}{2\cos 638^\circ + \cos 98^\circ}$;

(2) $\left[\dfrac{\sin(-560^\circ)}{\sin 470^\circ} - \dfrac{\tan(-1010^\circ)}{\cot 200^\circ}\right]\cos^2(-700^\circ)$;

(3) $\dfrac{[1+\sin 500^\circ \cos(-320^\circ)]\cos 2380^\circ}{(1-\cos 410^\circ \cos 2020^\circ)\sin(-580^\circ)\cot^2(-310^\circ)}$;

(4) $\left[\cos(-55^\circ)\sin 485^\circ - \dfrac{\cot(-415^\circ)\cos(-395^\circ)}{\sin 505^\circ}\right] \times \dfrac{1}{\sin^2 485^\circ}$.

1008. (1) $\tan(-3.1\pi)\cos 5.9\pi - \sin(-3.6\pi)\cot(-5.6\pi)$;

(2) $\dfrac{\sin(-3.4\pi)+\sin 5.6\pi \cos^2(-8.1\pi)}{\sin^3(-8.9\pi)+\sin 8.9\pi}$;

(3) $\dfrac{\sin(-4.8\pi)\sin(-5.7\pi)}{\cot(-5.2\pi)} + \dfrac{\cos(-6.7\pi)\cos(-5.8\pi)}{\tan(-6.2\pi)}$.

1009. Prove the correctness of the following equalities:

(1) $\sin 825^\circ \cos(-15^\circ) + \cos 75^\circ \sin(-555^\circ) + \tan 155^\circ \tan 245^\circ = 0$;

(2) $\dfrac{\sin 515^\circ \cos(-475^\circ)+\cot 222^\circ \cot 408^\circ}{\cot 415^\circ \cot(-505^\circ)+\tan 197^\circ \tan 73^\circ} = \dfrac{1}{2}\cos^2 25^\circ$;

(3) $\dfrac{\cos^2 696^\circ + \tan(-260^\circ)\tan 530^\circ - \cos^2 156^\circ}{\tan^2 252^\circ + \cot^2 342^\circ} = \dfrac{1}{2}\tan^2 18^\circ$;

(4) $\dfrac{\sin(-328^\circ)\sin 958^\circ}{\cot 572^\circ} - \dfrac{\cos(-508^\circ)\cos(-1022^\circ)}{\tan(-212^\circ)} = -1$;

(5) $\sin 395^\circ \sin 145^\circ - \cos(-505^\circ)\cos 755^\circ - \tan(-616^\circ)\tan 194^\circ = 2$;

(6) $\sin 585^\circ \cos(-315^\circ) + \tan(-382^\circ)\tan 608^\circ - \dfrac{1}{\cos 660^\circ \cos 480^\circ} = 2.5$.

1010. Prove the following identities:

(1) $\sin(\alpha-2\pi)\tan\left(\frac{3\pi}{2}-\alpha\right)+\cos(\alpha-\pi)+\sin(\alpha-\pi)=-\sin\alpha$;

(2) $\left[\frac{\tan(\pi-x)\tan(1.5\pi+x)}{\cos^2(x-1.5\pi)}-\frac{\cos(1.5\pi+x)}{\sin(\pi-x)}\right]\times\sin^2(2\pi-x)=$

$=\cos^2 x$;

(3) $\left[\sin\left(\frac{\pi}{2}-x\right)+\sin(\pi-x)\right]^2+$

$+[\cos(1.5\pi-x)+\cos(2\pi-x)]^2=2$;

(4) $\left[\tan\frac{17\pi}{4}+\tan\left(\frac{7\pi}{2}-x\right)\right]^2+\left[\cot\frac{13\pi}{4}+\cot(7\pi-x)\right]^2=\frac{2}{\sin^2 x}$.

1011. Determine:

(1) $\cos x$ if $\sin\left(x-\frac{\pi}{2}\right)+\sin\frac{\pi}{2}=\sin\left(x+\frac{\pi}{2}\right)$;

(2) $\tan x$ if $\cot 1.2\tan(4\pi+1.2)-\sin\left(x+\frac{\pi}{2}\right)\cos(2\pi-x)=0$;

(3) $\sin x$ if $\cot(x+\pi)-\tan\left(x-\frac{\pi}{2}\right)=\sin^2(-725^\circ)+\cos^2 365^\circ$;

(4) $\cot x$ if $\sin\left(\frac{3\pi}{2}-x\right)+2\cos(20\pi-x)=\sin 750^\circ$.

1012. Evaluate most rationally the following expressions:

(1) $4\sin(2.5\pi-\alpha)+2.5\sin(\alpha-0.5\pi)$ for $\alpha=1.3843$;

(2) $5\tan(630^\circ-\alpha)+3\cot(900^\circ-\alpha)$ for $\alpha=3.731$;

(3) $8\sin(450^\circ-\alpha)+5\sin(\alpha-90^\circ)$ for $\alpha=2.4458$;

(4) $\sin(\alpha-270^\circ)\sin(\alpha-180^\circ)$ for $\alpha=3.4545$.

1013. Prove that if α, β, and γ are the interior angles of a triangle, then the following relations hold true:

(1) $\sin\alpha=\sin(\beta+\gamma)$; (2) $\cos\alpha=-\cos(\beta+\gamma)$;

(3) $\sin\frac{\alpha+\beta}{2}=\cos\frac{\gamma}{2}$; (4) $\sin\frac{\alpha}{2}=\cos\frac{\beta+\gamma}{2}$;

(5) $\sin\alpha=-\sin(2\alpha+\beta+\gamma)$;

(6) $\sin\alpha=-\cos\frac{3\alpha+\beta+\gamma}{2}$;

(7) $\cos\beta = \sin\frac{\alpha+\gamma+3\beta}{2}$;

(8) $\cos\gamma = -\cos(\alpha+\beta+2\gamma)$.

Determine which of the below functions are even and which are odd:

1014. (1) $y = \sin x+3\tan x$; (2) $y = 5\sin x^2$;

(3) $y = 12x\tan^2 x$; (4) $y = \frac{1+\cos x}{1-\cos x}$;

(5) $y = \frac{1+\sin x}{1-\sin x}$; (6) $y = 3x^2+\cos^5 x$;

(7) $y = 5x^4-\sin x^3$; (8) $y = \cos(\tan x)$;

(9) $y = \sin(\sin x)$.

1015.

(1) $y = \cot x-\cos x$; (2) $y = \sin(\cos x)$;

(3) $y = \cos(\cos x)$; (4) $y = 3^{\cos x}$;

(5) $y = 2^{\sin x}$; (6) $y = \sqrt{\cos 2x}$;

(7) $y = \sqrt{\sin 2x}$; (8) $y = |\sin x|$; (9) $y = \sqrt[3]{\sin x}$.

1016. Find the roots of the given equations within the interval $[0, 2\pi]$:

(1) $\sin(\pi-x)-\sin(\pi+x) = 1$;

(2) $\cos(45^\circ-x)+\sin(45^\circ+x) = 0$;

(3) $\tan\left(\frac{\pi}{4}+2x\right)-\cot\left(\frac{3\pi}{4}+2x\right) = 2$;

(4) $\cos\left(\frac{\pi}{4}-x\right)+\cos\left(\frac{3\pi}{4}-x\right) = 0$;

(5) $\sin(90^\circ-x)+\sin(180^\circ-x) = 0$;

(6) $\cos(x-60^\circ)-\sin(x+30^\circ) = 0$.

CHAPTER XII

Inverse Trigonometric Functions and Trigonometric Equations

42. THE CONCEPT OF TWO MUTUALLY INVERSE FUNCTIONS

1017. The dependence of the area S of a sphere on the radius of this sphere is expressed by the formula $S=4\pi R^2$. Find the formula expressing the radius as a function of the area of the sphere.

1018. The equation $5x-4y-20=0$ expresses the functional dependence between the variables x and y. How is y expressed as a function of the variable x? How is x expressed as a function of y?

1019. Is it obligatory to denote the argument of a function by the letter x, and the function itself by the letter y? Do the functions $2x^5$, $2u^5$, $2t^5$ differ?

1020. Given the function $y=3x-4$. Its inverse will be the function $x=\frac{1}{3}y+\frac{4}{3}$. Construct on one drawing the graphs of these functions and consider their location in the system of coordinates.

1021. Find the inverse of the function $y=3x-4$, denoting the argument as usual by the letter x, and the function by the letter y. Construct the graphs of these functions on one drawing and consider their location in the coordinate system.

1022. Given the function $y=f(x)$. Find its inverse if:

(1) $y=3x+2$; (2) $y=4-x$; (3) $y=\frac{1}{x}$;

(4) $y = \frac{3}{2x}$; (5) $y = \frac{x}{x+1}$; (6) $y = \sqrt[3]{x}$;

(7) $y = \sqrt[3]{x-1}$; (8) $y = \frac{2x+3}{3x-2}$.

1023. Which of the given functions have inverses:

(1) $y = 5x-6$; (2) $y = \frac{3}{x+4}$; (3) $y = x^2$; (4) $y = 2x^3$;

(5) $y = |x|$; (6) $y = \frac{4}{3x}$; (7) $y = x^4$; (8) $y = \frac{x+1}{x-1}$?

1024. If a given function is an increasing (decreasing) one, then what function will be its inverse? Give some examples.

1025. Prove that (a) the inverse of the linear function $y = kx+b$ is a linear function, (b) the inverse of the linear-fractional function $y = \frac{ax+b}{cx+d}$ is a linear-fractional function.

1026. Given the function $y = 2-x$. Its inverse will be the function $x = 2-y$ or $y = 2-x$. In this case the given function and its inverse turned out to be equal. Give examples of other functions possessing the same property. What can be said about the graph of such functions?

1027. Unite the following functions into pairs of mutually inverse functions:

(1) $2x^3$; (2) $\frac{3}{x}$; (3) $4x$; (4) $-2x$; (5) $\sqrt[3]{\frac{x}{2}}$;

(6) $6-x$; (7) $\frac{7}{x}$; (8) $\frac{1}{4}x$; (9) $-\frac{1}{2}x$; (10) $6-x$.

43. THE INVERSE TRIGONOMETRIC FUNCTIONS

1028. Construct the following angles:

(1) $\arcsin \frac{4}{5}$; (2) $\arccos \frac{2}{3}$; (3) $\arctan \frac{3}{4}$;

(4) $\arctan \frac{5}{4}$; (5) $\operatorname{arcsec} \frac{7}{3}$; (6) $\operatorname{arccsc} \frac{6}{5}$;

(7) $\arcsin \left(-\frac{3}{5}\right)$; (8) $\arccos \left(-\frac{1}{2}\right)$; (9) $\arctan (-2)$.

1029. Find the following angles:

(1) $\arcsin 1$; (2) $\arccos \frac{1}{2}$; (3) $\arctan 0$;

(4) $\operatorname{arccot}(-1)$; (5) $\arcsin\left(-\frac{\sqrt{3}}{2}\right)$;

(6) $\arccos\frac{\sqrt{2}}{2}$; (7) $\arctan\frac{\sqrt{3}}{3}$;

(8) $\arcsin 0.3724$; (9) $\arccos(-0.4627)$.

1030. Rewrite the following equalities introducing the symbols for the inverse trigonometric functions:

(1) $\sin\frac{\pi}{6}=\frac{1}{2}$; (2) $\cos\frac{\pi}{4}=\frac{\sqrt{2}}{2}$;

(3) $\sin(-60^\circ)=-\frac{\sqrt{3}}{2}$; (4) $\cos 90^\circ=0$;

(5) $\tan\left(-\frac{\pi}{4}\right)=-1$; (6) $\cot\frac{\pi}{2}=0$.

1031. Rewrite the following equalities eliminating the symbols for the inverse trigonometric functions:

(1) $\arcsin\left(-\frac{1}{2}\right)=-\frac{\pi}{6}$; (2) $\arccos\frac{\sqrt{3}}{2}=30^\circ$;

(3) $\arctan 1=45^\circ$; (4) $\operatorname{arccot}\sqrt{3}=\frac{\pi}{6}$;

(5) $\arcsin 1=90^\circ$; (6) $\arcsin 0.7683=50^\circ 12'$.

1032. Within what intervals do the following arcs lie?

(1) $2\arcsin x$; (2) $2\arccos x$; (3) $\frac{1}{2}\arctan x$;

(4) $\pi-\arctan x$; (5) $\frac{\pi}{2}+\arcsin 2x$; (6) $\arccos\frac{x}{2}-\frac{\pi}{2}$;

(5) $\arctan x^2-\pi$; (8) $\operatorname{arccot} 3x+\pi$?

1033. With the aid of the inverse trigonometric functions express x as a function of y if the following functional relations between x and y are given:

(1) $y=\sin 2x$; (2) $y=\tan 3x$; (3) $y=4\cos 3x$;

(4) $y=5\cot\frac{x}{2}$; (5) $y=m\sin nx$; (6) $y=3\sin 2x$.

1034. With the aid of the trigonometric functions, express y as a function of x given the following functional relations:

(1) $x=\arcsin\frac{y}{3}$; (2) $x=4\arctan\frac{y}{2}$;

(3) $x = 6 \arccos 2y$; (4) $x-5 = 3 \arcsin y$;

(5) $x+3 = 4 \arctan 5y$; (6) $2x-4 = \frac{1}{2} \arccos \frac{y}{2}$.

Find the domain of definition of the following functions:

1035. (1) $y = \arcsin \frac{1}{2}x$; (2) $y = \arctan 2x$;

(3) $y = \arccos 2x$; (4) $y = \arctan \sqrt{x}$;

(5) $y = \arccos \sqrt{x}$; (6) $y = \arctan \frac{1}{2-x}$;

(7) $y = \operatorname{arccot} \sqrt{x-4}$; (8) $y = \arcsin \frac{1}{x+2}$;

(9) $y = \sqrt{\arcsin x}$.

1036. (1) $y = \arctan (1-x)$; (2) $y = \arctan \frac{1}{x}$;

(3) $y = \arcsin \frac{1}{2x}$; (4) $y = \arccos (2x-1)$;

(5) $y = \arcsin \sqrt{2x}$; (6) $y = \operatorname{arccot} \sqrt{x^2-3|x|+1}$.

Compute the following:

1037.

(1) $\arcsin \frac{1}{2}$; (2) $\operatorname{arcsec} (-2)$; (3) $\arccos \frac{1}{\sqrt{2}}$;

(4) $\arctan 1+\arctan \sqrt{3}$; (5) $3 \arctan \left(-\frac{1}{\sqrt{3}}\right)$;

(6) $\frac{1}{2} \arccos \left(-\frac{\sqrt{3}}{2}\right)$; (7) $-\arcsin(-1)$;

(8) $-\arccos \left(-\frac{\sqrt{2}}{2}\right)$; (9) $\operatorname{arccot} (-\sqrt{3})$;

(10) $\frac{1}{2} \operatorname{arccot} 0$.

1038.

(1) $\arccos 1+2 \arcsin \frac{1}{2}$; (2) $\operatorname{arccot} 1-\frac{\pi}{4}$;

(3) $\arctan (-1)-\frac{3\pi}{4}$; (4) $\arcsin \frac{1}{2}+\arccos \frac{1}{2}$;

(5) $\arccos\left(-\frac{1}{2}\right)-\arccos\frac{\sqrt{3}}{2}$;

(6) $\arccos(-1)+\pi$.

1039.

(1) $\frac{1}{3}\arctan\sqrt{3}+\frac{1}{2}\operatorname{arccot}(-\sqrt{3})$;

(2) $\arcsin\left(-\frac{\sqrt{3}}{2}\right)+\arccos\left(-\frac{\sqrt{3}}{2}\right)$;

(3) $\arccos x+\arccos(-x)$; (4) $\arcsin(-x)+\arcsin x$;

(5) $\arctan x+\arctan(-x)$; (6) $\operatorname{arccot} x+\operatorname{arccot}(-x)$.

1040.

(1) $\cos\left(2\arcsin\frac{\sqrt{2}}{2}\right)$; (2) $\tan\left(\arccos\frac{1}{2}\right)$;

(3) $\sin(\operatorname{arccot}\sqrt{3})$; (4) $\cot[\arctan(-1)]$;

(5) $\sin\left[\arccos\left(-\frac{1}{2}\right)\right]$; (6) $\cos\left[2\arcsin\left(-\frac{\sqrt{2}}{2}\right)\right]$;

(7) $\sin\left(\arcsin\frac{1}{2}+\arccos\frac{1}{2}\right)$;

(8) $\cos\left[\arccos\left(-\frac{\sqrt{3}}{2}\right)+\frac{1}{2}\arcsin\frac{\sqrt{3}}{2}\right]$.

1041.

(1) $\sin(\arcsin x)$; (2) $\tan(\arctan 1.8)$;

(3) $\sin\left(\arctan\frac{3}{4}\right)$; (4) $\cos[\arctan(-2)]$;

(5) $\cos(\arcsin 0.6)$; (6) $\tan\left(\arcsin\frac{8}{17}\right)$;

(7) $\cot\left[\arccos\left(-\frac{24}{25}\right)\right]$; (8) $\tan(\arcsin 0.912)$;

(9) $\cos[\operatorname{arccot}(-0.2839)]$.

1042.

(1) $\arcsin(\sin 70^\circ)$; (2) $\arcsin(\sin 210^\circ)$;

(3)* $\arcsin(\sin x)$; (4) $\arcsin\left(\sin\frac{9}{4}\pi\right)$;

(5) $\arccos(\cos 170^\circ)$; (6) $\arccos[\cos(-40^\circ)]$;

(7)* $\arccos(\cos x)$; (8) $\arccos(\cos 6\pi)$;

(9) $\arctan\left(\tan\frac{2\pi}{5}\right)$; (10) $\arctan\left(\tan\frac{4\pi}{5}\right)$.

1043. Prove the following identities:

(1) $\arcsin\sqrt{1-a^2} = \arctan\frac{\sqrt{1-a^2}}{|a|}$;

(2) $\arcsin\frac{1}{\sqrt{a^2+1}} = \operatorname{arcsec}\frac{\sqrt{a^2+1}}{|a|}$;

(3) $\arcsin\frac{a}{\sqrt{a^2+1}} = \operatorname{arccot} a$;

(4) $\arctan\frac{1}{\sqrt{a^2-1}} = \arccos\frac{\sqrt{a^2-1}}{|a|}$;

(5) $\sin(\arccos a) = \cos(\arcsin a)$;

(6) $\tan(\operatorname{arccot} a) = \cot(\arctan a)$.

1044. Express the following dependences between the variables x and y in the form of an explicit function y of x:

(1) $\arcsin\sqrt{1-x^2} = \arctan y$; (2) $\arccos\sqrt{1-x^2} = \arctan y$;

(3) $\arctan\frac{1}{\sqrt{x^2-1}} = \arccos y$;

(4) $\operatorname{arccot}\frac{x}{\sqrt{1-x^2}} = \arccos\frac{1}{y}$.

1045. Express (a) $\arcsin x$, (b) $\arccos x$, (c) $\arctan x$, (d) $\operatorname{arccot} x$ in terms of the remaining inverse trigonometric functions.

44. THE GENERAL FORM OF THE ANGLES CORRESPONDING TO A GIVEN VALUE OF A TRIGONOMETRIC FUNCTION

1046. Find the acute angle x if:

(1) $\sin x = 0.5556$; (2) $\cos x = 0.8393$; (3) $\tan x = 3$;

(4) $\cot x = 0.25$; (5) $\sin x = -0.8$; (6) $\cot x = -2$.

1047. Find the angles x between 0° and 360° if the following is given:

(1) $\sin x = 0.8452$; (2) $\sin x = -0.6824$; (3) $\cos x = -0.6688$;

(4) $\tan x = \frac{4}{3}$; (5) $\cos x = 0.7629$; (6) $\cot x = 5$.

1048. Determine the least (in absolute value) angle x if:

(1) $\sin x = 0.6815$; (2) $\cos x = -0.5$;

(3) $\sin x = \sin 70° - \cos 50°$; (4) $\cos x = \tan 45° - \tan 60°$.

1049. Write the general form of the angles if:

(1) $\sin x = m$; (2) $\cos x = m$; (3) $\tan x = m$;

(4) $\sin x = \frac{\sqrt{3}}{2}$; (5) $\cos x = -\frac{1}{2}$;

(6) $\tan x = \sqrt{3}$.

1050. 1. For what values of the argument x do the following functions take on the greatest value:

(a) $\sin x$; (b) $-2 \sin x$; (c) $\cos x$; (d) $-\frac{1}{2}\cos x$?

2. For what values of the argument x do the following functions take on the least value:

(a) $\sin x$; (b) $1+\cos x$; (c) $-\frac{1}{3}\sin x$;

(d) $\cos(-x)$?

1051. Find:

(a) $\operatorname{Arcsin} 0$, $\operatorname{Arccos} \frac{\sqrt{3}}{2}$, $\operatorname{Arccos}\left(-\frac{1}{2}\right)$, $\operatorname{Arctan}(-1)$;

(b) $\operatorname{Arccot} \sqrt{3}$, $\operatorname{Arccot}(-\sqrt{3})$, $\operatorname{Arcsin} 0.3051$, $\operatorname{Arccos}(-0.4)$.

1052. Solve the following equations:

(1) $\sin x = 0.9$; (2) $\cos x = \frac{1}{2}$; (3) $\tan x = 3.5$;

(4) $\cot x = -\frac{\sqrt{3}}{3}$; (5) $\sin x = -1$; (6) $\cos x = -1$.

1053. Solve the following inequalities:

(1) $\sin x > \frac{1}{2}$; (2) $\cos x < -\frac{1}{2}$; (3) $\tan x > 1$;

(4) $\cot x < \sqrt{3}$; (5) $\sin x < -0.28$; (6) $\cos x > 0.4112$.

45. TRIGONOMETRIC EQUATIONS

1054. Determine the value of x in the general form and from $0°$ to $360°$ (from 0 to 2π) for the following equations:

(1) $\sin x = \frac{\sqrt{3}}{2}$; (2) $\cos x = -\frac{1}{2}$;

(3) $\tan x = 2$; (4) $\cot x = \sqrt{3}$; (5) $\sin 3x = \frac{1}{2}$;

(6) $\cos\left(x+\frac{\pi}{6}\right) = \frac{\sqrt{2}}{2}$; (7) $\cot\frac{x}{2} = -1$;

(8) $\tan\frac{3x}{5} = -1$; (9) $\sin^2 x = \frac{1}{4}$.

Solve the following equations:

1055.

(1) $\sin x = 0$; (2) $\cos x = 0$; (3) $\tan x = 0$;

(4) $\cot x = 0$; (5) $\cos 5x = 0$; (6) $\tan(-2x) = 0$;

(7) $\sin\frac{x}{2} = 0$; (8) $\cos x = 1$.

1056.

(1) $\cos 2x = -1$; (2) $\sin\frac{x}{2} = -1$; (3) $\cos(2x-5) = 0$;

(4) $3\tan^2 2x = 1$; (5) $\sin x = 0.2818$;

(6) $\cos x = -0.8575$.

1057.

(1) $2\sin\left(x-\frac{\pi}{3}\right) = 1$; (2) $\cos\left(2x+\frac{\pi}{4}\right) = 0$;

(3) $3\tan\left(x-\frac{\pi}{6}\right) = -\sqrt{3}$;

(4) $\cot(3x+50°) = 1.25$; (5) $\cos(30°-2x) = \frac{1}{2}$;

(6) $\tan(28°-3x) = \sqrt{3}$.

1058.

(1) $4\cos^2 x-4\cos x+1=0$; (2) $\sin^2 x-3\sin x+2=0$;

(3) $\cos^2 x+\cos x-6=0$; (4) $5\sin^2 2x-3\cos^2 2x=0$;

(5) $7\sin^2 x-5\cos^2 x+2=0$; (6) $\tan^2 x+\cot^2 x=2$.

1059.

(1) $2\cos^2 x=3\sin x$; (2) $\tan^2 2x-7\tan 2x+10=0$;

(3) $3\tan^2 x+\sqrt{3}\tan x=0$; (4) $\tan 2x+\cot 2x=2$;

(5) $\sin^2 x-\cos^2 x=\cos x$; (6) $2-3\tan x\cos^2 x=\cos^2 x$.

1060.

(1) $\cos x\tan x=0.8706$; (2) $3\sin x\cot x=0.8127$;

(3) $\dfrac{3\sin x}{\tan x}=0.9981$; (4) $\dfrac{\cos x}{2\cot x}=0.3765$.

1061.

(1) $\sin x\cos 2x=0$; (2) $\sin 2x\cos\left(\dfrac{\pi}{3}-x\right)=0$;

(3) $5\sin^2 x=2\sin x$; (4) $\sin^2 x=2\sin x$;

(5) $\sin^3 2x-\sin 2x=0$; (6) $\cos^4 x-\cos^2 x=0$.

1062.

(1) $\tan^3 x+\tan^2 x-2\tan x-2=0$;

(2) $1-\cos x=\tan x-\sin x$;

(3) $\dfrac{1+\cos x}{\cos\dfrac{x}{2}}=\dfrac{\sin x}{1-\cos x}$; (4) $(\tan x+1)\cos x=0$;

(5) $\cot x-\cot x\cos x+\cos x-1=0$;

(6) $2\sin x\tan x-\sin x-2\tan x+1=0$.

1063*.

(1) $\tan x\cot 2x=0$; (2) $\sin 3x\cot x=0$;

(3) $\sin x\tan x\cos 2x=0$;

(4) $\sin 5x\tan 4x\cos 2x=0$;

(5) $\sin 6x\cot 4x\tan 3x=0$; (6) $\cos x\cot 2x=0$.

1064.

(1) $\sin x - \cos x = 0$; (2) $\sin 3x + \cos 3x = 0$;

(3) $\cos^2 x - 3\sin^2 x = 0$; (4) $\sin^2 x - 3\cos^2 x = 0$;

(5) $3\sin^2 x - 5\sin x \cos x = 0$; (6) $\sin^2 x - 4\cos^2 x = 0$.

1065. (1) $\sin^2 x - 10\sin x \cos x + 21\cos^2 x = 0$;

(2) $\sin^2 x - 8\sin x \cos x + 7\cos^2 x = 0$;

(3) $2\sin^4 2x - 9\sin^3 2x \cos 2x + 7\sin^2 2x \cos^2 2x = 0$;

(4) $3\sin^2 x \cos x - 4\sin x \cos^2 x + \cos^3 x = 0$;

(5) $\sin^3 x + \sin^2 x \cos x - 3\sin x \cos^2 x - 3\cos^3 x = 0$.

1066. (1) $\cos^2 x - 3\sin x \cos x + 1 = 0$;

(2) $8\sin^2 x + \sin x \cos x + \cos^2 x = 4$;

(3) $\sin^4 x - 5\sin^2 x \cos^2 x + 4\cos^4 x = 0$;

(4) $\sin^3 x + \cos^3 x = 0$;

(5) $9\sin^2 x - 30\sin x \cos x + 25\cos^2 x = 25$.

1067. (1) $\sin\left(x - \frac{\pi}{2}\right) - \sin\frac{\pi}{2} = \sin\left(x + \frac{\pi}{2}\right)$;

(2) $\cos^2\left(\frac{\pi}{2} + x\right) - 2\cos x + 2 = 0$;

(3) $2\sin^2 x = 3\cos(360^\circ - x)$;

(4) $9\sin^2 x - 27\cos(270^\circ - x) - 10 = 0$.

1068. (1) $\cos(x - 360^\circ) + 2\sin x \tan(180^\circ - x) + 5 = 0$;

(2) $2\sin^2(-x) + \cos^2(-x) + 3\sin x \cos(-x) = 0$;

(3) $3\cot(270^\circ - x) = \tan(90^\circ - x)$;

(4) $\tan^2(\pi - x) + \tan^2 x - 2 = 0$.

1069. (1) $\frac{3\sin x - 2\sin(90^\circ - x)}{\sin(180^\circ - x) - \cos(180^\circ - x)} = \frac{1}{2}$;

(2) $2\sin^2\frac{x}{2} + \cos\left(\frac{3\pi}{2} + \frac{x}{2}\right) = 0$;

(3) $3\sin^2(180^\circ - x) + 2\sin(90^\circ + x)\cos(90^\circ + x) -$

$-5\sin^2(270^\circ + x) = 0$;

(4) $\cos^2\left(\frac{\pi}{2}+2x\right)-\cos^2 2x-3\cos\left(\frac{\pi}{2}-2x\right)+2=0.$

1070. (1) $3\arcsin x-\pi=0$; (2) $4\arccos x+\pi=0$;

(3) $2\arcsin(4x^4-17x^2+3)+\pi=0$;

(4) $4\arctan(x^2+3x+1)+\pi=0$;

(5) $x=\arccos(\sin x)$.

1071.

(1) $\arcsin(x+1)=\frac{\pi}{6}$; (2) $\arccos(3x+2)=\frac{2\pi}{3}$;

(3) $\arctan\sqrt{3}=\arccos x$; (4) $\arctan\left(-\frac{1}{\sqrt{3}}\right)=-\arctan x$;

(5) $\arcsin\frac{3}{5}=\arccos x$; (6) $\arcsin\left(-\frac{4}{5}\right)=\arctan x$.

1072.

(1) $\arcsin\frac{3x}{2}=\arccos 2x$; (2) $\arcsin 8x=\arccos 15x$;

(3) $\arccos x=\arctan x$; (4) $\arcsin x=\arccos\sqrt{x+1}$.

1073. Find x and y if:

(1) $\arcsin 0.8=\arccos x=\arctan y$;
(2) $\arccos m=\arctan x=\operatorname{arcsec} y$.

CHAPTER XIII

Addition Theorems and Their Corollaries

46. THE SINE, COSINE AND TANGENT OF SUM AND DIFFERENCE OF TWO ANGLES

1074. In what quadrant is the terminal side of the angle $\alpha+\beta$ located if:

(1) $0^\circ < \alpha < 90^\circ$, $0^\circ < \beta < 90^\circ$, $\cos\alpha = 0.7986$, $\cos\beta = 0.9135$;

(2) $0^\circ < \alpha < 90^\circ$, $90^\circ < \beta < 180^\circ$, $\tan\alpha = 0.7538$, $\tan\beta = -14{,}3$;

(3) $0 < \alpha < \frac{\pi}{2}$, $\frac{\pi}{2} < \beta < \pi$, $\tan\alpha = 1.0724$, $\tan\beta = -0.7002$.

1075*. In what quadrant is the terminal side of the angle $\alpha-\beta$ located if:

(1) $0^\circ < \alpha < 90^\circ$, $0^\circ < \beta < 90^\circ$, $\sin^2\alpha+\tan^2\beta = \frac{5}{4}$, $\sin\alpha+\tan\beta = 1.5$;

(2) $0^\circ < \alpha < 90^\circ$, $270^\circ < \beta < 360^\circ$, $\cos\alpha = \cos\beta$, $\cos^2\alpha-\sin^2\beta = \frac{1}{2}$.

1076. Without using the Tables evaluate the trigonometric functions of the angles of 15°, 75°, 105°.

1077. Find $\sin(\alpha+\beta)$ and $\sin(\alpha-\beta)$ if:

(1) $\sin\alpha = 0.8$, $\cos\beta = 0.6$, $0 < \alpha < \frac{\pi}{2}$, $\frac{3\pi}{2} < \beta < 2\pi$;

(2) $\cos\left(\frac{\pi}{2}-\alpha\right) = \frac{5}{13}$, $\sin\left(\frac{\pi}{2}-\beta\right) = \frac{3}{5}$, $\frac{\pi}{2} < \alpha < \pi$, $0 < \beta < \frac{\pi}{2}$.

1078. Find $\cos(\alpha+\beta)$ and $\cos(\alpha-\beta)$ if:

(1) $\sin\alpha = \frac{3}{5}$, $\sin\beta = \frac{8}{17}$, $0 < \alpha < \frac{\pi}{2}$, $\frac{\pi}{2} < \beta < \pi$;

(2) $\sin(90^\circ-\alpha) = 0.8$, $\sin(90^\circ-\beta) = \frac{15}{17}$, $0 < \alpha < \frac{\pi}{2}$, $\frac{3\pi}{2} < \beta < 2\pi$.

1079. Find $\tan(\alpha+\beta)$ and $\tan(\alpha-\beta)$ if:

(1) $\cot\alpha = 0.25$, $\cot\beta = \frac{5}{6}$;

(2) $\cos\left(\frac{\pi}{2}-\alpha\right) = -\frac{12}{13}$, $\sin\left(\frac{\pi}{2}-\beta\right) = \frac{5}{13}$, $\frac{3\pi}{2} < \alpha < 2\pi$, $0 < \beta < \frac{\pi}{2}$.

1080. Find:

(1) $\sin(30^\circ-\alpha)$ if $\tan\alpha = 0.75$ and $180^\circ < \alpha < 270^\circ$;

(2) $\cos(30^\circ-\alpha)$ if $\cot\alpha = \frac{\sqrt{7}}{3}$ and $180^\circ < \alpha < 270^\circ$;

(3) $\tan(45^\circ+\alpha)$ if $\cos\alpha = 0.6$ and $\frac{3\pi}{2} < \alpha < 2\pi$.

1081. Find:

(1) $\sin(60^\circ+\alpha)$ and $\cos(60^\circ-\alpha)$ if $\sin\alpha = -\frac{8}{17}$ and $270^\circ < \alpha < 360^\circ$;

(2) $\sin(\alpha+\beta)$ and $\sin(\alpha-\beta)$ if $\tan\alpha = 3\frac{15}{16}$, $\cot\beta = -1\frac{23}{33}$, $0 < \alpha < 90^\circ$, $90^\circ < \beta < 180^\circ$;

(3) $\cos(\alpha+\beta)$ and $\cos(\alpha-\beta)$ if $\cot\alpha = -\frac{7}{24}$, $\tan\beta = 1\frac{7}{8}$, $270^\circ < \alpha < 360^\circ$, $180^\circ < \beta < 270^\circ$.

1082. Find:

(1) $\cos\beta$ if $\cos\alpha = \frac{3}{5}$, $\cos(\alpha+\beta) = 0$,
$0° < \alpha < 90°$, $180° < \beta < 270°$;

(2) $\sin\beta$ if $\sin\alpha = -\frac{7}{25}$, $\tan(\alpha-\beta) = \frac{1}{2}$,
$180° < \alpha < 270°$, $90° < \beta < 180°$.

1083. Prove that $\alpha+\beta = 45°$ if:

(1) $\tan\alpha = 0.5$, $\tan\beta = \frac{1}{3}$, α and β are positive acute angles;

(2) $\cot\alpha = 4$, $\cot\beta = \frac{5}{3}$, α and β are positive acute angles;

(3) $\tan\alpha = \frac{1}{m}$, $\tan\beta = \frac{m-1}{m+1}$, $m > 0$.

1084. 1. Prove that $\alpha+\beta = 90°$ if:

(a) $\sin\alpha = \frac{40}{41}$, $\sin\beta = \frac{9}{41}$;

(b) $\cos\alpha = \frac{60}{61}$, $\cos\beta = \frac{11}{61}$, α and β are positive acute angles.

2*. Prove that $\alpha-\beta = 30°$ if $\sin\alpha = \frac{3\sqrt{3}+4}{10}$, $\sin\beta = 0.6$.

1085. 1. Prove that $\alpha-\beta = 30°$ if $\tan\alpha = \frac{a\sqrt{3}}{4-a}$, $\tan\beta = \frac{a-1}{\sqrt{3}}$, $1 < a < 4$ and α and β are acute angles.

2. Prove that $\alpha-\beta = \gamma$ if $\tan\alpha = \frac{m(n+1)}{2}$, $\tan\beta = \frac{mn}{2}$, $\cot\gamma = \frac{2}{m}+\frac{mn(n+1)}{2}$, $m > 0$, $n > 0$ and α, β, γ are acute angles.

1086. 1. Let $\tan\alpha = (\sqrt{2}+1)\tan x$, $\tan\beta = (\sqrt{2}-1)\tan x$, $\tan\gamma = 2\sin x\cos x$. Prove that $\alpha-\beta = \gamma$ if α, β, γ, x are positive acute angles.

2*. Let $\tan\alpha = x$, $\tan\beta = \frac{1-x}{1+x}$, α, β are positive acute angles. Prove that (a) $\alpha+\beta = \frac{\pi}{4}$ if $x > -1$; (b) $\alpha+\beta = -\frac{3\pi}{4}$ if $x < -1$.

1087. 1. Prove that if α and β are positive acute angles, then $\sin(\alpha+\beta) < \sin\alpha+\sin\beta$.

2. Show that

(a) $\sin 75°+\sin 15° > 1$; (b) $\sin 15°+\sin 45° > 0.8$.

1088. 1. Prove that if $0 < \alpha < 45°$, $0 < \beta < 45°$, then $\tan(\alpha+\beta) > \tan\alpha + \tan\beta$.

2*. For what values of α and β does the equality $\tan(\alpha+\beta) = \tan\alpha+\tan\beta$ hold true?

1089. 1. The cosines of two angles of a triangle are equal to $\frac{21}{29}$ and 0.8. Find the sine of the exterior angle of the triangle which is not adjacent to the given two.

2*. The sines of two angles of a triangle are equal to $\frac{5}{13}$ and $\frac{99}{101}$. Find the cosine of the third angle.

1090. 1. Express $\sin(\alpha+\beta+\gamma)$, $\cos(\alpha+\beta+\gamma)$, and $\tan(\alpha+\beta+\gamma)$ in terms of trigonometric functions of the angles α, β, γ.

2. Prove that $\alpha+\beta+\gamma = \frac{\pi}{2}$ if

(a) $\sin\alpha = \frac{1}{3}$, $\sin\beta = \frac{1}{3\sqrt{11}}$, $\sin\gamma = \frac{3}{\sqrt{11}}$;

(b) $\cos\alpha = \frac{3}{5}$, $\cos\beta = \frac{12}{13}$, $\cos\gamma = \frac{63}{65}$.

Compute the following:

1091. (1) $\sin 17° \cos 13° + \cos 17° \sin 13°$;

(2) $\sin 49° \cos 11° + \cos 49° \sin 11°$;

(3) $\sin 57° \cos 12° - \cos 57° \sin 12°$;

(4) $\sin 104° \cos 14° - \cos 104° \sin 14°$.

1092. (1) $\cos 73° \cos 17° - \sin 73° \sin 17°$;

(2) $\cos 85° \cos 35° - \sin 85° \sin 35°$;

(3) $\cos 109° \cos 49° + \sin 109° \sin 49°$;

(4) $\cos 163° \cos 13° + \sin 163° \sin 13°$.

1093. (1) $\frac{\tan 31° + \tan 14°}{1 - \tan 31° \tan 14°}$;

(2) $\frac{\tan 74° - \tan 14°}{1 + \tan 74° \tan 14°}$;

(3) $\frac{\tan 200° + \tan 10°}{1 - \tan 20° \tan 10°}$;

(4) $\frac{1 + \tan 2° \tan 152°}{\tan 152° - \tan 2°}$.

1094. (1) $\sin 113^\circ \cos 323^\circ + \cos 247^\circ \cos 307^\circ$;

(2) $\sin 463^\circ \cos 373^\circ + \cos 823^\circ \sin 193^\circ$;

(3) $\cos(-53^\circ) \sin(-337^\circ) + \sin 307^\circ \sin 113^\circ$;

(4) $\dfrac{\cot 78^\circ - \cot 303^\circ}{1 + \tan(-192^\circ)\cot 237^\circ}$;

(5) $\dfrac{\tan 225^\circ - \cot 81^\circ \cot 69^\circ}{\cot 261^\circ + \tan 201^\circ}$.

1095. (1) $\cos 68^\circ \cos 78^\circ + \cos 22^\circ \cos 12^\circ - \cos 10^\circ$;

(2) $\sin 30^\circ - \sin 47^\circ \sin 13^\circ + \sin 43^\circ \sin 77^\circ$;

(3) $\sin 163^\circ \cos 347^\circ + \sin 73^\circ \sin 167^\circ$;

(4) $\sin 292^\circ \sin 293^\circ + \sin(-338^\circ) \sin 337^\circ$.

1096. (1) $\dfrac{\sin 20^\circ \cos 25^\circ + \cos 20^\circ \sin 25^\circ}{\cos 35^\circ \cos 10^\circ - \sin 35^\circ \sin 10^\circ}$;

(2) $\dfrac{\sin 34^\circ \sin 146^\circ + \sin 236^\circ \sin 304^\circ}{\cos 32^\circ \cos 28^\circ - \cos 302^\circ \sin 152^\circ}$;

(3) $\dfrac{\sin \frac{7\pi}{30} \cos \frac{4\pi}{15} + \cos \frac{7\pi}{30} \sin \frac{4\pi}{15}}{\sin \frac{7\pi}{12} \cos \frac{5\pi}{12} - \cos \frac{7\pi}{12} \sin \frac{5\pi}{12}}$;

(4) $\dfrac{\sin 34^\circ \cos 236^\circ - \sin 56^\circ \sin 124^\circ}{\cos 28^\circ \cos 88^\circ + \cos 178^\circ \sin 208^\circ}$.

1097. (1) $\cos(17^\circ + \alpha) \cos(13^\circ - \alpha) - \sin(17^\circ + \alpha) \sin(13^\circ - \alpha)$;

(2) $\sin(\alpha - 70^\circ) \cos(100^\circ - \alpha) + \cos(\alpha - 70^\circ) \sin(100^\circ - \alpha)$;

(3) $\cos(185^\circ - 3\alpha) \sin(4\alpha - 185^\circ) +$
$+ \cos(4\alpha - 185^\circ) \sin(185^\circ - 3\alpha) - \sin \alpha$;

(4) $\dfrac{\tan(39^\circ + \alpha) + \tan(6^\circ - \alpha)}{1 - \tan(39^\circ + \alpha)\tan(6^\circ - \alpha)} + \dfrac{1 + \cot(38^\circ - \beta)\tan(7^\circ + \beta)}{\tan(52^\circ + \beta) - \cot(83^\circ - \beta)}$.

1098. (1) $\dfrac{\sin(-1.8\pi)\cos 0.3\pi + \cos 0.2\pi(-1.7\pi)}{\cos 0.125\pi \cos\left(-\frac{43}{24}\pi\right) - \cos \frac{3\pi}{8} \sin \frac{5\pi}{24}}$;

(2) $\dfrac{\sin 0.3\pi \cos(-2.8\pi) + \cos 0.3\pi \sin(-2.8\pi)}{\cos 0.3\pi \cos 2.3\pi - \sin 0.3\pi \sin(-4.3\pi)}$;

(3) $\dfrac{\left(\sin\frac{4\pi}{9}\cos\frac{\pi}{9}-\sin\frac{\pi}{18}\cos\frac{7\pi}{18}\right)\left(1+\tan\frac{5\pi}{12}\tan\frac{\pi}{12}\right)}{\tan\frac{5\pi}{12}-\tan\frac{\pi}{12}}$;

(4) $\dfrac{\left(\cot\frac{\pi}{8}-\tan\frac{\pi}{8}\right)\left(\cot\frac{5\pi}{18}+\tan\frac{\pi}{9}\right)\left(1-\tan\frac{\pi}{12}\right)}{\left(1+\tan\frac{3\pi}{8}\tan\frac{\pi}{8}\right)\left(1-\tan\frac{2\pi}{9}\tan\frac{\pi}{9}\right)\left(1+\tan\frac{\pi}{12}\right)}$.

Simplify the following:

1099. (1) $\sin 2\alpha\cos\alpha-\cos 2\alpha\sin\alpha$;

(2) $\sin(\alpha+\beta)\sin\beta+\cos(\alpha+\beta)\cos\beta$;

(3) $\sin(10^\circ+\alpha)\cos(5^\circ-\alpha)+\cos(10^\circ+\alpha)\sin(5^\circ-\alpha)$;

(4) $\cos(\alpha+\beta)+2\sin\alpha\sin\beta$.

1100. (1) $\dfrac{\sin(\alpha+\beta)+\sin(\alpha-\beta)}{\sin(\alpha+\beta)-\sin(\alpha-\beta)}$;

(2) $\dfrac{\cos(\alpha+\beta)+\sin\alpha\sin\beta}{\cos(\alpha-\beta)-\sin\alpha\sin\beta}$;

(3) $\dfrac{\sin(\alpha+\beta)-\sin(\alpha-\beta)}{\cos(\alpha+\beta)+\cos(\alpha-\beta)}$;

(4) $\dfrac{\sin(30^\circ+\alpha)-\cos(60^\circ+\alpha)}{\sin(30^\circ+\alpha)+\cos(60^\circ+\alpha)}$.

1101. (1) $\cos(\alpha+\beta)\cos(\alpha-\beta)+\sin(\alpha+\beta)\sin(\alpha-\beta)$;

(2) $\sin(\alpha+45^\circ)\cos(\alpha-45^\circ)-\cos(\alpha+45^\circ)\sin(\alpha-45^\circ)$;

(3) $\cos\alpha+\cos(120^\circ-\alpha)+\cos(120^\circ+\alpha)$;

(4) $\sin(90^\circ+\alpha)\cos(90^\circ+\beta)+\cos(90^\circ+\alpha)\sin(90^\circ+\beta)$.

1102. (1) $\sin 53^\circ\sin 67^\circ-\cos 14^\circ+\cos 37^\circ\cos 23^\circ$;

(2) $\sin 20^\circ+\sin 13^\circ\sin 57^\circ-\sin 33^\circ\sin 77^\circ$;

(3) $\sin 200^\circ\sin 310^\circ+\cos 340^\circ\cos 310^\circ$.

1103. (1) $\sin^2(30^\circ-\alpha)+\sin^2(30^\circ+\alpha)-\sin^2\alpha$;

(2) $\cos^2(30^\circ-\alpha)+\cos^2(\alpha+30^\circ)+\sin^2\alpha$;

(3) $\cos^2(60^\circ+\alpha)+\cos^2(\alpha-60^\circ)+\cos^2\alpha$;

(4) $\sin^2\alpha+\sin^2(120^\circ+\alpha)+\sin^2(\alpha-120^\circ)$.

1104. (1) $\tan\alpha\tan\beta+(\tan\alpha+\tan\beta)\cot(\alpha+\beta)$;

(2) $\tan(45^\circ+\alpha)-\dfrac{1+\tan\alpha}{1-\tan\alpha}$;

(3) $\dfrac{\tan(\alpha-\beta)+\tan\beta}{\tan(\alpha+\beta)-\tan\beta}$;

(4) $(\tan\alpha-\tan\beta)\cot(\alpha-\beta)-\tan\alpha\tan\beta$.

1105. (1) $\dfrac{\tan\alpha-\tan\beta}{\tan\alpha+\tan\beta}$; (2) $\dfrac{1+\tan\alpha\tan\beta}{1-\tan\alpha\tan\beta}$;

(3) $\dfrac{\cot\alpha-1}{\cot\alpha+1}-\cot(45^\circ+\alpha)$;

(4) $\dfrac{3-\tan^2 15^\circ}{3\tan^2 15^\circ-1}$;

(5) $\dfrac{\tan\left(\frac{\pi}{4}+\alpha\right)+\tan\left(\frac{\pi}{4}-\alpha\right)}{\cot\left(\frac{\pi}{4}+\alpha\right)+\cot\left(\frac{\pi}{4}-\alpha\right)}$;

(6) $\dfrac{\tan(45^\circ-\alpha)+\tan\alpha}{1-\tan(45^\circ-\alpha)\tan\alpha}$.

1106. (1) $\cos 2\alpha-\sin 2\alpha\tan\alpha$;

(2) $\sin 4\alpha\cot 2\alpha-\cos 4\alpha$;

(3) $\cos\dfrac{\alpha}{2}\cot\dfrac{\alpha}{4}-\sin\dfrac{\alpha}{2}$;

(4) $\dfrac{2\sin(\alpha+\beta)}{\cos(\alpha+\beta)+\cos(\alpha-\beta)}-\tan\beta$;

(5) $2\cos\left(\frac{\pi}{4}+\alpha\right)\cos\left(\frac{\pi}{4}-\alpha\right)$;

(6) $(\sin\alpha+\sin\beta)^2+(\cos\alpha+\cos\beta)^2$.

Prove the following identities:

1107. (1) $\sin(45^\circ+\alpha)=\cos(45^\circ-\alpha)$;

(2) $\cos(45^\circ+\alpha)=\sin(45^\circ-\alpha)$;

(3) $\cos(45^\circ+\alpha)\cos(45^\circ-\alpha)-$
$-\sin(45^\circ+\alpha)\sin(45^\circ-\alpha)=0$.

1108. (1) $\sin 15^\circ+\tan 30^\circ\cos 15^\circ=\dfrac{\sqrt{6}}{3}$;

(2) $0.5\left(\cos\alpha+\sqrt{3}\sin\alpha\right)=\sin(30^\circ+\alpha)$;

(3) $\sin^2(30^\circ+\alpha)-\sin^2(30^\circ-\alpha)=\sqrt{3}\sin\alpha\cos\alpha$;

(4) $\sin^2(30^\circ+\alpha)+\sin^2(30^\circ-\alpha)-\sin^2\alpha=0.5$;

(5) $\cos^2(\alpha+30^\circ)+\cos^2(\alpha-30^\circ)+\sin^2\alpha=1.5$.

1109. (1) $\sin(\alpha+\beta)\sin(\alpha-\beta)=\sin^2\alpha-\sin^2\beta$;

(2) $\cos(\alpha+\beta)\cos(\alpha-\beta)=\cos^2\alpha-\sin^2\beta$;

(3) $\sin 2\alpha+\cos 2\alpha\cot\alpha=\cot\alpha$;

(4) $\sin 2\alpha-\cos 2\alpha\tan\alpha=\tan\alpha$.

1110. (1) $\dfrac{\sin(\alpha\pm\beta)}{\cos\alpha\cos\beta}=\tan\alpha\pm\tan\beta$;

(2) $\dfrac{\tan\alpha+\cot\beta}{\cot\beta-\tan\alpha}=\dfrac{\cos(\alpha-\beta)}{\cos(\alpha+\beta)}$;

(3) $\dfrac{\cos(\alpha\pm\beta)}{\cos\alpha\cos\beta}=1\mp\tan\alpha\tan\beta$;

(4) $\dfrac{\tan^2\alpha-\tan^2\beta}{1-\tan^2\alpha\tan^2\beta}=\tan(\alpha+\beta)\tan(\alpha-\beta)$.

1111. (1) $\sin\alpha-\cos\alpha\tan\dfrac{\alpha}{2}=\tan\dfrac{\alpha}{2}$;

(2) $\tan(\alpha+\beta)-\tan\alpha-\tan\beta=\tan\alpha\tan\beta\tan(\alpha+\beta)$;

(3) $\dfrac{\cot(\alpha+\beta)-\cot\beta}{\cot(\alpha-\beta)+\cot\beta}=\dfrac{\sin(\beta-\alpha)}{\sin(\alpha+\beta)}$;

(4) $\dfrac{\sin\alpha+2\sin(60^\circ-\alpha)}{2\cos(30^\circ-\alpha)-\sqrt{3}\cos\alpha}=\dfrac{\sqrt{3}}{\tan\alpha}$.

1112. (1) $\dfrac{\sqrt{2}\cos\alpha-2\sin(45^\circ-\alpha)}{2\sin(60^\circ+\alpha)-\sqrt{3}\cos\alpha}=\sqrt{2}$;

(2) $\dfrac{\cos(30^\circ-\alpha)-\cos 330^\circ\cos\alpha}{\sin(30^\circ-\alpha)+\sin 120^\circ\sin\alpha}=\tan\alpha$;

(3) $\sin^2(\alpha+\beta)=\sin^2\alpha+\sin^2\beta+2\sin\alpha\sin\beta\cos(\alpha+\beta)$;

(4) $\dfrac{\cot\alpha\cos\beta-\sin\beta}{\cos\beta+\cot\alpha\sin\beta}=\cot(\alpha+\beta)$.

1113. (1) $\sqrt{3}\cos\alpha-\sin^3\alpha-\cos^2\alpha\sin\alpha=2\cos(\alpha+30^\circ)$;

(2) $\dfrac{(1+\tan\alpha)\sin(45^\circ-\alpha)}{1-\tan\alpha}=\sin(45^\circ+\alpha)$;

(3) $\dfrac{(1-\tan\alpha)\cos(45^\circ-\alpha)}{1+\tan\alpha}=\cos(45^\circ+\alpha)$;

(4) $\dfrac{2\cos\alpha\sin\beta-\sin(\alpha+\beta)}{2\cos\alpha\cos\beta-\cos(\alpha+\beta)}=\tan(\beta-\alpha)$.

1114. (1) $(\tan\alpha+\tan\beta)\cot(\alpha+\beta)+$
$+(\tan\alpha-\tan\beta)\cot(\alpha-\beta)=2$;

(2) $(\cot\alpha+\cot\beta)\cot(\alpha+\beta)-$
$-(\cot\alpha-\cot\beta)\cot(\alpha-\beta)=-2$;

(3) $\dfrac{\tan\alpha+\tan\beta}{\tan(\alpha+\beta)}+\dfrac{\tan\alpha-\tan\beta}{\tan(\alpha-\beta)}=2$;

(4) $\cot 15^\circ-\tan 15^\circ=2\tan 60^\circ$.

Solve the following equations:

1115. (1) $\sin(35^\circ+x)+\sin(35^\circ-x)=0$;

(2) $\sin(m+x)-\sin(m-x)=0$;

(3) $\cos(m+x)-\cos(m-x)=0$;

(4) $\cos(m+\sin x)+\cos(m-\sin x)=0$;

(5) $1+\sin x\sin 2x=\cos x\cos 2x$;

(6) $\tan 20^\circ-\tan x=1+\tan 20^\circ\tan x$.

1116. (1) $\cos 3x+\sin x\sin 2x=0$;

(2) $\sin 5x-\sin 2x\cos 3x=0$;

(3) $\sin(45^\circ+2x)\sin(60^\circ+x)+$
$+\sin(45^\circ-2x)\sin(30^\circ-x)=1$;

(4) $\cos(90^\circ+x)\cos(90^\circ-x)=-0.5$.

1117. (1) $\sin(60^\circ+x)+\sin(60^\circ-x)=-\dfrac{\sqrt{6}}{2}$;

(2) $\cos(60^\circ-x)+\cos(60^\circ+x)=0.58$;

(3) $\cos(\alpha+x)-\cos(\alpha-x)=\sin\alpha$;

(4) $\cos^2(60^\circ+x)-\cos^2(60^\circ-x)=\dfrac{\sqrt{3}}{2}$.

Compute the following functions without using the trigonometric tables:

1118. (1) $\tan\left(\arctan 2+\arctan\frac{1}{2}\right)$;

(2) $\tan\left(3\arctan\frac{4}{3}\right)$;

(3) $\sin\left(\arcsin\frac{3}{5}+\arcsin\frac{4}{5}\right)$;

(4) $\cos\left[\arcsin\left(-\frac{8}{17}\right)-\arccos\left(-\frac{3}{5}\right)\right]$.

1119. (1) $\sin[\arctan 2.4-\operatorname{arccot}(-1.05)]$;

(2) $\tan\left[\arccos\frac{9}{\sqrt{82}}-\arcsin\frac{4}{\sqrt{41}}\right]$;

(3) $\tan\left(\arcsin\frac{1}{5\sqrt{2}}+\arctan\frac{1}{9}\right)$;

(4) $\cot(\arccos 0.8-\arctan 2)$.

1120. 1. Express the following in terms of arcsin:

(a) $\arcsin\frac{8}{17}-\arccos\frac{12}{37}$; (b) $\operatorname{arccot}\frac{15}{8}-\arctan\frac{5}{12}$.

2. Express the following in terms of arccos:

(a) $\arcsin\frac{4}{5}-\arccos\frac{5}{13}$; (b) $\arctan\frac{9}{40}+\arccos\frac{7}{25}$.

3. Express the following in terms of arctan:

(a) $\pi-\operatorname{arccot}(-2)$; (b) $0.5\pi-\operatorname{arccot}(-2)$.

Prove the validity of the following equations:

1121. (1) $\arcsin\frac{4}{5}=\arccos\frac{3}{5}$;

(2) $\arcsin\frac{5}{13}+\arcsin\frac{12}{13}=\frac{\pi}{2}$;

(3) $\operatorname{arccot}2+\operatorname{arccot}3=\frac{\pi}{4}$;

(4) $\arcsin x+\arccos x=\frac{\pi}{2}$;

(5) $\arctan x+\operatorname{arccot}x=\frac{\pi}{2}$;

(6) $\arccos\left(-\frac{3}{5}\right)-\arcsin\frac{3}{5}=\frac{\pi}{2}$.

1122. (1) $\arcsin \frac{9}{41} - \arccos \frac{4}{5} = -\arcsin \frac{84}{205}$;

(2) $\arctan \frac{1}{3} + \arctan \frac{1}{5} + \arctan \frac{1}{7} + \arctan \frac{1}{8} = \frac{\pi}{4}$;

(3) $\arccos \frac{3}{7} + \arccos \frac{9}{11} = \arccos \left(-\frac{13}{77}\right)$;

(4) $\arccos \frac{\sqrt{6}+1}{2\sqrt{3}} - \arccos \sqrt{\frac{2}{3}} = -\frac{\pi}{6}$.

1123. (1) $\arctan \frac{1}{\sqrt{3}} + \arctan \frac{1}{2+\sqrt{3}} = \frac{\pi}{4}$;

(2) $\arcsin \frac{\sqrt{2}}{2} + \arctan \frac{\sqrt{2}}{2} = \arctan (\sqrt{2}+1)^2$;

(3) $\tan (\arcsin x) \cos (\arcsin x) = x$;

(4) $\arctan \frac{2}{5} - \arctan 1\frac{1}{8} = \arctan \left(-\frac{1}{2}\right)$.

Prove the following identities:

1124. (1) $\sin (\arccos a - \arccos b) = b\sqrt{1-a^2} - a\sqrt{1-b^2}$;

(2) $\sin (\arcsin a + \arcsin b) - \sin (\arccos a + \arccos b) = 0$;

(3) $\cos (\arcsin a - \arcsin b) = \sqrt{1-a^2}\sqrt{1-b^2} + ab$.

1125. (1) $\cot (\arctan a - \arctan b) = \frac{1+ab}{a-b}$;

(2) $\tan (\arccos a) + \tan (\arccos b) = \frac{a\sqrt{1-b^2} + b\sqrt{1-a^2}}{ab}$;

(3) $\tan (\arcsin a) + \tan (\arcsin b) = \frac{a\sqrt{1-b^2} + b\sqrt{1-a^2}}{\sqrt{1-a^2}\sqrt{1-b^2}}$.

1126. Solve the following equations:

(1) $\arcsin x + \arcsin x\sqrt{3} = \frac{\pi}{2}$;

(2) $\operatorname{arccsc} \frac{\sqrt{3}}{x} + \arccos \sqrt{1-x^2} = \frac{\pi}{2}$;

(3) $\arctan (x-1) + \arctan (x+1) = \frac{\pi}{4}$;

(4) $\arcsin 2x + \arcsin x = \frac{\pi}{3}$.

47. DOUBLE- AND HALF-ANGLE TRIGONOMETRIC FUNCTIONS

1127. Compute $\sin 2\alpha$, $\cos 2\alpha$, $\tan 2\alpha$, $\cot 2\alpha$ if:

(1) $\cos\alpha = 0.6$ and $0 < \alpha < 90°$; (2) $\tan\alpha = 1-\sqrt{2}$ and $\frac{\pi}{2} < \alpha < \pi$.

1128. Rewrite the following functions using the double-angle formulas:

(a) $\sin\alpha$, $\cos\alpha$, $\tan\alpha$; (b) $\sin 24°$, $\cos 70°$, $\tan 65°$;

(c) $\sin 3\alpha$, $\cos 4\alpha$, $\tan\frac{\alpha}{2}$.

1129. 1. Compute $\sin\alpha$, $\cos\alpha$ and $\tan\alpha$ if $\cos\frac{\alpha}{2} = -\frac{15}{17}$ and $\sin\frac{\alpha}{2} > 0$.

2. Compute $\sin\alpha$ and $\cos\alpha$ if $\tan\frac{\alpha}{2} = -\frac{5}{12}$ and $270° < \alpha < 360°$.

1130. 1. Find $\sin 2\alpha$, $\cos 2\alpha$, $\tan 2\alpha$ if $\tan\alpha = 2$, α lies in the third quadrant.

2. Find $\sin 2\alpha$, $\cos 2\alpha$, $\tan 2\alpha$, $\cot 2\alpha$ if $\cot\alpha = \sqrt{2}+1$.

1131. (1) Find $\tan 2\alpha$ and $\tan 4\alpha$ if $\tan\alpha = -1.5$;

(2) Find $\tan(45°+\alpha)$ if $\tan\frac{\alpha}{2} = 2$;

(3) Find $\sin(\alpha-2\beta)$ and $\cos(2\alpha+\beta)$ if $\tan\alpha = 2.4$, $\tan\beta = -0.75$, $0° < \alpha < 90°$, $90° < \beta < 180°$.

1132. 1. Express $\cot 2\alpha$ in terms of (a) $\cot\alpha$, (b) $\tan\alpha$.

2. Show that all trigonometric functions of the angle α are expressed rationally in terms of $\tan\frac{\alpha}{2}$;

3. Find: $\sin\alpha$, $\cos\alpha$, $\tan\alpha$, $\cot\alpha$ if $\tan\frac{\alpha}{2} = 0.3$.

1133. 1. Evaluate $\frac{\sin\alpha}{3-2\cos\alpha}$ if $\tan\frac{\alpha}{2} = 2$.

2. Evaluate $\frac{\tan\alpha+\sin\alpha}{\tan\alpha-\sin\alpha}$ if $\tan\frac{\alpha}{2} = \frac{2}{15}$.

1134. In an isosceles trapezoid the smaller base is equal to the side. The tangent of the angle between the diagonal and the greater base is equal to $\frac{3}{4}$. Find the trigonometric functions of the angles at the smaller base of the trapezoid.

1135. 1. Express the functions $\sin 3\alpha$, $\cos 3\alpha$, $\tan 3\alpha$ in terms of $\sin \alpha$, $\cos \alpha$, $\tan \alpha$, respectively.

2. Express the functions $\sin 4\alpha$ and $\cos 4\alpha$ in terms of $\sin \alpha$ and $\cos \alpha$.

3*. Prove the formula $\tan 3\alpha = \tan (60° - \alpha) \tan \alpha \tan (60° + \alpha)$.

1136. 1. Prove that $\sin 2\alpha < 2 \sin \alpha$ if $0° < \alpha < 180°$. For what values of α are the equalities $\sin 2\alpha = \sin \alpha$ and $\sin 2\alpha = 2 \sin \alpha$ correct?

2. Is it possible that $2 \cos \frac{\alpha}{2}$ exceeds unity?

1137. For what values of x (between 0° and 360°) does the function $y = 4 \cos x \sin x$ attain the greatest value, the least value, vanish? Find the greatest and the least values.

1138. 1. Given $\sin \alpha = \frac{2\sqrt{ab}}{a+b}$, $ab > 0$. Find $\sin 2\alpha$, $\cos 2\alpha$, $\tan 2\alpha$.

2. Given $\cos \alpha = \frac{2a}{1+a^2}$. Find $\sin 2\alpha$, $\cos 2\alpha$, $\tan 2\alpha$.

3. Given $\sin \alpha + \cos \alpha = \frac{5}{4}$. Find $\sin 2\alpha$, $\cos 2\alpha$, $\tan 2\alpha$.

1139. 1. Find $\tan 2\beta$, $\cot (2\alpha - 2\beta)$ if $\tan \alpha = \frac{1}{4}$, $\tan \beta = 2$.

2. Find $\sin (2\alpha + \beta)$, $\cos (\alpha - 2\beta)$, $\sin (2\alpha + 2\beta)$ if $\cos \alpha = \frac{2}{3}$, $\sin \beta = \frac{1}{3}$, $270° < \alpha < 360°$, $90° < \beta < 180°$.

1140. Find the angle α of a triangle if:

(1) $\sin^2 \alpha - \cos^2 \alpha = \frac{1}{2}$; (2) $\sin \alpha \cos \alpha = \frac{1}{4}$.

1141. 1. Find $\sin 4x$ if $\tan x = 3$.

2. Find $\cos 4x$, if $\cot x = \frac{1}{2}$.

1142. Evaluate the following expressions using the most rational method:

(1) $2 \sin 10° \cos 10°$; (2) $\sin 25° \sin 65°$;

(3) $\cos^2 20° - \sin^2 20°$; (4) $\sin^2 65° - \cos^2 65°$;

(5) $\sin^2 28^\circ - \sin^2 62^\circ$; (6) $2 \sin 0.6034 \cos 0.6034$;

(7) $\dfrac{2 \tan 40^\circ}{1 - \tan^2 40^\circ}$; (8) $\dfrac{4 \tan 22^\circ 30'}{1 - \tan^2 22^\circ 30'}$;

(9) $\dfrac{1 - \tan^2 50^\circ}{\tan 50^\circ}$; (10) $\cos^2 0.5 - \sin^2 0.5$;

(11) $2 \cos^2 1.3963 - 1$; (12) $1 - 2 \sin^2 1.1715$.

1143. Calculate orally:

(1) $2 \sin 15^\circ \cos 15^\circ$; (2) $\sin 22^\circ 30' \cos 22^\circ 30'$;

(3) $\cos^2 67^\circ 30' - \sin^2 67^\circ 30'$;

(4) $1 - 2 \sin^2 15^\circ$; (5) $\dfrac{\tan 15^\circ}{1 - \tan^2 15^\circ}$;

(6) $\dfrac{2 \cos^2 15^\circ - 1}{1 - 2 \sin^2 22^\circ 30'}$.

Simplify the following expressions:

1144. (1) $4 \sin \alpha \cos \alpha \cos 2\alpha$; (2) $\cos^4 2\alpha - \sin^4 2\alpha$;

(3) $2 \cos^2 \alpha - 1$; (4) $\cos^2 \alpha - 4 \sin^2 \dfrac{\alpha}{2} \cos^2 \dfrac{\alpha}{2}$;

(5) $1 - 8 \sin^2 \alpha \cos^2 \alpha$; (6) $\dfrac{\cos 2\alpha}{\cos \alpha} - \dfrac{\sin 2\alpha}{\sin \alpha}$.

1145. (1) $\cos 2\alpha - 2 \cos^2 \alpha$; (2) $\cos 2\alpha + 2 \sin^2 \alpha$;

(3) $\sin 2\alpha - (\sin \alpha + \cos \alpha)^2$;

(4) $\cos^4 \alpha - 6 \cos^2 \alpha \sin^2 \alpha + \sin^4 \alpha$.

1146. (1) $2 \cos 40^\circ \cos 50^\circ$;

(2) $2 \sin \left(45^\circ - \dfrac{\alpha}{2}\right) \sin \left(45^\circ + \dfrac{\alpha}{2}\right)$;

(3) $(\sin 20^\circ + \sin 70^\circ)(\cos 70^\circ - \cos 20^\circ)$;

(4) $\left[\sin \left(\dfrac{\pi}{4} + 2\alpha\right) + \sin \left(\dfrac{\pi}{4} - 2\alpha\right)\right]^2 - \cos 4\alpha$.

1147. (1) $\dfrac{(\sin \alpha + \cos \alpha)^2}{1 + \sin 2\alpha}$; (2) $\dfrac{1 - \sin \alpha}{\left(\cos \dfrac{\alpha}{2} - \sin \dfrac{\alpha}{2}\right)^2}$;

(3) $\tan \dfrac{\alpha}{2} - \cot \dfrac{\alpha}{2}$; (4) $\dfrac{1}{\tan \alpha + \cot \alpha}$;

(5) $\dfrac{1 - \tan^2 \alpha}{1 + \tan^2 \alpha}$; (6) $\dfrac{1}{1 - \tan \alpha} - \dfrac{1}{1 + \tan \alpha}$.

1148. Is there an angle α such that

(1) $\sin\alpha\cos\alpha = \sin 20°$; (2) $\sin\alpha\cos\alpha = \sin 40°$;

(3) $\sin^2\alpha+\cos 2\alpha-\cos 30° = 0.5$?

Prove the following equalities:

1149. (1) $\dfrac{2\tan 15°}{1-\tan^2 15°} = \dfrac{\sqrt{3}}{3}$;

(2) $4\sin 18°\cos 36° = 1$;

(3) $\sin 70°\sin 50°\sin 10° = \dfrac{1}{8}$;

(4) $\sin 40°\sin 50° = \dfrac{1}{2}\cos 10°$;

(5) $2\cos(45°+\alpha)\cos(45°-\alpha) = \cos 2\alpha$;

(6) $\sin^4\dfrac{\alpha}{2}-\cos^4\dfrac{\alpha}{2} = -\cos\alpha$.

1150.

(1) $1+\cos 2\alpha+2\sin^2\alpha = 2$;

(2) $\sin\alpha-\cos\alpha = \pm\sqrt{1-\sin 2\alpha}$;

(3) $4\sin^4\alpha+\sin^2 2\alpha = 4\sin^2\alpha$;

(4) $\tan 15°+\tan 75° = 4$;

(5) $\sin 2\alpha-\tan\alpha = \cos 2\alpha\tan\alpha$;

(6) $\sin 2\alpha-\cot\alpha = -\cos 2\alpha\cot\alpha$.

1151. (1) $\dfrac{\sin 3\alpha}{\sin\alpha}-\dfrac{\cos 3\alpha}{\cos\alpha} = 2$;

(2) $\dfrac{\cos 2\alpha}{1+\sin 2\alpha} = \dfrac{1-\tan\alpha}{1+\tan\alpha}$;

(3) $\dfrac{\tan\dfrac{\alpha}{2}}{1+\tan\dfrac{\alpha}{2}}+\dfrac{\tan\dfrac{\alpha}{2}}{1-\tan\dfrac{\alpha}{2}} = \tan\alpha$;

(4) $\cos^2(\alpha+\beta)+\cos^2(\alpha-\beta)-\cos 2\alpha\cos 2\beta = 1$;

(5) $16\cos 20°\cos 40°\cos 60°\cos 80° = 1$;

(6) $8\cos 10°\cos 20°\cos 40° = \cot 10°$.

1152. 1. Compute $\sin\frac{\alpha}{2}$, $\cos\frac{\alpha}{2}$, $\tan\frac{\alpha}{2}$ if $\sin\alpha = -0.8$, $180° < \alpha < 270°$.

2*. Compute $\sin\frac{\alpha}{2}$, $\cos\frac{\alpha}{2}$, $\tan\frac{\alpha}{2}$ if $\cot\alpha = -\frac{4}{3}$.

1153. Applying the half-angle formulas find the sine, cosine and tangent of the angles 15° and 22°30′.

1154. 1*. Determine $\sin\frac{\alpha}{4}$ and $\cos\frac{\alpha}{4}$ if $540° < \alpha < 630°$ and $\sin\alpha = -\frac{336}{625}$.

2. Determine $\tan\frac{\alpha}{2}$ if $\pi < \alpha < \frac{3\pi}{2}$ and $\tan\alpha = \frac{3}{4}$.

1155. In an isosceles triangle the sine of the vertex angle is equal to $\frac{24}{25}$. Determine the sine and cosine of the base angle.

1156. The cosine of a central angle subtended by an arc smaller than 90° is equal to $\frac{3}{8}$. Find the sine, cosine and tangent of the inscribed angle subtended by the same arc.

1157. The cosine of the angle of a rhomb equals $-\frac{23}{25}$. Find the sine and cosine of the angles formed by the diagonals of the rhomb and its sides.

1158. 1. If $\sin\alpha = \frac{9}{41}$ and $\cos\beta = \frac{60}{61}$, and α and β are positive acute angles, then $\sin^2\frac{\alpha-\beta}{2} = \frac{1}{41\times61}$. Check on the result.

2*. Verify the equality $\tan 7°30' = \sqrt{6}-\sqrt{3}+\sqrt{2}-2$.

Simplify the following expressions:

1159. (1) $\sqrt{\frac{1-\cos 4\alpha}{2}}$; (2) $\sqrt{\frac{1+\cos\frac{\alpha}{4}}{2}}$;

(3) $\sqrt{\frac{1+\cos\frac{\alpha}{6}}{1-\cos\frac{\alpha}{6}}}$; (4) $\sqrt{\frac{1-\cos 3\alpha}{1+\cos 3\alpha}}$;

(5) $\sqrt{\frac{\sin\left(\alpha-\frac{3\pi}{2}\right)+1}{1-\sin\left(\frac{\pi}{2}+\alpha\right)}}$; (6) $2\cos^2\frac{\alpha}{2}-\cos\alpha$.

1160. (1) $\dfrac{\cos^2\alpha}{\cot\frac{\alpha}{2}-\tan\frac{\alpha}{2}}$;

(2) $\dfrac{1-\tan^2(45^\circ-\alpha)}{1+\tan^2(45^\circ-\alpha)}$;

(3) $\tan(\alpha+45^\circ)+\tan(\alpha-45^\circ)$;

(4) $\sin^2 2x+4\cos^2 x+4\cos^4 x$.

1161. (1) $\sqrt{2+2\cos 2\alpha}$, where $\frac{\pi}{2}<\alpha<\pi$;

(2) $\sqrt{0.5-0.5\cos 4\alpha}$, where $\frac{\pi}{4}<\alpha<\frac{\pi}{2}$;

(3) $\sqrt{\frac{1}{2}(1-\sin\alpha)}$; (4) $\sqrt{2(1+\sin 2\alpha)}$;

(5) $\cot\alpha(\cos 2\alpha-1)$; (6) $\tan\alpha(\cos 2\alpha+1)$.

1162. (1) $\sin 2\alpha+2\sin^2(45^\circ-\alpha)$;

(2) $\sin 2\alpha-2\cos^2(45^\circ-\alpha)$;

(3) $\sqrt{\frac{1}{2}+\frac{1}{2}\sqrt{\frac{1}{2}+\frac{1}{2}\cos\alpha}}$, where $0\leqslant\alpha\leqslant\pi$;

(4) $\sqrt{\frac{1}{2}-\frac{1}{2}\sqrt{\frac{1}{2}+\frac{1}{2}\cos\alpha}}$, where $0\leqslant\alpha\leqslant\pi$.

1163. Is there an angle α such that

(1) $\cos^2\alpha-\cos^2 35^\circ=\sin^2(45^\circ+\alpha)-0.5\sin 2\alpha$;

(2) $\sin^2\left(45^\circ-\frac{\alpha}{2}\right)-\cos^2\frac{\alpha}{4}+\frac{1}{2}\sin\alpha=\cos 282^\circ$?

Prove the following identities:

1164. (1) $1+\sin\alpha=2\cos^2\left(45^\circ-\frac{\alpha}{2}\right)$;

(2) $1-\sin\alpha=2\sin^2\left(\frac{\alpha}{2}-45^\circ\right)$;

(3) $\dfrac{1+\sin 2\alpha}{\cos 2\alpha}=\tan\left(\frac{\pi}{4}+\alpha\right)$;

(4) $\dfrac{1+\sin\alpha}{1-\sin\alpha}=\cot^2\left(\frac{\pi}{4}-\frac{\alpha}{2}\right)$;

(5) $\dfrac{\sin 2\alpha - 2\sin\alpha}{\sin 2\alpha + 2\sin\alpha} = -\tan^2\dfrac{\alpha}{2}$;

(6) $\dfrac{1+\cos 2\alpha}{\sin 2\alpha}\cdot\dfrac{1+\cos\alpha}{\cos\alpha} = \cot\dfrac{\alpha}{2}$.

1165. (1) $\cot\dfrac{\alpha}{2} - 2\cot\alpha = \tan\dfrac{\alpha}{2}$;

(2) $\sin\alpha - \cos\alpha\tan\dfrac{\alpha}{2} = \tan\dfrac{\alpha}{2}$;

(3) $2(1+\cos 2\alpha) - \sin^2 2\alpha = 4\cos^4\alpha$;

(4) $4\sin\alpha\cos^3\alpha - 4\sin^3\alpha\cos\alpha = \sin 4\alpha$;

(5) $\dfrac{\cot 2\alpha}{\cot\alpha - \cot 2\alpha} = \cos 2\alpha$;

(6) $2(\sin^6\alpha + \cos^6\alpha) - 3(\sin^4\alpha + \cos^4\alpha) + 1 = 0$.

1166. (1) $\dfrac{2\cos 2\alpha - \sin 4\alpha}{2\cos 2\alpha + \sin 4\alpha} = \tan^2(45^\circ - \alpha)$;

(2) $\dfrac{1+\sin 2\alpha - \cos 2\alpha}{1+\sin 2\alpha + \cos 2\alpha} = \tan\alpha$;

(3) $\sin\alpha(\sin\alpha + \sin\beta) + \cos\alpha(\cos\alpha + \cos\beta) = 2\cos^2\dfrac{\alpha-\beta}{2}$;

(4) $\sin\alpha(\sin\alpha - \sin\beta) + \cos\alpha(\cos\alpha - \cos\beta) = 2\sin^2\dfrac{\alpha-\beta}{2}$;

(5) $(\sin\alpha + \sin\beta)^2 + (\cos\alpha + \cos\beta)^2 = 4\cos^2\dfrac{\alpha-\beta}{2}$;

(6) $(\sin\alpha - \sin\beta)^2 + (\cos\alpha - \cos\beta)^2 = 4\sin^2\dfrac{\alpha-\beta}{2}$.

1167. Compute the following (without using the Tables):

(1) $\cot 22^\circ 30' - \cot 67^\circ 30'$; (2)* $8\cos 10^\circ\cos 50^\circ\cos 70^\circ$;

(3) $4\sin\dfrac{\pi}{10}\sin\dfrac{13\pi}{10}$; (4) $\sin^4\dfrac{3}{8}\pi - \cos^4\dfrac{3}{8}\pi$.

1168. Find the periods of the following functions:

(1) $y = \sin x\cos x$; (2) $y = \cos^2 x - \sin^2 x$;

(3) $y = \dfrac{\cos x}{1-\sin x}$; (4) $y = \cos 2x - 2\sin^2 x$;

(5) $y = 2\cot x - \cot\dfrac{x}{2}$.

1169. Find the greatest and the least values of the following functions:

(1) $y = \sin(x+45°)\sin(x-45°)$;

(2) $y = \cos^4 x - \sin^4 x$; (3) $y = 1-8\sin^2 x\cos^2 x$;

(4) $y = 1+\tan x\tan 2x$; (5) $y = 2\sin^2 x-\cos 2x$;

(6) $y = \cos^2(45°+x)+(\sin x-\cos x)^2$.

1170. Prove that of all right triangles with a given hypotenuse the isosceles triangle has the greatest area.

Solve the following equations:

1171. (1) $\sin x\cos x = \frac{\sqrt{3}}{4}$;

(2) $\sin^2 x-\cos^2 x = 0.5$; (3) $\sin x = \sin 2x$;

(4) $\cos 2x = \cos x$; (5) $2\sin^2 x = \cos 2x$;

(6) $\tan 2x = 3\tan x$.

1172. (1) $\sin(45°-x)\sin(45°+x) = 0.5$;

(2) $1-\cos x = 2\sin\frac{x}{2}$;

(3) $1+\cos x = 2\cos\frac{x}{2}$;

(4) $1-\cos x = \sin x\sin\frac{x}{2}$;

(5) $\sin x+\cos x = 1$; (6) $\sin x-\cos x = 1$.

1173. (1) $2\cos 2x+2\cos 4x+3\sin^2 2x = 1$;

(2) $\sin 2x = \tan x$; (3) $3\cos 2x = 2\cos x$;

(4) $4\cos 2x = 5\sin x+1$;

(5) $(\cos x-\sin x)^2 = \sin 2x$;

(6) $\sin 2x = \tan x$.

1174. (1) $\tan 2x = 3\sin x$;

(2) $4\sin^2 x(1+\cos 2x) = 1-\cos 2x$;

(3) $4\cos^2 x(1-\cos 2x) = 1+\cos 2x$;

(4) $(\cos x+\sin x)^2 = 1-\cos 2x$;

(5) $1+\cos x = (\cos x+\sin x)^2$;

(6) $\sin 2x = 3(\sin x+\cos x)^2$.

1175. (1) $\sin^4 x+\cos^4 x = 1$;

(2) $(1-2\sin x)\sin x = 2\cos 2x-1$;

(3) $\tan\frac{x}{2} = 3\cot\frac{2\pi-x}{4}$;

(4) $8\cos^4 x-\cos 4x = 1$;

(5) $\sin^2 x-2\cos^2 x+\frac{1}{2}\sin 2x = 0$.

1176. (1) $\cos 7x+\sin^2\frac{3x}{2} = \cos^2\frac{3x}{2}-\cos x$;

(2) $\tan x-2\sin^2\frac{x}{2} = \sin x$;

(3) $\tan 2x+\frac{\cos 2x}{1+\sin 2x} = 2$;

(4) $\cot^2 x = \frac{1+\sin x}{1+\cos x}$;

(5) $\tan(45^\circ+x) = 1+\sin 2x$;

(6) $\cot^2 x-\tan^2 x = 16\cos 2x$.

Compute the following without using the tables of values of trigonometric functions:

1177. (1) $\sin\left(2\arcsin\frac{3}{5}\right)$; (2) $\cos\left(\frac{1}{2}\arccos\frac{4}{5}\right)$;

(3) $\cos\left[2\arccos\left(-\frac{7}{25}\right)\right]$;

(4) $\sin\left(3\arccos\frac{1}{4}\right)$; (5) $\sin\left(4\arccos\frac{2}{3}\right)$;

(6) $\tan\left[2\arctan\left(-\frac{3}{4}\right)\right]$.

1178. (1) $\sin\left[2\arccos\left(-\frac{24}{25}\right)\right]$;

(2) $2\cos^2\left(\frac{1}{2}\arccos\frac{5}{8}\right)$;

(3) $\cot\left(\frac{1}{2}\arccos\frac{3}{5}\right)$; (4) $\cot^2\left(\frac{1}{2}\arcsin\frac{4}{7}\right)$;

1179. (1) $2\cos^2\left(\frac{1}{2}\arcsin\frac{5}{13}\right)-1$;

(2) $1-2\sin^2\left(\frac{1}{2}\arccos\frac{5}{13}\right)$;

(3) $\sin\left(2\arcsin\frac{15}{17}\right)+\sin\left(2\arccos\frac{15}{17}\right)$;

(4) $\cos\left(4\arccos\frac{2}{3}\right)-\cos\left(4\arcsin\frac{2}{3}\right)$;

(5) $\sin\left[3\arccos\left(-\frac{1}{3}\right)\right]+\cos\left[3\arcsin\left(-\frac{1}{3}\right)\right]$;

(6) $\cos^2\left(\frac{1}{2}\arccos\frac{4}{5}\right)-\sin^2\left(\frac{1}{2}\arccos\frac{4}{5}\right)$.

1180. Prove the validity of the following equalities:

(1) $2\arctan\frac{1}{5}+\arctan\frac{1}{4}=\arctan\frac{32}{43}$;

(2) $2\arccos x=\arccos(2x^2-1)$ for $0\leqslant x\leqslant 1$;

(3) $2\arcsin x=\arccos(1-2x^2)$ for $0\leqslant x\leqslant 1$.

Prove the following identities:

1181. (1) $2\cos^2\left(\frac{1}{2}\arccos a\right)=1+a$;

(2) $2\sin^2\left(\frac{1}{2}\arccos a\right)=1-a$;

(3) $\sin(2\arcsin a)=2a\sqrt{1-a^2}$;

(4) $\cos(2\arcsin a)=1-2a^2$.

1182. (1) $\cos\left(\frac{1}{2}\arccos a\right)=\sqrt{\frac{1+a}{2}}$;

(2) $\tan\left(\frac{1}{2}\arccos a\right)=\sqrt{\frac{1-a}{1+a}}$.

1183. (1) $\sin(3\arcsin a)=3a-4a^3$;

(2) $\cos(3\arccos a)=4a^3-3a$;

(3) $\sin(3\arccos a)+\cos(3\arcsin a)=0$.

Solve the following equations:

1184. (1) $2x=\arctan(\cot x)$;

(2) $\arccos(x-1)=2\arccos x$;

(3) $\arctan x=2\operatorname{arccot} x$; (4) $\arcsin 2x=2\arcsin x$.

1185. (1) $\arcsin 3x = 2\arcsin 2x$;

(2)* $2\arccos x = \arccos \frac{3x}{\sqrt{2}}$;

(3) $2\arcsin x = \arccos \sqrt{1-4x^2}$;

(4)* $2\arcsin x\sqrt{3} = \arcsin 2x\sqrt{2}$.

1186. (1) $2\arcsin 6x = \arccos 28x^2$;

(2) $\arctan 2x = 2\arctan x$;

(3) $\operatorname{arccot}\frac{4}{x} = 2\operatorname{arccot} x$;

(4) $\arccos x - \frac{1}{2}\arccos x\sqrt{3} = \frac{\pi}{4}$.

48. TRANSFORMING A SUM OR A DIFFERENCE OF TRIGONOMETRIC FUNCTIONS INTO A PRODUCT

Transform the sums into products:

1187. (1) $\sin 20^\circ + \sin 40^\circ$; (2) $\sin 24^\circ - \sin 36^\circ$;

(3) $\sin 110^\circ - \sin 130^\circ$; (4) $\cos 70^\circ + \cos 50^\circ$;

(5) $\cos 30^\circ - \cos 90^\circ$; (6) $-\cos 80^\circ - \cos 40^\circ$.

1188. (1) $\cos 100^\circ + \sin 10^\circ$; (2) $\sin 25^\circ + \cos 65^\circ$;

(3) $\sin 35^\circ - \cos 45^\circ$; (4) $\sin 100^\circ + \cos 130^\circ$;

(5) $\sin(-70^\circ) - \cos(-70^\circ)$;

(6) $\sin(45^\circ + \alpha) - \cos\alpha$.

1189.

(1) $\sin(\alpha + 20^\circ) + \sin(\alpha - 60^\circ)$;

(2) $\cos(40^\circ + \alpha) + \cos(40^\circ - \alpha)$; (3) $\sin 3\alpha + \sin 2\alpha$;

(4) $\cos 2\alpha - \cos 6\alpha$; (5) $\cos(\alpha + \beta) + \cos(\alpha - \beta)$;

(6) $\cos(60^\circ + \alpha) + \cos(60^\circ - \alpha) + \cos 3\alpha$.

1190. (1) $\sin\alpha + \cos\alpha$; (2) $\sin\alpha - \cos\alpha$;

(3) $\cos\alpha - \sin\beta$; (4) $\sin^2\alpha - \sin^2\beta$;

(5) $\cos^2\alpha - \cos^2\beta$; (6) $\frac{\sin 75^\circ - \sin 15^\circ}{\cos 75^\circ + \cos 15^\circ}$;

(7) $\frac{\sin 7\alpha - \sin 5\alpha}{\sin 7\alpha + \sin 5\alpha}$; (8) $\frac{\cos 4\alpha + \cos 2\alpha}{\cos 4\alpha - \cos 2\alpha}$.

1191. (1) $\sin 28^\circ+\sin 8^\circ+\sin 20^\circ$;

(2) $\sin 70^\circ-\sin 20^\circ+\sin 50^\circ$;

(3) $\cos 46^\circ-\cos 22^\circ-2\cos 78^\circ$;

(4) $\sin 54^\circ-\sin 40^\circ+\sin 14^\circ$.

1192. (1) $1+\cos\alpha$; (2) $1-\cos\alpha$; (3) $1+\sin 2\alpha$;

(4) $1-\cos 29^\circ$; (5) $\cos 32^\circ-1$; (6) $1+\sin 40^\circ$.

1193. (1) $1+\cos\left(\frac{\pi}{4}+\alpha\right)$; (2) $1-\sin\left(\frac{\pi}{3}+\alpha\right)$;

(3) $\cos 17^\circ-1$; (4) $2\sin^2\alpha-1$;

(5) $2\cos^2\alpha-1$; (6) $1-2\sin^2 40^\circ$.

1194. (1) $\frac{\sin 80^\circ-\sin 50^\circ}{\cos 80^\circ+\cos 50^\circ}$; (2) $\frac{1+\sin\alpha}{1-\sin\alpha}$;

(3) $\frac{1-\cos 40^\circ}{\cos 50^\circ}$; (4) $\frac{1-\cos(\alpha-\beta)}{1+\cos(\alpha-\beta)}$.

1195. (1) $1+\cos\alpha+\sin\alpha$; (2) $1-\cos\alpha+\sin\alpha$;

(3) $1-\cos\alpha+\cos 2\alpha$; (4) $1+\sin\alpha-\cos 2\alpha$;

(5) $1-2\cos\alpha+\cos 2\alpha$; (6) $1-\sin\alpha-\cos\alpha$.

1196. (1) $\frac{1+2\cos\alpha+\cos 2\alpha}{1-2\cos\alpha+\cos 2\alpha}$; (2) $\frac{1+2\sin\alpha-\cos 2\alpha}{1-2\sin\alpha-\cos 2\alpha}$;

(3) $\cos 44^\circ-\cos 22^\circ-2\cos 79^\circ$;

(4) $1-\sin 50^\circ+\cos 80^\circ$.

1197. (1) $\sin\alpha+2\sin 2\alpha+\sin 3\alpha$;

(2) $\cos 2\alpha+\cos 3\alpha+\cos 4\alpha$;

(3) $\sin\alpha+\sin\beta+\sin(\alpha+\beta)$;

(4) $\sin(3\alpha+\beta)+\sin(\alpha+\beta)+\sin 2\alpha$.

1198. (1) $1+\cos\alpha+\cos 2\alpha+\cos 3\alpha$;

(2) $1-\sin\alpha+\sin 3\alpha+\cos 4\alpha$;

(3) $\sin 16^\circ-2\cos 26^\circ+\sin 64^\circ+2\cos 66^\circ$;

(4) $1-\cos 10^\circ+\cos 70^\circ-\cos 80^\circ$.

1199. (1) $\sqrt{1+\cos\alpha}+\sqrt{1-\cos\alpha}$ if $90^\circ < \alpha < 180^\circ$;

(2) $\sqrt{1-\cos\alpha}+\sqrt{1+\cos\alpha}$ if $180^\circ < \alpha < 360^\circ$;

(3) $\sqrt{2+2\sin 2\alpha}-\sqrt{2-2\sin 2\alpha}$ if $45^\circ < \alpha < 135^\circ$;

(4)* $\sqrt{1-\sin 2\alpha}+\sqrt{1+\sin 2\alpha}$ if $-45^\circ < \alpha < 45^\circ$.

1200. (1) $\tan\alpha+\tan\beta$; (2) $\tan\alpha-\tan\beta$;

(3) $\tan\alpha+\cot\beta$; (4) $\tan\alpha-\cot\beta$;

(5) $\cot\alpha+\cot\beta$; (6) $\cot\alpha-\cot\beta$;

(7) $\cot\alpha+\tan\alpha$; (8) $\tan\alpha-\cot\alpha$;

(9) $\tan\alpha+\tan 60^\circ$.

1201. (1) $\tan 12^\circ+\tan 18^\circ$; (2) $\tan\frac{5\pi}{24}-\tan\frac{7\pi}{24}$;

(3) $\cot\frac{\pi}{5}-\tan\frac{4\pi}{5}$; (4) $\tan\alpha-\sqrt{3}$;

(5) $3\cot\alpha+\sqrt{3}$; (6) $1-\cot\alpha$.

1202. (1) $\tan\alpha+\cot 3\alpha$; (2) $1+\tan\alpha$;

(3) $\tan 22^\circ 30'-\cot 22^\circ 30'$;

(4) $\cot(45^\circ+\alpha)-\cot(45^\circ-\alpha)$;

(5) $\tan^2\alpha-\tan^2\beta$; (6) $\cot^2\alpha-\cot^2\beta$.

1203. (1) $\dfrac{\tan(15^\circ+\alpha)+\tan(15^\circ-\alpha)}{\tan(15^\circ+\alpha)-\tan(15^\circ-\alpha)}$;

(2) $\dfrac{\cot(45^\circ+\alpha)+\cot(45^\circ-\alpha)}{\tan(45^\circ+\alpha)-\tan(45^\circ-\alpha)}$;

(3) $\dfrac{\cot 2\alpha-\tan\alpha}{\cot 2\alpha+\tan\alpha}$;

(4) $\dfrac{1}{\tan\alpha-\tan 2\alpha}-\dfrac{1}{\cot\alpha-\cot 2\alpha}$.

1204. (1) $\sin\alpha+\tan\alpha$; (2) $\cos\alpha+\cot\alpha$;

(3) $1+\sin\alpha+\cos\alpha+\tan\alpha$;

(4) $1-\sin\alpha+\cos\alpha-\tan\alpha$;

(5) $1-\tan\alpha\cot\beta$; (6) $\tan\alpha+\tan 2\alpha-\tan 3\alpha$.

1205. (1) $\tan 30^\circ+\tan 40^\circ+\tan 50^\circ+\tan 60^\circ$;

(2) $1-\tan^2\alpha\tan^2\beta$; (3) $1-\cot^2\alpha\cot^2\beta$;

(4)* $\sqrt{\tan\alpha+\sin\alpha}+\sqrt{\tan\alpha-\sin\alpha}$.

Transform the given sums into products by introducing auxiliary angles:

1206. (1) $\frac{1}{2}+\sin\alpha$; (2) $\cos 2\alpha-\frac{\sqrt{3}}{2}$;

(3) $2\sin 50^\circ-\sqrt{3}$; (4) $\sin\alpha+\sqrt{3}\cos\alpha$;

(5) $\sqrt{3}\sin\alpha-\cos\alpha$; (6) $4\sin\alpha+3$.

1207. (1) $3\sin\alpha-4\cos\alpha$; (2) $3\sin 40^\circ+\sqrt{3}$;

(3) $3\sin 40^\circ+4\cos 40^\circ$;

(4) $\cos 72^\circ-\sqrt{3}\sin 72^\circ$;

(5) $15\sin(45^\circ-\alpha)+8\sin(45^\circ+\alpha)$.

1208. (1) $4\cos^2\alpha-3$; (2) $\sin^2\alpha-0.36$;

(3) $-0.5-\cot\alpha$; (4) $2+3\tan\alpha$;

(5) $3-\tan^2\alpha$; (6)* $\frac{b\sin\alpha-c}{b\sin\alpha+c}$.

1209. Check on the following equalities:

(1) $\sin 25^\circ+\sin 35^\circ=\cos 5^\circ$;

(2) $\cos 58^\circ-\cos 2^\circ=-\sin 28^\circ$;

(3) $\tan 20^\circ+4\sin 20^\circ=\sqrt{3}$;

(4) $4\cos 20^\circ-\sqrt{3}\cot 20^\circ=-1$;

(5) $\sqrt{6}-2\cos 15^\circ=2\sin 15^\circ$.

1210. Prove the following identities:

(1) $\sin^2(\alpha+\beta)-\sin^2(\alpha-\beta)=\sin 2\alpha\sin 2\beta$;

(2) $\cos^2(\alpha-\beta)-\cos^2(\alpha+\beta)=\sin 2\alpha\sin 2\beta$;

(3) $\frac{1-\tan\alpha}{1+\tan\alpha}=\tan(45^\circ-\alpha)$;

(4) $\frac{\sin\alpha+\sin 3\alpha+\sin 5\alpha+\sin 7\alpha}{\cos\alpha+\cos 3\alpha+\cos 5\alpha+\cos 7\alpha}=\tan 4\alpha$.

Transform the given products into sums:

1211. (1) $\sin 42^\circ \cos 12^\circ$; (2) $\sin 40^\circ \sin 14^\circ$;

(3) $\sin 22^\circ \sin 8^\circ$; (4) $2 \sin 18^\circ \cos 12^\circ$;

(5) $\cos 20^\circ \cos 40^\circ$; (6) $\cos 18^\circ \cos 56^\circ$.

1212. (1) $\sin \frac{\pi}{8} \cos \frac{3\pi}{8}$; (2) $\cos \frac{\pi}{10} \cos \frac{\pi}{5}$;

(3) $\sin \frac{\pi}{5} \sin \frac{2\pi}{5}$; (4) $\sin 3\alpha \cos 2\alpha$;

(5) $2 \cos 3\alpha \cos \alpha$; (6) $\sin 4\alpha \sin 3\alpha$.

1213. (1) $\sin (\alpha+\beta) \cos (\alpha-\beta)$;

(2) $2 \sin 40^\circ \cos 40^\circ \cos 50^\circ$;

(3) $4 \cos 18^\circ \cos 20^\circ \cos 16^\circ$;

(4) $4 \cos \alpha \cos 3\alpha \cos 5\alpha$;

(5) $2 \sin \alpha \sin 2\alpha \sin 3\alpha$;

(6) $8 \cos (\alpha-\beta) \cos (\alpha-\gamma) \cos (\beta-\gamma)$.

1214. 1. Evaluate the following expressions without using the Tables:

(a) $2 \cos 45^\circ \sin 15^\circ$; (b) $\cos 105^\circ \cos 75^\circ$;

(c) $\sin \frac{\pi}{24} \sin \frac{5\pi}{24}$.

2. Transform the following products to linear sums of trigonometric functions:

(a) $\sin^2 2\alpha$; (b) $2 \cos^2 (45^\circ-\alpha)$;

(c) $2 \sin^2 (45^\circ-\alpha)$; (d) $4 \cos \alpha \sin^2 \frac{\alpha}{2}$;

(e) $\sin 4\alpha \cos^2 \left(2\alpha+\frac{\pi}{4}\right)$;

(f) $4 \cos^4 \alpha$; (g) $16 \sin^2 \alpha \cos^3 \alpha$;

(h) $32 \sin^3 \cos^5 \alpha$.

1215. Evaluate the following expressions using two methods: by direct multiplication of the values taken from the Tables and by firstly transforming the product to a sum. Compare the results:

(1) $\sin 22^\circ \cos 8^\circ$; (2) $\sin 18^\circ \sin 34^\circ$;

(3) $\cos 0.263 \cos 0.445$; (4) $2\cos^2 32°24'$;

(5) $\sin^2 36°34'$; (6) $4\cos^2 10° \cos 70°$.

Simplify the following expressions:

1216. (1) $2\cos 10° \cos 40° - \cos 50°$;

(2) $\cos 20° - 2\sin 20° \sin 40°$;

(3) $\sin\alpha\,(1+2\cos 2\alpha)$;

(4) $\cos 3\alpha - 2\cos\alpha \cos 2\alpha$;

(5) $\sin\alpha + 2\sin\left(\frac{5\pi}{12}-\frac{\alpha}{2}\right)\cos\left(\frac{5\pi}{12}+\frac{\alpha}{2}\right)$;

(6) $\cos 2\alpha + 2\sin(\alpha+30°)\sin(\alpha-30°)$.

1217. (1) $\sin\left(\alpha+\frac{\pi}{3}\right)\sin\left(\alpha-\frac{\pi}{3}\right)-\sin^2\alpha$;

(2) $\cos^2\alpha - \cos(\alpha+30°)\cos(\alpha-30°)$;

(3) $\cos 4\alpha + 2\cos 2\alpha - 4\cos 2\alpha \cos^2\alpha$;

(4) $\sin 1° + \sin 91° - 2\sin 157°(\sin 112° + \sin 158°)$;

(5) $\sin 55° - \sin 145° + 2\sin 185°(\sin 130° + \sin 140°)$.

1218.

(1) $\frac{\tan 2\alpha \tan\alpha}{\tan 2\alpha - \tan\alpha}$; (2) $\frac{1}{\tan 2\alpha \tan\alpha + 1}$;

(3) $\frac{\cos\alpha + \cos 3\alpha}{\sin\alpha + \sin 3\alpha}$;

(4) $(\sin\alpha + \sin 2\alpha)^2 + (\cos\alpha + \cos 2\alpha)^2$;

(5) $\frac{\cos\alpha - \cos 3\alpha + \cos 5\alpha - \cos 7\alpha}{\sin\alpha + \sin 3\alpha + \sin 5\alpha + \sin 7\alpha}$.

1219. Find the greatest and the least values of the given functions and their periods:

(1) $y = a\sin x + b\cos x$; (2) $y = \sin x + \cos x$;

(3) $y = \sin x \cos\left(\frac{\pi}{4}-x\right)$;

(4) $y = \cos^4 x + \sin^4 x$;

(5) $y = \sin 2x - \sqrt{3}\cos 2x$; (6) $y = \sin^6 x + \cos^6 x$.

1220*. Compute without using the Tables:

(1) $\frac{1}{\sin 10^\circ}-4\sin 70^\circ$; (2) $\cos 72^\circ-\sin 54^\circ$;

(3) $\cos\frac{2\pi}{7}+\cos\frac{4\pi}{7}+\cos\frac{6\pi}{7}$;

(4) $\cos 20^\circ \cos 40^\circ \cos 80^\circ$.

1221. Prove the validity of the following equalities:

(1) $\cos 45^\circ+\cos 75^\circ-\cos 15^\circ = \cos(45^\circ-\alpha)-\sin(45^\circ+\alpha)$;

(2)* $16\sin 20^\circ \sin 40^\circ \sin 60^\circ \sin 80^\circ = 3$;

(3)* $8\sin 20^\circ \cos 40^\circ \cos 80^\circ = \tan 20^\circ$;

(4)* $\tan 20^\circ \tan 40^\circ \tan 60^\circ \tan 80^\circ = 3$.

Prove the following identities:

1222. (1) $\frac{\cos\alpha+\sin\alpha}{\cos\alpha-\sin\alpha} = \tan(45^\circ+\alpha)$;

(2) $\frac{1+\sin 2\alpha}{1-\sin 2\alpha} = \cot^2(45^\circ-\alpha)$;

(3) $\frac{\cot\alpha-1}{\cot\alpha+1} = \tan(45^\circ-\alpha)$;

(4) $\frac{\cot\alpha+\cot\beta}{\cot\alpha-\tan\beta} = \tan(\alpha+\beta)\cos\beta$;

(5) $(\sin\alpha+\sin\beta)^2+(\cos\alpha+\cos\beta)^2 = 4\cos^2\frac{\alpha-\beta}{2}$.

1223. (1) $\frac{\tan\left(\frac{\pi}{4}+\alpha\right)-\tan\left(\frac{\pi}{4}-\alpha\right)}{\cot\left(\frac{\pi}{4}+\alpha\right)+\cot\left(\frac{\pi}{4}-\alpha\right)} = \sin 2\alpha$;

(2) $\cos^2\alpha-\sin^2\left(\frac{\pi}{4}+\alpha\right) = \frac{\sqrt{2}}{2}\sin\left(\frac{\pi}{4}-2\alpha\right)$;

(3) $\cos^2\alpha+\cos^2\beta-\sin^2(\alpha+\beta) =$

$= 2\cos\alpha\cos\beta\cos(\alpha+\beta)$;

(4) $\sqrt{4\sin^4\alpha+\sin^2 2\alpha}+4\cos^2\left(45^\circ-\frac{\alpha}{2}\right) = 2$,

$180^\circ < \alpha < 270^\circ$.

1224. Prove that for the condition $\alpha+\beta+\gamma = 180°$ (for instance, for the angles of a triangle) the following relations are valid:

(1) $\sin\alpha+\sin\beta+\sin\gamma = 4\cos\frac{\alpha}{2}\cos\frac{\beta}{2}\cos\frac{\gamma}{2}$;

(2) $\cos\alpha+\cos\beta+\cos\gamma = 1+4\sin\frac{\alpha}{2}\sin\frac{\beta}{2}\sin\frac{\gamma}{2}$;

(3) $\sin\alpha+\sin\beta-\sin\gamma = 4\sin\frac{\alpha}{2}\sin\frac{\beta}{2}\cos\frac{\gamma}{2}$;

(4) $\cos\alpha+\cos\beta-\cos\gamma = -1+4\cos\frac{\alpha}{2}\cos\frac{\beta}{2}\sin\frac{\gamma}{2}$;

(5) $\tan\alpha+\tan\beta+\tan\gamma = \tan\alpha\tan\beta\tan\gamma$;

(6) $\cot\frac{\alpha}{2}+\cot\frac{\beta}{2}+\cot\frac{\gamma}{2} = \cot\frac{\alpha}{2}\cot\frac{\beta}{2}\cot\frac{\gamma}{2}$;

(7) $\sin^2\alpha+\sin^2\beta+\sin^2\gamma = 2+2\cos\alpha\cos\beta\cos\gamma$;

(8) $\cos^2\alpha+\cos^2\beta+\cos^2\gamma = 1-2\cos\alpha\cos\beta\cos\gamma$.

Solve the following equations:

1225. (1) $\sin 4x\cos 2x = \sin 5x\cos x$;

(2) $\cos 5x\cos x = \cos 6x\cos 2x$;

(3) $\sin 2x\sin 6x = \sin 3x\sin 5x$;

(4)* $\cot 2x-\tan x = 0$;

(5) $\cos 3x = \sin x$.

1226. (1) $\sin 3x+\sin x = 0$; (2) $\cos 2x+\cos x = 0$;

(3) $\cos 5x = \cos 3x$; (4) $\cos 10x = \sin 5x$;

(5) $\sin(40°-x) = \cos(40°+x)$;

(6) $1-\cos 2x = 2\sin x$.

1227. (1) $1-\sin 2x = \frac{1-\cot x}{1+\cot x}$;

(2)* $\tan x+\tan 2x-\tan 3x = 0$;

(3) $\tan 2x = \sin 4x$;

(4) $\tan(x+45°)\tan x\tan(x-45°) = \tan 2x$.

1228. (1) $\sin x+\cos x = 1$; (2) $\cos x-\sin x = \frac{\sqrt{2}}{2}$;

(3) $\sin x - \cos x = 0.7071$;

(4) $\sqrt{3} \sin x - \cos x = \sqrt{2}$;

(5) $2 \sin x + 9 \cos x = 7$; (6) $\cos x - 8 \sin x = 9$.

1229. (1) $\sin (x+47^\circ) - \sin (x-13^\circ) = -0.7432$;

(2) $\sin (x+94^\circ) - \cos (86^\circ - x) = 0$;

(3) $\sin^2 (450^\circ + x) + \cos (45^\circ + x) \sin (45^\circ + x) = 0$;

(4) $\sin^2 x + 2 \sin (45^\circ + x) \sin (45^\circ - x) = 0$;

(5) $\cos^2 x - 2 \cos (45^\circ + x) \cos (45^\circ - x) = 0$.

1230. Solve the following inequalities:

(1) $\cos x \cos 2x < \sin x \sin 2x$;

(2) $\sin x + \sqrt{3} \cos x < 0$;

(3) $\sin 3x \cos x > \sin x \cos 3x$;

(4)* $\sin x \sin \left(\frac{\pi}{3} - x\right) \sin \left(\frac{\pi}{3} + x\right) \leqslant \frac{1}{8}$.

1231. Derive relations for the arguments α and β for which the following equalities hold:

(1) $\sin \alpha = \sin \beta$; (2) $\cos \alpha = \cos \beta$; (3) $\tan \alpha = \tan \beta$;

(4) $\sin \alpha = \cos \beta$; (5) $\sin (\alpha + \beta) = \sin \alpha + \sin \beta$.

1232. The base of an isosceles triangle equals b, the base angle, α. Find the perimeter of the triangle if $b = 20$ cm, and $\alpha = 44^\circ$.

49. GRAPHICAL REPRESENTATION OF TRIGONOMETRIC FUNCTIONS

1233. Graph the functions $y = \sin x$, $y = \cos x$, $y = \tan x$, $y = \cot x$ and show how their properties have been used in constructing the graphs.

1234. 1. Construct in one drawing the graphs of the functions:

(a) $y = \sin x$ and $y = \sin \left(x - \frac{\pi}{3}\right)$; (b) $y = \sin x$ and $y = \sin \left(x + \frac{\pi}{2}\right)$. What can be said of the shape and location of these graphs in the coordinate system? Are these conclusions correct for

the graphs of the functions: $y = f(x)$ and $y = f(x+a)$, where $y = f(x)$ is an arbitrary function?

2. Construct the graphs of the given functions with the aid of stencils:

$$y = \sin\left(x + \frac{\pi}{4}\right); \quad y = \cos\left(x - \frac{\pi}{6}\right); \quad y = \tan\left(x + \frac{\pi}{3}\right);$$

$$y = \cot\left(x - \frac{\pi}{2}\right).$$

1235. 1. Graph on one drawing the following functions:

(a) $y = \sin x$ and $y = -\sin x$; (b) $y = \tan x$ and $y = -\tan x$.

What can be said of the shape and location of the graphs in the coordinate system? Are these conclusions correct when speaking of the graphs of the functions $y = f(x)$ and $y = -f(x)$, where $y = f(x)$ is an arbitrary function?

2. With the aid of stencils construct the graphs of the functions

$$y = -\cos x, \quad y = -\cot x.$$

1236. 1. Graph in one drawing the following functions:

(a) $y = \sin x$ and $y = \sin(-x)$; (b) $y = \cos x$ and $y = \cos(-x)$.

What can be said about the shape and location of these graphs in the coordinate system? Draw a conclusion on the shape and situation of the graphs of the functions $y = f(x)$ and $y = f(-x)$ in the coordinate system.

2. Using a stencil, construct the graphs of the functions $y = \tan(-x)$ and $y = \cot(-x)$.

1237. 1. On one drawing graph the functions $y = \sin x$ and $y = \sin x + 2$. Consider the shape of the graphs and their location in the coordinate system. Draw a conclusion on the shape and location of the graphs of the functions $y = f(x)$ and $y = f(x) + a$ in the coordinate system.

2. Using stencils, construct the graphs of the following functions:

(a) $y = \sin x - 1$, $y = \cos x + \frac{1}{2}$, $y = \tan x - 3$;

(b) $y = \cot x + 3$, $y = -\sin x + 1$, $y = -\cos x - 2$.

1238. 1. Construct on one drawing the graphs of the functions $y = \sin x$ and $y = |\sin x|$. How should the graph of the function

$y = \sin x$ be changed to obtain the graph of the function $y = |\sin x|$? How should the graph of the function $y = f(x)$ be changed to obtain the graph of the function $y = |f(x)|$?

2. Using stencils, construct the graphs of the following functions:

(a) $y = |\cos x|$, $y = |\tan x|$, $y = |\cot x|$;

(b) $y = \left|\sin\left(x - \frac{\pi}{3}\right)\right|$, $y = \left|\cos\left(x + \frac{\pi}{4}\right)\right|$, $y = |\sin x - 1|$.

1239. 1. Construct the graphs of the functions $y = \sin x$ and $y = \sin |x|$ on one and the same drawing. How should the graph of the function $y = \sin x$ be changed in order to obtain the graph of the function $y = \sin |x|$? How should the graph of the function $y = f(x)$ be changed to obtain the graph of the function $y = f(|x|)$?

2. Using stencils, construct the graphs of the functions

$$y = \cos |x|, \quad y = \tan |x|, \quad y = \sin \left|x + \frac{\pi}{3}\right|, \quad y = \sin |x| + 1.$$

1240. 1. Construct on one drawing the graphs of the functions $y = \sin x$ and $y = 2 \sin x$. How should the graph of the function $y = \sin x$ be deformed to obtain that of $y = 2 \sin x$? How should the graph of the function $y = f(x)$ be deformed into the graph of the function $y = Af(x)$?

2. Plot the graphs of the following functions:

(a) $y = \frac{1}{2} \sin x$, $y = 3 \cos x$, $y = -\frac{1}{3} \tan x$,

$y = 2 \sin\left(x + \frac{\pi}{3}\right)$;

(b) $y = |2 \cot x|$, $y = -2 \sin x$, $y = 1 - 2 \cos x$,

$y = \frac{1}{2} \tan x + 2$.

1241. 1. Construct on one drawing the graphs of the functions $y = \sin x$ and $y = \sin 2x$. How should the graph of the function $y = \sin x$ be deformed to obtain that of $y = \sin 2x$? How should the graph of the function $y = f(x)$ be deformed to obtain that of $y = f(ax)$?

1242. 1. Construct on one drawing the graphs of the functions: $y = \sin x$ and $y = 3 \sin\left(2x + \frac{\pi}{3}\right)$. What should be done with the graph of the function $y = \sin x$ to obtain the graph of the function

$y = 3 \sin \left(2x + \frac{\pi}{3}\right)$? How should the graph of the function $y = f(x)$ be transformed to get the graph of the function $y = Af(ax+b)$?

2. Plot the graphs of the following functions:

(a) $y = \frac{1}{2} \sin \left(2x - \frac{\pi}{2}\right)$; (b) $y = 3 \cos \left(\frac{1}{2}x + \frac{\pi}{6}\right)$;

(c) $y = 2 \sin \left(x - \frac{\pi}{4}\right) + 1$; (d) $y = \frac{1}{3} \tan \left(2x - \frac{\pi}{3}\right)$.

Find their periods.

1243. Plot the graphs of the following functions:

(1) $y = \sin x + \cos x$; (2) $y = \sin^2 x$;

(3) $y = \sin x^2$; (4) $y = \sin^2 x + \cos^2 x$;

(5) $y = \sin x + \sin 2x$; (6) $y = \sin x + x$.

1244. Solve graphically the following equations:

(1) $\sin x = \frac{1}{3}$; (2) $\sin x = x$; (3) $\cos x = |x|$;

(4) $\tan x = 2$; (5) $\cos x + x = 0$.

1245. Given the function $y = A \sin (\omega t + \varphi_0)$.

1. Determine the greatest and the least values of the function. What is the physical meaning of the parameter A?

2. Show that the number $T = \frac{2\pi}{\omega}$ is the period of this function.

3. Find out the physical meaning of φ_0.

1246. Given three simple harmonic oscillations:

$$y_1 = \sin 2t; \quad y_2 = 2 \sin 2t; \quad y_3 = 4 \sin 2t.$$

1. What period has each of these oscillations?

2. In what respect do they differ from one another?

1247. Find the amplitude, the period, the frequency and the initial phase of each of the following oscillations:

(1) $y = 2 \sin t$; (2) $y = 0.5 \sin \left(2t + \frac{\pi}{3}\right)$;

(3) $y = 3 \sin \left(0.5t - \frac{\pi}{6}\right)$;

(4) $y = \sqrt{2} \sin \left(\frac{\pi}{2} t+3\right)$;

(5) $y = \frac{1}{3} \sin 2\pi(t-3)$; (6) $y = \cos \left(\pi t+\frac{\pi}{4}\right)$.

1248. A point performs simple harmonic oscillation along the line segment A_1A_2 equal to 12 cm; the distance A_1A_2 is covered by the point in 6 seconds.

1. Set up the equation of the motion, taking the point A_1 as its initial position.

2. Determine at what distance from its initial position the moving point will be found in 3 seconds; 12 seconds; 21 seconds.

1249. Transform the function $y = A \cos ax+B \sin ax$ to $y = C \sin (ax+\alpha)$:

(1) $y = 3 \sin x+4 \cos x$; (2) $y = -5 \sin 2x+12 \cos 2x$;

(3) $y = -0.6 \sin 3x-0.8 \cos 3x$;

(4) $y = 0.5 \sin 4x-0.8 \cos 4x$.

1250. 1. Do the graphs of the functions $\sin x$ and $\tan x$ have the axes of symmetry?

2. What is the centre of symmetry of the graphs of the functions $\sin x$ and $\cot x$?

3. Indicate the centres of symmetry of the graphs of the functions $\cos x$ and $\tan x$.

50. TRIGONOMETRIC EQUATIONS OF VARIOUS TYPES

1251. (1) $\sin x+\sin 2x = \cos x+2 \cos^2 x$;

(2) $\cos x-\sqrt{2} \sin \frac{x}{2} = 1$;

(3) $\cos^2 \left(\frac{\pi}{8}-x\right)-\cos^2 \left(\frac{\pi}{8}+x\right) = \frac{1}{2}$;

(4) $2 \cos 2x = \sqrt{6}(\cos x-\sin x)$.

1252. (1) $(2 \sin x-\cos x)(1+\cos x) = \sin^2 x$;

(2) $2(\sin x+\cos x)^2(\sin x+\cos x)^2 = \tan (45^\circ+x)$;

(3) $\sin 3x+\sin 2x+\sin x = 0$;

(4) $\frac{1}{\cos x}+\frac{1}{\sin x} = \sin x \tan x+\cos x \cot x$.

1253. (1) $\cos x+\sin x=\sqrt{2}\cos 2x$;

(2) $\cos 4x-\cos 2x+\sin x=0$;

(3) $\cos x-\cos 2x=\sin 3x$;

(4) $\cos 3x=-2\cos x$.

1254. (1) $\sin 2x+\sin 4x+\sin 6x=1+\cos 2x+\cos 4x$;

(2) $2-\tan 2x=\dfrac{\cos 2x}{1+\sin 2x}$;

(3) $\csc x=\sin x+\cos x$;

(4) $\dfrac{\cos 2x}{1-\sin 2x}=\cos x+\sin x$;

(5)* $\tan 2x-2\sin^2 x=\sin 2x$.

1255. (1) $1-\cos 2x=\tan x$;

(2) $\cos x-\sin x=1-\sin 2x$;

(3) $1-\sin x=\sin^2\left(\dfrac{\pi}{4}-\dfrac{x}{2}\right)$;

(4) $\sin^3 x+\cos^3 x=\sin x$.

1256*. (1) $\dfrac{\cos 2x}{1-\tan x}=0$; (2) $\dfrac{\cos 2x}{1+\sin 2x}=0$;

(3) $\dfrac{\sin x}{\sin 2x}=\dfrac{\cos 2x}{\cos x}$; (4) $\dfrac{2\cos x}{1+\cos 2x}=\dfrac{1-\cos 2x}{\sin 2x}$;

(5) $\dfrac{1-\cos 2x}{2\sin x}=\dfrac{\sin 2x}{1+\cos 2x}$; (6) $\dfrac{1-\tan x}{1-\cot x}=2\sin x$.

1257*. (1) $\tan 5x+\tan 3x=0$; (2) $\tan 4x-\tan x=0$;

(3) $\cot 3x+\tan x=0$; (4) $\tan 5x-\cot 2x=0$;

(5) $\dfrac{\tan x}{\tan 2x}+\dfrac{\tan 2x}{\tan x}=-2$.

1258. (1) $\dfrac{\tan(x+29^\circ)}{\tan(x-1^\circ)}=\dfrac{1}{3}$; (2) $\dfrac{\tan(x+42^\circ)}{\tan(x+12^\circ)}=3$;

(3) $\dfrac{\cot(x+30^\circ)}{\cot(x+10^\circ)}=2.2$; (4) $\dfrac{\sin(x-15^\circ)}{\sin(x+45^\circ)}=2$;

(5) $\dfrac{\sin(x+20^\circ)}{\cos(50^\circ+x)}=4$; (6) $\dfrac{\cos(x-5^\circ)}{\cos(95^\circ-x)}=2+\sqrt{3}$.

1259. (1) $\tan(x+20^\circ)\cot(x-10^\circ) = -0.5$;

(2) $\tan(40^\circ+x)\tan(10^\circ-x) = 1.5$;

(3) $\tan(x-50^\circ) = -0.25\cot(x+10^\circ)$;

(4) $\tan\left(\frac{\pi}{4}+x\right) = 3\tan\left(\frac{\pi}{4}-x\right)$;

(5) $\cot x+\cot(45^\circ+x) = 2$.

1260. (1) $\tan x+\cot x = 8\sin 2x$; (2) $\cot x-\cot 2x = 2$;

(3) $\tan(\pi+2x)-\cot\left(\frac{3\pi}{2}+3x\right) = 2\tan 2x$;

(4) $\tan x+\cot x = \frac{1}{\cos 70^\circ}$.

1261. (1) $\tan 2x = \sin 4x$; (2) $8\tan^2\frac{x}{2} = 1+\frac{1}{\cos x}$;

(3) $\cos 2x-3\cos x = 4\cos^2\frac{x}{2}$;

(4)* $\tan x+\tan 2x = \tan 3x$.

1262. (1) $\sin^2 x+\sin^2 2x = \sin^2 3x$;

(2)* $\sin(\pi\cos x)-\cos(\pi\sin x) = 0$;

(3) $3-3\sin^4 x-5\cos^4 x = 0$;

(4) $4\sin 2x\sin 4x\sin 6x = \sin 4x$;

(5) $2\cos 3x\cos 4x+2\sin 2x\sin 5x = \cos 2x+\cos 4x$.

1263*. (1) $2\sin x\cos\frac{x}{2} = \sqrt{\frac{1+\cos x}{2}}$;

(2) $\sin\left(\frac{\pi}{4}+\frac{3x}{2}\right) = 3\sin\left(\frac{\pi}{4}-\frac{x}{2}\right)$;

(3) $\sin^2 x+\frac{1}{4}\sin^2 3x = \sin x\sin^2 3x$;

(4) $\sin^4 x+\cos^4 x = \frac{5}{8}$.

1264. 1*. Show that the equation $2\cos^2\frac{x}{2}\sin^2 x = x^2+\frac{1}{x^2}$ has no solution.

2. Solve the equation $2\cos\frac{x}{2} = 3^x+3^{-x}$.

1265. Eliminate x from the given systems of equations:

(1) $\begin{cases} \sin x \cos x = a \\ \sin x - \cos x = b; \end{cases}$ (2) $\begin{cases} \tan x + \cot x = a \\ \tan^2 x + \cot^2 x = b. \end{cases}$

1266. Find $\tan x$ and $\tan y$ if $x+y = 45°$ and $\tan x + \tan y = 4$.

1267. Determine positive acute angles from the given systems:

(1) $\begin{cases} \sin x \cos y = 0.46 \\ \cos x \sin y = 0.24; \end{cases}$ (2) $\begin{cases} x+y = 67° \\ \cos x + \cos y = 1.1228; \end{cases}$

(3) $\begin{cases} x+y = 96°38' \\ \sin x : \sin y = 5 : 3; \end{cases}$ (4) $\begin{cases} x+y = 77°42' \\ \tan x \tan y = 0.3216. \end{cases}$

1268. Determine the angles of a rhomb if its perimeter is 1.5 times the sum of its diagonals.

1269. Determine the base angle of an isosceles triangle if its altitude equals $\frac{2}{3}$ of the radius of the circumscribed circle.

1270. In an isosceles trapezoid each of the lateral sides is equal to the smaller base, and the ratio of the greater base to its altitude is 3. Find the acute angles of the trapezoid.

1271. Determine the angles of a triangle, knowing that the ratio of two of them is 1 : 3, and the bisector of the third quadrant divides the area of the triangle in the ratio 5 : 2.

1272. A circle whose centre is at the vertex of the right angle of a right triangle touches the hypotenuse. Determine the angles of the triangle if this circle divides its area into two equal parts.

1273. A plane whose speed is v carries out its flight with the wind blowing from east to west at a speed of v_1. What should the direction of the longitudinal axis of the plane be so that it flies exactly northward?

CHAPTER XIV

Progressions

51. NUMBER SEQUENCES

1274. Write the first five terms of the sequence of (a) even numbers, (b) odd numbers, (c) numbers multiple of 3, 5, 8, (d) numbers which, when divided by 4 yield the remainder 3. Write the formula of the general term for each of these sequences.

1275. Establish the rule of setting up the given sequences and continue them according to this rule. Write the formula of the general term of the given sequences

(1) $1, 2, 3, \ldots$; (2) $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots$;

(3) $1, 4, 9, 16 \ldots$; (4) $1, 1, 01, 1, 0101, \ldots$;

(5) $\frac{1}{2}, -\frac{2}{3}, \frac{3}{4}, -\frac{4}{5}, \ldots$;

(6) $27, 22, 17, \ldots$; (7) $-1, \frac{1}{2}, -\frac{1}{4}, \frac{1}{8}, \ldots$;

(8) $\frac{1}{1\times2}, \frac{1}{2\times3}, \frac{1}{3\times4}, \ldots$;

(9) $\frac{1\times3}{2\times4}, \frac{3\times5}{4\times6}, \frac{5\times7}{6\times8}, \ldots$;

(10) $1, 1\times2, 1\times2\times3, 1\times2\times3\times4, \ldots$;

(11) $\frac{5}{3}, \frac{10}{9}, \frac{15}{27}, \ldots;$

(12) $\frac{3}{7}, \frac{7}{13}, \frac{11}{19}, \ldots.$

Which of these sequences (a) increase, (b) decrease, (c) increase without bound, (d) decrease without bound, (e) fluctuate?

1276. 1. Write the first five terms of the sequence of approximate values of $\sqrt{3}$ accurate to $\frac{1}{10^n}$ with deficit.

2. Write the first ten terms of the sequence of simple numbers. Are the formulas expressing the general term of each of these sequences known?

1277. Write the first five terms of the number sequence whose general term is expressed by the formula:

(1) $a_n = 3n-1$; (2) $a_n = n^2-1$; (3) $a_n = \frac{1}{n}$;

(4) $a_n = \frac{(-1)^{n-1}}{2n}$; (5) $a_n = \frac{n}{2n+1}$;

(6) $a_n = \frac{2^n-1}{3n+1}$; (7) $a_n = \left(\frac{n}{n+1}\right)^{(-1)^n}$.

Which of these sequences are decreasing, increasing, fluctuating? Give graphical representation of each of these sequences.

1278. Using the formula of the general term of the given sequence, compute a_3, a_6, a_{10}:

(1) $a_n = 2^n$; (2) $a_n = \frac{n-1}{n+1}$; (3) $a_n = \frac{1}{n!}$;

(4) $a_n = 3+4(n-1)$; (5) $a_n = 3\times 2^{n-1}$;

(6) $a_n = \frac{1+(-1)^n}{n}$.

1279. Find the third, the sixth and the eighth terms of the sequence if

(1) $a_n = \begin{cases} 2, \text{ where } n \text{ is an odd number} \\ \frac{n+1}{n}, \text{ where } n \text{ is an even number;} \end{cases}$

(2) $a_n = \begin{cases} \frac{1}{(n+1)^2}, \text{where } n \text{ is an odd number} \\ \frac{1}{n}, \text{ where } n \text{ is an even number.} \end{cases}$

1280. The general term of a number sequence is expressed by the formula $a_n = 2n^2 - 1$. Find out whether each of the following numbers is one of the terms of this sequence, and if it is, then what is its ordinal number: 241, 22 049, 7921, 15 841.

1281*. Compute the sum:

(1) $\frac{1}{1\times 2}+\frac{1}{2\times 3}+\frac{1}{3\times 4}+\ldots+\frac{1}{99\times 100}$;

(2) $\frac{3}{2\times 5}+\frac{3}{5\times 8}+\frac{3}{8\times 11}+\frac{3}{11\times 14}+\ldots+\frac{3}{35\times 38}$;

(3) $\frac{1}{1\times 3}+\frac{1}{3\times 5}+\frac{1}{5\times 7}+\ldots+\frac{1}{19\times 21}$.

52. ARITHMETIC PROGRESSION

1282. Given below are sequences of numbers generated according to a certain law:

(1) 1, 3, 5, 7, ...; (2) $1\times 2, 2\times 3, 3\times 4, \ldots$;

(3) 2, -3, 4, -5, ...; (4) $1+2, 2+3, 3+4, \ldots$;

(5) 9, 6, 3, 0, ...; (6) $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \ldots$;

(7) -3, 2, 0, 5, 3, ...; (8) 2, 1, 5, 1, 0, 5, 0, ...

Indicate which of these sequences constitute an arithmetic progression.

1283. To convert degrees Fahrenheit to degrees Centigrade the formula $C = \frac{5}{9}(F - 32)$ is used, where F is the number of degrees Fahrenheit, and C is the corresponding number of degrees Centigrade. Compute C if F = 41, 42, 43, 44, 45. Do the obtained numbers form an arithmetic progression?

1284. 1. What sequence of numbers do the values of the function $y = 8 - 5x$ form, if x takes on successively the values 0, 1, 2, 3, ...?

2. Find the sums of the interior angles of a triangle, quadrilateral, pentagon, etc. What sequence of numbers do these sums form?

1285. Determine the difference and the sixth term of the progression 2, 5, 8, 11, ...

1286. Write the arithmetic progressions consisting of six terms if (1) $a_1 = 4$ and $d = +2$; (2) $a_1 = 5$ and $d = -3$.

1287. 1. From the formula $a_n = a_1 + d(n-1)$ find a_1, d and n.

2. Determine the last term of an arithmetic progression if:

(a) $a_1 = 10$, $d = 4$, $n = 4$; (b) $a_1 = -4$, $d = 2$, $n = 10$;

(c) $a_1 = m_1$, $d = m$, $n = m$; (d) $a_1 = k$, $d = 3$, $n = 10$.

1288. Determine the first term of an arithmetic progression if:

(a) $a_n = -10$, $n = 6$, $d = -3$; (b) $a_n = \frac{10k}{3}$, $n = 5$, $d = \frac{k}{3}$.

1289. Determine the number of terms of an arithmetic progression if:

(a) $a_1 = 1$, $a_n = 61$, $d = 5$; (b) $a_1 = 15$, $a_n = -15$, $d = -6$.

1290. 1. Find a_1 if $a_n = \frac{4+\tan^2\alpha}{1+\tan^2\alpha}$, $n = 5$ and $d = \cos^2\alpha$.

2. Find a_1 if $a_n = \frac{5\cos^2\alpha+1}{\cos^2\alpha}$, $n = 6$ and $d = 1$.

3. Find n if $a_1 = \frac{a+b}{2}$, $a_n = 4a-3b$, $d = \frac{a-b}{2}$.

4. Find n if $a_1 = \tan\alpha$, $a_n = \frac{2\sin(60°+\alpha)}{\cos\alpha}$ and $d = \frac{\sqrt{3}}{5}$.

1291. 1. Determine the sum of all natural numbers (a) from 1 to 100, (b) from 1 to n.

2. Determine the sum of all the odd numbers (a) from 1 to 99; (b) from 1 to $2n-1$. Make sure that the sum of the first n odd numbers is a perfect square.

3. Determine the sum of all the even numbers from 2 to 100; from 2 to $2n$.

1292. The problems involving the arithmetic progression usually deal with the following five quantities: a_1, d, n, a_n, S_n. How many independent equations, interconnecting these quantities, exist? What are these equations? How many of the five quantities is it sufficient to know in order to calculate the remaining ones?

1293. Find the unknown quantities in the following table:

No.	a_1	a_n	d	n	S_n	No.	a_1	a_n	d	n	S_n
1	−1		5	8		8	18	−27			−72
2	31		−7	10		9	54			6	264
3		30	4	10		10	15			7	−21
4		−50	−6	10		11		−30		10	−120
5		67	2/3	100		12		23		12	474
6	7	26	1			13			−4	8	−24
7	13	−8	−3			14	31		−4		136

1294. The second term of an arithmetic progression is equal to −5, and the difference between the sixth and the fourth one to 6. Find the sum of the first ten terms of the progression.

1295. The third term of an arithmetic progression is equal to 11, and the sixth term to 23. Find the first term and the difference of the progression.

1296. Find the sum of the seven terms of an arithmetic progression if the product of the third term by the fifth one equals the second term, and the sum of the first and the eighth terms is equal to 2.

1297. The sum of the first and the fifth terms of an arithmetic progression is equal to −2, and that of the second and sixth terms to 2. Find the progression.

1298. The sum of the second and the fourth terms of an arithmetic progression is equal to 7, and the product of these terms to 10. Find the progression.

1299. The sum of the first and the fourth terms of a progression is equal to 2, and the sum of their squares to 20. Find the sum of eight terms of the progression.

1300. Find the arithmetic progression if:

(1) $a_2-a_6+a_4 = -7$ and $a_8-a_7 = 2a_4$;

(2) $S_4-S_6+2a_5+a_3 = 0$ and $S_2+S_3 = 2$;

(3) $S_3 = -3$, $S_5 = 10$.

1301. Find the arithmetic progression if:

(1) $a_1 = 1$ and $S_5 = \frac{1}{4}(S_{10} - S_5)$;

(2) $a_1 a_2 a_3 = 120$, $a_1 + a_2 + a_3 = 15$.

1302. 1. In an arithmetic progression $a_1 = 2$, $a_5 = 14$. How many terms should be taken to obtain the sum equal to 77?

2. In an arithmetic progression $a_1 = 1$, $S_m : S_n = \frac{m^2}{n^2}$. Find a_n.

1303*. Determine the sum of n terms of the series:

(a) $1 - 3 + 5 - 7 + \ldots$; (b) $1 - 2 + 3 - 4 + \ldots$.

1304. 1. Find S_{20} in an arithmetic progression in which $a_6 + a_9 + a_{12} + a_{15} = 20$.

2. In an arithmetic progression $a_1 + a_3 + a_5 + \ldots + a_{11} = 72$. Compute the sum $a_1 + a_6 + a_{11}$.

1305. Find the arithmetic progression, consisting of 10 terms if the sum of the terms occupying the even places is equal to 15, and the sum of those occupying the odd places to 12.5.

1306. The first term of an arithmetic progression is equal to 2, the second and the third terms are respectively equal to the squares of two successive natural numbers. Find the progression.

1307. Find the arithmetic progression if the sum of its n terms equals $2n^2 - 3n$.

1308. In an arithmetic progression $a_1 = -3$, $d = \frac{1}{2}$. How many terms has the progression if the sum of all the terms is equal to the sum of the same number of addends, each of them being equal to 3?

1309. Between the numbers 7 and 35 insert six arithmetic means, i.e. six numbers such that, together with the given ones, form an arithmetic progression.

1310. Determine the number of arithmetic means inserted between 1 and 31 if the seventh of them is to the last but one as 5 : 9.

1311. Find the arithmetic progression in which the arithmetic mean of the first n terms is equal to their number for any n.

1312. An arithmetic progression consists of n terms, its middle term is m. Find the sum of all its terms.

1313. The fifth term of an arithmetic progression equal to 10 is its middle term. Find the sum of all the terms of the progression.

1314. Six points A, B, C, D, E, and F lying on one straight line are spaced so that AB, BC, CD, DE, and EF form an arithmetic progression. $AC = 16$ cm and $CE = 24$ cm. Determine the distances between the neighbouring points.

1315. The interior angles of a polygon constitute an arithmetic progression whose difference is 5°. The least angle is equal to 120°. How many sides has the polygon?

1316. Determine the three-digit number whose digits form an arithmetic progression. When divided by the sum of its digits, this number yields the quotient 48, and the difference between this number and 198 is a number having the same digits as the sought-for number but written in the reverse order.

1317*. Find the sum of all non-reducible fractions with the denominator 3 contained between the numbers 5 and 8.

1318*. The sides of a right triangle form an arithmetic progression. Determine the angles of the triangle.

1319*. Solve the equations:

(1) $1+7+13+ \ldots +x = 280$;

(2) $(x+1)+(x+4)+(x+7)+ \ldots +(x+28) = 155$;

(3)* $1+3+5+ \ldots +(1+2x)+\left(3.5+5+6.5+ \ldots +\frac{7+3x}{2}\right) = 105$.

1320. Given an arithmetic progression. The corresponding terms of another arithmetic progression are added to, or subtracted from, the terms of the given progression. Will the obtained sequence be an arithmetic progression?

1321. The difference of an arithmetic progression with an infinite number of terms is positive. Are there terms in this progression which exceed an arbitrarily specified positive number? Give a graphical explanation of the answer.

1322. For what condition will all the terms of an arithmetic progression turn out to be numbers of equal parity?

1323. For what condition is the sum of the terms of an arithmetic progression equal to the number of its terms?

1324. Are there simple three-digit numbers whose digits form an arithmetic progression?

1325. Do the positive roots of the equations (a) $\sin x = 0$, (b) $\sin x = 0.5$, (c) $\tan x = \sqrt{3}$, arranged in the increasing order, form an arithmetic progression?

1326. For what values of a do the roots of the equation $\cos x = a$, arranged in the increasing order, form an arithmetic progression?

1327. Prove that if the numbers $\frac{2}{b+c}, \frac{2}{a+c}, \frac{2}{a+b}$ form an arithmetic progression, then the numbers a^2, b^2, c^2 also constitute an arithmetic progression.

1328. Prove that for any arithmetic progression the following equalities hold true:

(1) $a_1 - 2a_2 + a_3 = 0$; (2) $a_1 - 3a_2 + 3a_3 - a_4 = 0$;

(3) $a_1 - 4a_2 + 6a_3 - 4a_4 + a_5 = 0$.

1329. Prove that the cube of any whole number n is the sum of n terms of an arithmetic progression in which

(a) $a_1 = 1$, $d = 2(n+1)$; (b) $a_1 = n$, $d = 2n$;

(c) $a_1 = n^2 - n + 1$, $d = 2$.

1330. As is known, a freely falling body (air friction neglected) covers 4.9 m during the first second of its fall, then it continues falling with an acceleration of 9.8 m/s^2. Bearing this in mind, fill in the following table:

Time, s	1	2	3	4	5
Distance covered per second, m					
Distance covered from the beginning of the fall, m					

1. What sequence is formed by the numbers entering the second row of the table?

2. What can be said about the numbers forming the third row in connection with this sequence?

1331. Two bodies start from one point at an interval of 5 seconds and move in one and the same direction. During the first second each body covered 10 m, during each successive second it did by 10 m more than during the preceding one. When will the distance between the bodies be equal to 900 m?

1332. A train leaving the terminal increases its speed which in ten minutes becomes equal to 60 km/h. Determine its acceleration.

1333. Two bodies 32.2 m apart started moving to meet each other. During the first second the first body covered 1.5 m and then it accelerated at a rate of 0.5 m/s^2. The second body covered 0.7 m during the first second, accelerating then at a rate of 0.3 m/s^2. In how many seconds will they meet?

1334. Two balls are rolling along an inclined plane in one direction. During the first second they covered 0.5 m and 0.3 m, accelerating then at a rate of 0.6 m/s^2 and 0.4 m/s^2, respectively. What was the distance between the balls in ten seconds if they start rolling simultaneously?

1335. A cyclist started riding from point A. During the first hour he covered 10 km and then accelerated at a rate of 1 km/h^2. Another cyclist started simultaneously with him from point B 7.5 km distant from the point A. During the first hour he covered 12 km, accelerating then at a rate of 1.5 km/h^2. When will the second cyclist catch up with the first one?

1336. Two bodies started moving from point A in opposite directions. One of them covered 3 m during the first minute and then accelerated at a rate of 6 m/min^2. The other body left A 5 minutes later, covered 54 m during the first minute, and then accelerated at a rate of 3 m/min^2. In how many minutes (after the start of the first body) will the bodies be at equal distances from the point A?

1337. The ground floor of a house is 1 m above the ground. The stairs leading from the ground to the first floor has 18 steps each 20 cm high. Write the general formula expressing the height of any step above the ground level. What is the elevation of the first floor above the ground level?

1338. A worker attends 16 automatic lathes. The output of each lathe is a pieces per hour. The worker switched on the first lathe at 8 o'clock in the morning and then successively all the remaining lathes after each 5 minutes. How many pieces does the worker manufacture during the first two hours?

1339. A team of workers undertook to dig a well. It was agreed upon that each successive metre of depth would be paid for by 2 roubles dearer than the preceding one. Due to this agreement the last metre and the third from the bottom metre taken together cost as much as would cost the digging of the whole well, if each metre, independently of its depth, would cost as much as the first one. The average cost of digging one metre equals 17 roubles. Compute the depth of the well.

1340. Balls are arranged to form a triangle so that the first row

consists of one ball, the second of two, the third of three balls, and so on. In how many rows are the balls arranged if their total number equals 120?

1341. Balls are arranged in rows to form an equilateral triangle. The first row consists of one ball, the second of 2 balls, and so on. If 669 more balls are added, then all the balls can be arranged in the shape of a square, and each of its sides then contains 8 balls less than each side of the triangle did. Determine the initial number of balls.

53. THE GEOMETRIC PROGRESSION

1342. Show that a sequence of numbers forms a geometric progression if its general term is expressed by the formula:

(a) $a_n = \frac{2}{3} \times 2^n$; (b) $a_n = \frac{3}{2^n}$; (c) $a_n = \left(-\frac{1}{3}\right)^n$.

1343. (Orally). Find the ratio of a geometric progression if:

(1) $a_1 = 9$, $a_2 = 12$; (2) $a_4 = 20$, $a_5 = 50$;

(3) $a_7 = -30$, $a_9 = -90$.

1344. Compute the seventh term of the geometric progression:

(a) 5, 10, 20, ...; (b) 1100, 110, 11, ...; (c) 6, 3, 1, 5, ...; (d) $5\frac{5}{8}$, $-3\frac{3}{4}$, $2\frac{1}{2}$, ...

1345. A geometric progression consists of n terms. Which number has the fifth term when counted from the end, the kth term from the end?

1346. Find the first term of a geometric progression in which:

(a) $a_6 = 486$, $q = 3$; (b) $a_9 = \frac{4}{9}$, $q = -\frac{1}{3}$; (c) $a_7 = -11\frac{25}{64}$, $q = \frac{3}{2}$.

1347. Find the sum of the terms of a geometric progression in which:

(1) $a_1 = 2$, $q = 3$, $n = 5$; (2) $a_1 = 6$, $q = \frac{1}{2}$, $n = 6$;

(3) $a_1 = \frac{1}{2}$, $q = -2$, $n = 7$;

(4) $a_1 = -2.5$, $q = 1.5$, $n = 8$.

1348. Determine the ratio and the sum of the terms of a geometric progression in which:

(1) $a_1 = 76\frac{4}{5}$, $n = 6$, $a_6 = -\frac{12}{5}$;

(2) $a_1 = -20\frac{1}{4}$, $n = 5$, $a_5 = -\frac{1}{4}$;

(3) $a_1 = 1.5$, $n = 4$, $a_4 = 96$.

1349. Determine the first term and the sum of all terms of a geometric progression in which:

(1) $q = 4$, $n = 8$, $a_8 = 49\,152$;

(2) $q = 2\frac{1}{2}$, $n = 6$, $a_6 = -3125$;

(3) $q = -\frac{1}{2}$, $n = 10$, $a_{10} = -7$;

(4) $q = \frac{2}{3}$, $n = 7$, $a_7 = \frac{2}{3}$.

1350. Determine the first and the last terms of a geometric progression in which:

(1) $q = 2$, $n = 9$, $S_9 = 1533$; (2) $q = \frac{1}{2}$, $n = 5$, $S_5 = 3\frac{7}{8}$;

(3) $q = \frac{2}{3}$, $n = 5$, $S_5 = -422$;

(4) $q = -2$, $n = 12$, $S_{12} = -1365$.

1351. Determine the number of terms of a geometric progression in which:

(1) $a_1 = 1$, $a_n = -512$, $S_n = -341$;

(2) $a_1 = \frac{3}{2}$, $a_n = -\frac{1}{1458}$, $S_n = 1\frac{91}{729}$;

(3) $a_1 = -3$, $a_4 = -192$, $S_n = -1023$;

(4) $q = 2$, $a_n = 96$, $S_n = 189$.

1352. 1. Between the numbers 4 and 108 place two numbers such that the four numbers form a geometric progression.

2. Between the numbers 240 and 3.75 place five geometric means.

1353. In the following problems, given three quantities determine the remaining two:

No	a_1	q	n	a_n	S_n	No	a_1	q	n	a_n	S_n
1		1/2	8	2		4	1/2			$\frac{1}{128}$	$\frac{127}{128}$
2	1		7	729		5	−1/3	2/3		$-\frac{16}{243}$	
3		−2/3	6		$-\frac{133}{160}$	6		−3	4	121.5	

1354. Write a geometric progression in which:

(1) $a_5-a_1 = 160$, $a_4-a_2 = 48$;

(2) $a_5+a_2-a_4 = 66$, $a_6+a_3-a_5 = -132$.

1355. Determine the first term, the ratio, and the number of terms of a geometric progression in which:

(1) $a_6-a_4 = -\frac{32}{27}$, $a_5+a_4 = \frac{16}{9}$, $S_n = 53\frac{241}{243}$;

(2) $a_6-a_4 = 216$, $a_3-a_1 = 8$, $S_n = 40$.

1356. What geometric progression has the sum:

(1) $\frac{3-3\times 4^{10}}{1-4}$; (2) $\frac{100\left[1-\left(\frac{2}{5}\right)^6\right]}{1-\frac{2}{5}}$; (3) $\frac{q^n-1}{q-1}$?

1357. 1. Express the product of the first n terms of a geometric progression in terms of its first term and the ratio.

2. Compute the product of the first five terms of the progression 1, 4, 16, ...

1358. Prove that in a geometric progression consisting of even number of terms the ratio of the sum of terms occupying even places to the sum of terms occupying odd places is equal to the ratio of the progression.

1359. Show that if the numbers ab, b^2, c^2 form an arithmetic progression, then the numbers b, c, $2b-a$ constitute a geometric progression.

1360. Prove that the terms of a geometric progression a_k, a_l, a_m, a_n are related by the equality $a_k a_l = a_m a_n$ if $m+n = k+l$.

1361*. 1. Can the numbers 1, $\sqrt{2}$, $\sqrt{3}$ be the terms of a geometric or arithmetic progression (not obligatory successive ones)?

2. Is it possible that the three numbers a_1, a_2, a_3 are simultaneously the first, the second, and the third terms of both an arithmetic and a geometric progressions?

1362. Given the progression $a_1, a_2, a_3, \ldots, a_n$. Do the following numbers form a progression:

(1) $a_1, a_4, a_7, \ldots, a_{3n-1}$; (2) $a_2, a_4, a_6, \ldots, a_{2n}$;

(3) $\frac{1}{a_1}, \frac{1}{a_2}, \ldots, \frac{1}{a_n}$;

(4) $\sqrt{a_1}, \sqrt{a_2}, \ldots, \sqrt{a_n}$?

Consider two cases: the given progression is (a) arithmetic; (b) geometric.

1363. In a geometric progression $a_m = k$, $a_n = l$. Find a_p if all terms of the progression are positive.

1364*. Find the sum of n numbers of the form 2, 22, 222, ...

1365. The sides of a right triangle form a geometric progression. Find the cosines of its acute angles.

1366. Find three numbers forming a geometric progression if their sum is equal to 26, and the sum of their reciprocals to $\frac{13}{18}$.

1367. The sum of the first three terms of a geometric progression is equal to 7, and the sum of their squares to 21. Find the progression.

1368. Break the number 155 into three parts so that the obtained numbers form a geometric progression, the first term being less than the third one by 120.

1369. The legend says that an Indian shakh allowed the inventor of chess to choose the reward himself. The inventor asked for the following: let one weat grain be placed on the first square of the chessboard, two grains on the second, four grains on the third, and so on, i.e. let a double amount of grains be placed on the next square as compared with the preceding one. Find the number of grains to be received by the inventor of chess.

1370*. A geometric progression consists of an even number of terms. The sum of all its terms is five times the sum of the terms occupying odd places. Find the ratio of this progression.

1371. Find the geometric progression consisting of 6 terms if the sum of its first three terms is equal to 168, and the sum of the last three terms to 21.

1372. Determine the four numbers forming a geometric progression if the sum of its extreme terms is equal to 27, and the sum of its middle terms to 18.

1373*. The sum of the first ten terms of an arithmetic progression is equal to 155, and the sum of the first two terms of a geometric progression to 9. Find these progressions if the first term of the arithmetic progression equals the ratio of the geometric progression, and the first term of the geometric progression equals the difference of the arithmetic progression.

1374. An arithmetic progression consists of 11 terms. Its first, fifth and eleventh terms form a geometric progression. Write all the terms of the arithmetic progression if the first term equals 24.

1375. In an arithmetic progression, consisting of 7 terms, the first term is equal to unity, and the sum of all the terms to 91. A geometric progression also consists of 7 terms, the first and the last terms coinciding with the terms of the given arithmetic progression. Find its tenth term.

1376*. Given an arithmetic and a geometric progression. The first two terms of the geometric progression coincide respectively with the first and the second terms of the arithmetic progression, and the third term of the geometric progression exceeds the third term of the arithmetic progression by 8. Find these progressions.

1377*. Three numbers form a geometric progression. If 8 is added to the second number, then these numbers will constitute an arithmetic progression. If then 64 is added to the third number, the obtained numbers will form a geometric progression once again. Find these three numbers.

1378*. In an arithmetic progression the second term is a mean proportional between the first and the fourth terms. Show that the fourth, sixth and ninth terms of this progression form a geometric progression. Find its ratio.

1379. Given an arithmetic and a geometric progression, and in each of them three positive terms are taken, the first and third terms being equal. Determine in which progression the sum of these terms is greater.

1380. Find the arithmetic and the geometric progression, if the first term of each progression equals 2, the third terms of both progressions are equal to each other, and the eleventh term of the arithmetic progression is equal to the fifth term of the geometric progression.

1381. The first terms of an arithmetic and a geometric progres-

sion are equal to 3. The second term of the arithmetic progression exceeds that of the geometric progression by 6; their third terms are equal to each other. Find these progressions.

1382. The sum of three numbers forming an arithmetic progression is equal to 33. If unity is added to the first term, 2 to the third, and unity is subtracted from the second term, then the obtained numbers will constitute a geometric progression. Find these numbers.

1383. The sum of three numbers forming an arithmetic progression equals 21. If 2, 3, and 9 are respectively added to them, then the obtained numbers will form a geometric progression. Find these numbers.

1384. The sum of three numbers forming a geometric progression is equal to 65. If unity is subtracted from the least of these numbers and 19 from the greatest of them, then the obtained numbers will constitute an arithmetic progression. Find these numbers.

1385. Find the four numbers of which the first three form an arithmetic, and the last three a geometric progression. It is also known that the sum of the two extreme numbers is equal to 22, and the sum of the two middle numbers to 20.

1386. Four numbers form an arithmetic progression. If 2, 5, 7, 7 are respectively subtracted from them, then the newly obtained numbers will form a geometric progression. Find these numbers.

1387. Four numbers form a geometric progression. If 3, 9, 11 and 1 are respectively added to them, then the numbers thus obtained will constitute an arithmetic progression. Find these numbers.

1388*. A full tank contains 729 litres of pure acid. First a litres of acid was poured out and the tank was filled to the brim with water. After the content was thoroughly mixed up, a litres of the solution was poured out, and the tank was filled with water once again, and a thorough stirring was carried out. After this operation was repeated 6 times the solution in the tank contained 64 litres of pure acid. Determine a.

1389. A solution contains 20% of admixtures (from the total volume of the solution). What is the minimum number of filters through which it is to be passed so that the final content of the impurities does not exceed 0.01%, if each filter absorbs 80% of the impurities.

1390. (1) Solve the equation

$$1+a+a^2+a^3+\ \ldots\ +a^x = (1+a)\,(1+a^2)\,(1+a^4).$$

(2)* Find the sum

$$\left(a+\frac{1}{a}\right)^2+\left(a^2+\frac{1}{a^2}\right)^2+\ \ldots\ +\left(a^n+\frac{1}{a^n}\right)^2.$$

CHAPTER XV

The Exponential Function and Logarithms

54. THE EXPONENTIAL FUNCTION

1391. How is the function u^v called if:

(1) u is an argument and v is a constant;

(2) u is a constant (positive), and v is an argument?

1392. What values does the power $a^{\frac{m}{n}}$ attain for $a < 0$ if:

(a) m is even, n is odd;

(b) m is odd, n is odd;

(c) m is odd, n is even?

1393. Find the domain of definition and the domain of variation of the given functions. Determine also which of the functions are even, and which are odd.

(1) $y = 3^x$; (2) $y = 4 \times 3^{x^2}$; (3) $y = 2^{\frac{1}{x}}$;

(4) $y = 5 \times 2^{\sin x}$; (5)* $y = \left(\frac{1}{2}\right)^{\sqrt{x^2-4}}$;

(6)* $y = 2^{\arcsin x}$; (7) $y = \sqrt[3]{3^x - 27}$;

(8)* $y = \sqrt{0.5^x - 4}$; (9) $y = 2^{\sqrt{x}}$.

1394. Find the greatest and the least values of the following functions:

(1) $y = 2^{\sqrt{1-x}}$; (2) $y = 2^{\sqrt{1-x^2}}$; (3) $y = \left(\frac{1}{3}\right)^{\sqrt{4-x^2}}$;

(4) $y = \frac{1}{2^{|x|}}$; (5) $y = 4^{\sin x}$; (6) $y = 4^{|\sin x|}$;

(7) $y = 3^{1-\cos x}$; (8) $y = 2+2^{1+\cos x}$;

(9)* $y = 3^{2x}-2\times 3^{x+2}+85$.

1395. What values does the function $y = 5^x$ take on if x attains successively the following values

$$0, 1, 2, 3, -1, -2, -3, \frac{1}{2}, \frac{2}{3}, \frac{1}{4}, \frac{3}{4}, \sqrt{2}?$$

1396. Which of the below exponential functions are increasing, and which are decreasing?

(1) $y = 3^x$; (2) $y = (0.3)^x$; (3) $y = 10^x$; (4) $y = \left(\frac{1}{2}\right)^x$.

1397. 1. Which of the exponential functions, $y = 2^x$ or $y = (\sqrt{2})^x$, increases more rapidly with an increase in the argument x if $x > 0$?

2. Which of the exponential functions, $y = \left(\frac{1}{2}\right)^x$ or $y = \left(\frac{1}{3}\right)^x$, decreases more rapidly with an increase in x if $x < 0$?

1398. How are the graphs of the exponential functions

(a) $y = 2^x$ and $y = 10^x$; (b) $y = \left(\frac{1}{2}\right)^x$ and $y = \left(\frac{1}{3}\right)^x$

arranged with respect to each other? Consider the cases $x > 0$, $x = 0$, $x < 0$.

1399. How is it possible to construct the graphs of the given functions with the aid of the graph of the function $y = a^x$?

(1) $y = -a^x$; (2) $y = 2a^x$; (3) $y = \left(\frac{1}{a}\right)^x$;

(4) $y = |a^x|$; (5) $y = a^{x-2}$; (6) $y = a^x+1$;

(7) $y = -2a^x$; (8) $y = a^{2x}$; (9) $y = a^{\frac{1}{2}x}$?

1400. 1. Is it possible to single out from all the values of the function $y = 2^x$ (a) its greatest value, (b) its least value?

2. Solve the same problem for the functions:

$$y = 2^{|x|} \text{ and } y = 3^x+3^{-x}.$$

1401. Analyse the following functions (the domains of definition and variation, the property of being even or odd, the least and the greatest values, the points of discontinuity, the intervals of increasing and decreasing, their roots, and the intervals of constant sign) and construct their graphs:

(1) $y = 2^x - 1$; (2) $y = \frac{1}{2} \times 3^{x-1}$;

(3) $y = \frac{1}{3} \times 2^{-x}$; (4) $y = 2^{\sqrt{x^2}}$;

(5)* $y = 2^x + 2^{-x}$; (6)* $y = \frac{2^x - 2^{-x}}{2}$.

1402. 1. What graph would the exponential function $y = a^x$ have if $a = 1$? How is the graph of the exponential function $y = a^x$ (for $a \neq 1$) situated with respect to that of the function $y = 1^x$ for $a > 1$, for $a < 1$?

2. How are the graphs of the functions $y = a^x$ and $y = a$ situated for $a > 1$ and for $a < 1$?

1403. Solve the following inequalities:

(1) $2^x > \frac{1}{4}$; (2) $2^{2x} < \frac{1}{8}$; (3) $\left(\frac{1}{3}\right)^x < 9$;

(4) $5^{x^2+3x+1.5} < 5\sqrt{5}$; (5) $\left(\frac{1}{3}\right)^{4x} > \sqrt{3}$;

(6) $(0.02)^{7(x^2-1)+6x-7x^2} < 1$; (7) $(0.04)^{3x-2} > 5^{2-x}$.

1404. Solve graphically the following equations:

(1) $2^x = x^2$; (2) $2^x = -4x$; (3) $\left(\frac{1}{2}\right)^x = 2x + 1$.

1405. Compare the following powers with unity:

$$\left(\frac{2}{5}\right)^{\frac{2}{3}}, \left(\frac{5}{3}\right)^{\frac{3}{4}}, \left(\frac{1}{2}\right)^{-\frac{6}{7}}, \left(\frac{3}{2}\right)^{-\frac{4}{5}}, (0.21)^{0.1}, (\sqrt{2})^{-1.2}.$$

1406. Compare the numbers m and n if:

(1) $\left(\frac{3}{5}\right)^m < \left(\frac{3}{5}\right)^n$; (2) $(1.5)^m > (1.5)^n$;

(3) $(0.3)^m > (0.3)^n$.

1407. Compare the given powers pairwise:

(1) $\left(\frac{3}{5}\right)^2$ and $\left(\frac{3}{5}\right)^{-1}$; (2) $(2.5)^{-\frac{4}{5}}$ and $(2.5)^{-\frac{5}{6}}$;

(3) $\left(\frac{1}{2}\right)^{-5.3}$ and $\left(\frac{1}{2}\right)^{-6.3}$; (4) $(\sqrt{3})^{-9}$ and $(\sqrt{3})^{1.2}$;

(5) $\left(\frac{3}{4}\right)^{\frac{7}{6}}$ and $\left(\frac{3}{4}\right)^{\frac{8}{7}}$; (6) $(1.5)^{-2}$ and $(1.5)^0$.

1408. What conclusion can be drawn with respect to the exponent x if:

(1) $10^x = 7$; (2) $\left(\frac{5}{7}\right)^x = \frac{3}{4}$; (3) $\left(\frac{2}{3}\right)^x = 2.5$; (4) $\left(\frac{7}{3}\right)^x = 0.5$.

1409. Conclude on the positive base a if:

(1) $a^{\frac{2}{5}} > a^{\frac{5}{2}}$; (2) $a^{\frac{2}{3}} > a$; (3) $a^{-\frac{5}{6}} < a^{\frac{7}{6}}$;

(4) $a^{0.3} > 1$; (5) $a^{\frac{3}{4}} < a^{\frac{4}{5}}$; (6) $a^{-0.7} > a^{-\frac{4}{5}}$.

1410. A light beam, passing through a plastic plate, loses one third of its intensity. What is the intensity of the beam after it has passed n such plates?

1411. What sequence of numbers do the values of the exponential function $\left(\frac{1}{3}\right)^x$ corresponding to the values of the argument $x = 1, 2, 3, \ldots$, form?

1412. Given the geometric progression 1, 4, 16, 64, ... The values of what exponential function do the terms of this progression represent and for what values of the argument?

55. THE CONCEPT OF LOGARITHM OF A NUMBER

1413. Rewrite the following equalities in the logarithmic form:

(1) $2^4 = 16$; (2) $10^3 = 1000$; (3) $3^0 = 1$;

(4) $125^{\frac{1}{3}} = 5$; (5) $3^{-3} = \frac{1}{27}$; (6) $(2\sqrt{2})^{-\frac{2}{3}} = \frac{1}{2}$.

1414. Rewrite the following equalities in the exponential form:

(1) $\log_2 32 = 5$; (2) $\log_{10} 10{,}000 = 4$; (3) $\log_3 \frac{1}{81} = -4$.

1415. Check the validity of the following equalities:

(1) $\log_3 729 = 6$; (2) $\log_{\frac{1}{2}} \sqrt[3]{4} = -\frac{2}{3}$;

(3) $\log_{10} \frac{1}{10\sqrt{10}} = -\frac{3}{2}$;

(4) $\log_a a^2 \sqrt[3]{a} = 2\frac{1}{3}$; (5) $\log_a \sqrt{a\sqrt{a\sqrt{a}}} = \frac{7}{8}$;

(6) $\log_{3\sqrt{3}} \frac{1}{3} = -\frac{2}{3}$.

1416. Find the logarithms of the following numbers to the base 3:

$$27,\ 1,\ \frac{1}{9},\ \frac{1}{\sqrt[3]{3}},\ \sqrt[3]{9},\ \frac{9}{\sqrt[4]{3}},\ 3^{\sqrt{3}},\ 2187,\ \frac{1}{3\sqrt{3}}.$$

1417. Find the logarithms of the given numbers:

(1) 2, 4, $\frac{1}{4}$, 32, $\frac{1}{64}$ to the base $\frac{1}{2}$;

(2) $\frac{2}{5}$, 6.25, 1, $\frac{4}{25}$, $15\frac{5}{8}$ to the base 2.5.

1418. Proceeding from the definition of logarithm, find x if (a) $\log_x 3 = 3$; (b) $\log_4 x = -\frac{1}{2}$; (c) $\log_{25} 125 = x$.

1419. Find:

(1) $\log_6 1296$; (2) $\log_2 \frac{1}{16}$; (3) $\log_{\frac{1}{7}} \sqrt[3]{49}$;

(4) $\log_{49} \sqrt[3]{\frac{1}{7}}$; (5) $\log_{\frac{1}{16}} \sqrt[5]{64}$; (6) $-\log_8 \log_4 \log_2 16$.

Solve the following equations:

1420. (1) $\log_x 2401 = 4$; (2) $\log_x \frac{1}{81} = 4$;

(3) $\log_x \sqrt{8} = \frac{3}{4}$; (4) $\log_x \frac{1}{8} = \frac{3}{2}$;

(5) $\log_x \frac{1}{125} = -2$; (6) $\log_x \frac{1}{2} = -\frac{2}{3}$;

(7) $\log_7 x = 0$; (8) $\log_{\frac{1}{2}} x = -1$.

1421. (1) $\log_{0.1} x = -1$; (2) $\log_{\sqrt[3]{3}} x = -\frac{3}{2}$;

(3) $\log_{\frac{3}{2\sqrt{5}}} x = -6$; (4) $\log_{\frac{1}{25}} 5 = x$;

(5) $\log_{3\sqrt{3}} 27 = x$; (6) $\log_{5\sqrt[3]{5}} \frac{1}{625} = x$;

(7) $\log_{2\sqrt{2}} \frac{1}{16} = x$;

(8) $\log_{0.32} \left(\frac{2}{5}\sqrt{2}\right) = x$.

1422. 1. Find the number whose logarithm to the base a equals a.

2. Find x if (a) $\log_x \frac{1}{2} = \frac{1}{2}$; (b) $\log_x N = N$.

1423. On the base of the identity $a^{\log_a b} = b$ find:

(1) $2^{\log_2 8}$; (2) $3^{-\log_3 3}$; (3) $3^{\log_3 7.5}$;

(4) $25^{\log_5 3}$; (5) $\left(\frac{1}{2}\right)^{\log_{1/2} 1}$; (6) $(2^{\log_2 5})^2$;

(7) $3^{3\log_3 2}$; (8) $25^{-\log_5 10}$; (9) $49^{0.5\log_7 0.25}$;

(10) $2^{2-\log_2 5}$; (11) $5^{\log_5 10-2}$; (12) $2^{2\log_2 5+\log_2 3}$;

(13) $(3.6)^{\log_{3.6} 10+1}$; (14) $8^{\log_2 \sqrt[3]{121}+\frac{1}{3}}$;

(15) $27^{\log_3 \sqrt[3]{12}-\frac{2}{3}}$.

1424. Compute:

(1) $2\log_5 \sqrt{5}+3\log_2 8$; (2) $\log_2 \log_2 \log_2 2^{16}$;

(3) $\log_2 \log_3 81$; (4) $5\times 3^{\log_3 0.2}$; (5) $25^{-\log_5 2}$;

(6) $\log_3 5^{-2\log_5 3}$; (7) $\log_a \tan 40^\circ \log_a \tan 41^\circ \ldots \log_a \tan 50^\circ$.

1425. Solve the following equations:

(1) $3\log_3 x-2\log_3 x-2 = 0$;

(2) $5\log_5 x-3\log_3 9 = 2\log_5 x$;

(3) $(\log_2 x^2)-3\log_2 x+2 = 0$;

(4) $(\log_4 x)^3-(\log_4 x)^2-2\log_4 x = 0$;

(5) $(\log_{10} x)^4-3(\log_{10} x)^2+2 = 0$.

1426. 1. Prove that $\frac{\log_a N}{\log_a M} = \frac{\log_b N}{\log_b M}$.

2. Prove that the ratio of logarithms of one and the same number to different bases is independent of the number.

3. Prove the equalities:

(a) $\log_a b \log_b a = 1$; (b) $\log_{a^k} N^k = \log_a N$;

(c) $\log_a \frac{1}{N} = \log_{1/a} N$; (d) $\log_{a^k} N = \frac{1}{k} \log_a N$;

(e) $\log_{1/a} N = -\log_a N$; (f) $N = \log_a a^N$.

1427. 1. Prove that $b^{\frac{\log_a (\log_a b)}{\log_a b}} = \log_a b$.

2. Compute:

(a) $\log_2 5 \log_{25} 8$; (b) $3^{\frac{2}{\log_5 3}}$; (c) $3^{\log_{3\sqrt{3}} 10}$;

(d) $\log_3 4 \log_4 5 \log_5 6 \log_6 7 \log_7 8 \log_8 9$.

1428*. Logarithms of four numbers to the base 2 constitute an arithmetic progression. The product of its extreme terms is equal to -8, the product of the middle terms to zero. Find the numbers.

56. LOGARITHMIC FUNCTIONS

1429. Find the inverses of the given functions:

(a) $y = 2^x$; (b) $y = 2^{3x}$; (c) $y = 2^{-\frac{1}{2}x}$; (d) $y = 3^{x+1}$;

(e)* $y = 3^{|x|}$; (f) $y = \left(\frac{1}{3}\right)^{x^2}$; (g) $y = 5^{\sin x}$;

(h) $y = 5^{x^2+1}$.

1430. 1. What root has any logarithmic function $y = \log_a x$ what value does it assume for $x = a$?

2. Find the roots of the functions:

(a) $y = \log_2 x$; (b) $y = \log_2 (x+1)$;

(c) $y = \log_2 (x^2-3)$; (d) $y = \log_a \sin x$;

(e) $y = \log_3 \cos 2x$; (f) $y = \log_{10} \arcsin x$.

Find the domains of definition of the given functions:

1431. (1) $y = \log_a x$; (2) $y = \log_a(-x)$; (3) $y = \log_a(4-x)$;
(4) $y = \log_2(x^2-4)$; (5) $y = \log_{1/2}(3-x^2)$;
(6) $y = \log_5 x^2$; (7) $y = \log_a |x|$;
(8) $y = \log_a \sqrt{x}$; (9) $y = \log_a(x^2+2x)$.

1432. (1) $y = \log_a(3x^2-4x-7)$;
(2) $y = \log_a(x^2+x+1)$;
(3) $y = \log_a(4x^2-4x+1)$;
(4) $y = \log_a(x^4-5x^2+4)$;
(5) $y = \log_a \sqrt[3]{x^2-x-2}$;
(6) $y = \log_a \frac{1}{x^2+1}$; (7) $y = \log_a \frac{1}{x^2-1}$;
(8) $y = \log_a \log_a x$.

1433. (1) $y = \log_a \sin x$; (2) $y = \log_a \tan x$;
(3) $y = \log_a \cos 2x$; (4) $y = \log_a |\sin x|$;
(5) $y = \log_a \operatorname{arcsin} x$; (6) $y = \log_a \operatorname{arccot} x$.

1434. Which of the below listed functions are increasing and which are decreasing?

(1) $y = \log_2 x$; (2) $y = \log_5 x$; (3) $y = \log_{0.3} x$;
(4) $y = \log_3(x-1)$; (5) $y = \log_3(5-x)$;
(6) $y = \log_{1/2} |x|$; (7) $y = \frac{1}{\log_{10} x}$;
(8) $y = \frac{2}{\log_{1/5} x}$; (9) $y = \log_a \cos x$.

1435. 1. Given the graph of the function $y = a^x$, construct the graph of the function $y = \log_a x$.

2. Plot the graph of the function $y = \log_2 x$, make stencil from it. Using the stencil construct the graphs of the following functions:

(a) $y = \log_{1/2} x$; (b) $y = \log_2 x - 1$; (c) $y = \log_2(x-1)$;
(d) $y = -\log_2 x$; (e) $y = |\log_2 x|$; (f) $y = \log_2 |x|$.

For each of the above functions find the domain of definition, its value for $x = 1$, the intervals of increase and the intervals of

decrease, the intervals within which the function is positive and on which it is negative, the greatest and the least values.

1436. Plot on one drawing the graphs of the following functions:

(1) $y = \log_2 x$ and $y = 2\log_2 x$;

(2) $y = \log_2 x$ and $y = \frac{1}{2}\log_2 x$;

(3) $y = \log_2 x$ and $y = \log_2 2x$;

(4) $y = \log_2 x$ and $y = \log_2 \left(\frac{1}{2}x\right)$.

1437. 1. How are the graphs of the given functions situated with respect to each other: (a) $y = \log_2 x$ and $y = \log_3 x$; (b) $y = \log_{1/2} x$ and $y = \log_{0.1} x$?

2. Does the graph of the function $y = x$ differ from that of the function $y = a^{\log_a x}$?

1438. Solve graphically the following equations:

(1) $2x^2 = \log_2 x$; (2) $\log_3 x = -x+4$; (3) $2^x = \log_{1/2} x$.

1439. Determine between what integers are the following logarithms found:

(1) $\log_2 6$; (2) $\log_{10} 27$; (3) $\log_{10} 0.54$; (4) $\log_2 \frac{1}{5}$.

1440. Determine which is greater:

(1) $\log_2 5$ or $\log_2 8$; (2) $\log_{1/2} 6$ or $\log_{1/2} 3$;

(3) $\log_7 8$ or $\log_5 8$; (4) $\log_{1/2} 9$ or $\log_{1/3} 9$;

(5) $\log_2 3$ or $\log_3 2$; (6)* $\log_a 5$ or $\log_5 a$.

1441. 1. Compare the numbers m and n if:

(a) $\log_4 m < \log_4 n$; (b) $\log_{1/2} m < \log_{1/2} n$.

2*. Draw the conclusion about the number m if:

(a) $\log_2 m = -0.22$; (b) $\log_{1/3} m = \frac{6}{5}$.

3. Compare:

(a) $\log_2 \frac{6}{5}$ and $\log_2 \frac{7}{6}$; (b) $\log_{1/2} 10$ and $\log_{1/2} 20$.

1442. Which of the below listed numbers are positive and which are negative?

(1) $\log_{10} 4.5$, $\log_{10} 0.45$, $\log_2 25.3$, $\log_{1/3} 9.6$;

(2) $\log_{0.3} 0.6$, $\log_{0.3} 3$, $-\log_2 \frac{15}{7}$, $-\log_{15} 0.6$;

(3) $\log_3 8-1$, $1-\log_4 9$, $\log_{2/3} 8-\log_{1/3} 8$;

(4) $\log_{3/4} 10-\log_{3/4} 7$, $\log_2 \frac{4}{5}-\log_2 \frac{5}{6}$.

1443. Compare the following pairs of numbers:

(1) $3 \log_{10} 296$ and $\log_{10} 296$;

(2) $7 \log_a 1$ and $\log_a 1$;

(3) $\log_3 3.9$ and $\log_3 3.89$;

(4) $\log_{1/3} 5.8$ and $\log_{1/3} 5.82$;

(5) $2 \log_{10} 0.723$ and $\log_{10} 0.723$;

(6) $\log_9 7$ and $\log_7 9$.

1444. Compare the number x with unity if:

(1) $\log_2 x = -0.1$; (2) $\log_{1/2} x = 1.4$;

(3) $\log_{10} x = 0.2$.

1445. For what values of x are the following inequalities correct?

(1) $\log_x 8 < \log_x 10$; (2) $\log_x \frac{3}{4} < \log_x \frac{1}{2}$;

(3) $\log_3 x-1 > 0$; (4) $\log_{3/4} x-1 > 0$;

(5) $\log_a x \leqslant 3 \log_a x$; (6) $\log_x |x| \geqslant 3 \log_a |x|$.

57. TAKING LOGARITHMS OF ALGEBRAIC EXPRESSIONS. RAISING ALGEBRAIC EXPRESSIONS TO A POWER

1446. Give a numerical example to illustrate that (a) the logarithm of a sum is not equal to the sum of the logarithms of the addends, (b) the logarithm of a difference is not equal to the difference of the logarithms of the minuend and the subtrahend.

1447. Knowing that $\log_{10} 2 = 0.3010$, $\log_{10} 3 = 0.4771$, $\log_{10} 5 = 0.6990$, find:

$\log_{10} 6$, $\log_{10} 10$, $\log_{10} 12$, $\log_{10} 16$, $\log_{10} \frac{3}{5}$, $\log_{10} 4\frac{4}{5}$, $\log_{10} 0.8$.

1448. How will the logarithm of a given number change if without changing the base, (1) the number is cubed; (2) the number is squared; (3) a square root is extracted from the given number; (4) the number is multiplied by 10; (5) the number is divided by 5?

Take the logarithms of the following expressions:

1449. (1) $x = 2a$; (2) $x = 3ab$;

(3) $x = 10(a^2 - b^2)$; (4) $x = 5 \sin \alpha$;

(5) $x = 5abcd$; (6) $x = \frac{a}{b}$;

(7) $x = \frac{4ab}{3c}$; (8) $x = \frac{\sin \alpha}{3}$;

(9) $x = \frac{3(a+b)}{a(a-b)}$; (10) $x = \frac{p(p-a)(p-b)(p-c)}{2abc}$;

(11) $x = a^2$; (12) $x = 5a^2b$;

(13) $x = 17a^4b^3cd^2$; (14) $x = \frac{2a^2}{3b^4}$;

(15) $x = \frac{a^4 \tan^2 \alpha}{5b^3}$; (16) $x = \frac{10(a^2+b^2)}{3c^3}$.

1450. (1) $x = \sqrt{a}$; (2) $x = a\sqrt[3]{b}$; (3) $x = 3a\sqrt[3]{4b^2}$;

(4) $x = \sqrt[4]{\frac{a^3}{b}}$; (5) $x = \sqrt[5]{a^3(a+b)^2}$;

(6) $x = \frac{a^2b^3}{c\sqrt{d}}$; (7) $x = 4a^4\sqrt[5]{\frac{c^2}{d}}$;

(8) $x = \frac{3mn^2}{4\sqrt{5mn}}$; (9) $x = \frac{p^2(m-n^2)^3}{\sqrt{q}}$.

1451. (1) $x = \left(\sqrt[5]{\frac{a}{2b}}\right)^3$; (2) $x = \frac{a^2\sqrt{\sin \alpha}}{b^3\sqrt[4]{c}}$;

(3) $x = \sqrt[-5]{\frac{a^3b}{\sqrt{c}}}$; (4) $x = m^{-2}n^3$;

(5) $x = 3a^{-3}\sqrt{\cos 2\alpha}$; (6) $x = \frac{1}{3}a^{-\frac{1}{2}}\sqrt[3]{4\tan^2\alpha}$;

(7) $x = 2a^{\frac{1}{3}}b^{\frac{3}{4}}$; (8) $x = a^{-2}b^{\frac{4}{3}}$;

(9) $x = \left(\frac{m}{n}\right)^{\frac{m}{n}}$.

1452. (1) $x = \frac{a^{-2}\sqrt[3]{4\sin^2\alpha}}{5b^3}$;

(2) $x = \sqrt[4]{\frac{1}{a^3}}\sqrt[3]{\frac{1}{b^2}}$;

(3) $x = \frac{1}{5}\sqrt{a\sqrt{b}}$; (4) $x = \sqrt{2\sqrt{6\sqrt{15}}}$;

(5) $x = \frac{3}{5}\sqrt[3]{a\sqrt{b}}$; (6) $x = \sqrt[3]{3\sqrt[3]{21\sqrt[3]{6}}}$;

(7) $x = \sqrt[3]{\frac{a^2b}{\sqrt[5]{c^3}}}$; (8) $x = \sqrt[5]{\left(\frac{1}{a^2b^3\sqrt[4]{c}}\right)^3}$.

1453. (1) $x = \sqrt{\frac{a}{\sqrt[3]{ab}}\sqrt{\frac{a}{b}}}$; (2) $x = \sqrt[3]{\frac{5a^2}{\sqrt[5]{\left(\frac{a}{b^2}\right)^2}}}$;

(3) $x = \frac{a^{\frac{2}{5}}b^{-3}}{c^{-\frac{3}{4}}}$; (4) $x = \sqrt{\frac{\sqrt[3]{ab}}{b}\sqrt[-2]{\frac{b}{a^3}}}$;

(5) $x = \sqrt[n]{a\sqrt[n]{b\sqrt[n]{c}}}$; (6) $\sqrt[-n]{a\sqrt[-n]{b\sqrt[-n]{c}}}$.

1454. (1) $x = 2^{\sqrt[3]{4}}$; (2) $x = (0.5)^{\sqrt[5]{0.5}}$;

(3) $x = 0.6^{\sqrt{0.5}}$; (4) $x = \log(a^2)$;

(5) $x = \log^2 a$; (6) $x = \log(\sqrt{ab})^3$.

1455. (1) $x = (\sqrt{2})^{\sqrt{2}}$; (2) $x = \log(\sqrt{2}^{\sqrt{2}})$;

(3) $x = \log(\sqrt[3]{3}^{\sqrt[3]{3}})$; (4) $x = \log(10^{ab})$;

(5) $x = \frac{10\log a}{\log(a^3)}$; (6) $x = \log\sqrt{\log 5}$.

Find x given its logarithm:

1456. (1) $\log x = \log 2 + \log 5$; (2) $\log x = \log a + \log b$;
(3) $\log x = \log a - \log 3$; (4) $\log x = 4 \log a$;
(5) $\log x = 2 \log a + \log b$; (6) $\log x = 3 \log a - 2 \log b$.

1457. (1) $\log x = 2 \log a + 3 \log b - 4 \log c$;
(2) $\log x = 2 \log (a-b) - 3 \log (a+b)$;
(3) $\log x = \frac{1}{5} \log 11 - \log 2$;
(4) $\log x = \frac{2}{3} \log a + \frac{3}{4} \log b$.

1458. (1) $\log x = \log 9 - \log 5 + \log 2$;
(2) $\log x = 2 \log 5 + 3 \log 3$;
(3) $\log x = 9 \log 5 - 3 \log 4$;
(4) $\log x = 3 \log 5 - 2 \log 25 + \log 4$.

1459. (1) $\log_2 x = 2 \log_2 6 + \log_2 3 - 3$;
(2) $\log_5 x = \frac{1}{2} \log_5 10 - \log_5 2 + \frac{1}{2}$;
(3) $\log_{1/2} x = \log_{1/2} 4 + \log_{1/2} 3 - 2$;
(4) $2 \log_7 x = 2 - 3 \log_7 2$.

1460. (1) $\log x = \frac{1}{2} \log (a-b) - \frac{2}{5} \log (a+b) - \frac{2}{3} \log b$;
(2) $\log x = \frac{2}{5} (\log a + \log b)$;
(3) $\log x = \frac{4}{7} (\log a - \log b)$;
(4) $\log x = \frac{3}{4} (\log 2 + \log a - \log 4)$.

1461. (1) $\log x = \log (a+b) - \frac{2}{3} \left(2 \log a + \frac{3}{4} \log b\right)$;
(2) $\log x = \log b - \frac{1}{n} \log (b-c) - n \log (b+c)$;
(3) $\log x = -2 \log a + \frac{1}{3} \left[\log (a+b) + \right.$
$\left. + \frac{3}{4} \log (a-b) - \log b - \frac{1}{2} \log c\right]$;

(4) $\log x = -\frac{2}{3}\log b + \frac{3}{4}\left[\log 2 - 3\log c - \right.$

$\left. -\log(a+b) + \frac{3}{5}\log(a-b)\right]$.

1462. (1) $\log x = 5\log a + \frac{1}{2}[\log(a+b) - \log m - 3\log n]$;

(2) $\log x = -\log 5 + \frac{2}{5}\left[2\log a + \frac{1}{2}\log 3 - \right.$

$\left. -\frac{1}{3}(\log a - \log b) - \log a\right]$;

(3) $\log x = \frac{2}{3}\left[\log m + \frac{3}{4}\log(m+n) - \right.$

$\left. -2\log(m-n)\right] - \frac{1}{2}\log n$;

(4) $\log x = -\frac{1}{3}\log a + \frac{1}{4}\left[\log b - \frac{2}{3}\log a + \right.$

$\left. + \frac{2}{3}\log(a+b) - \frac{1}{2}\log(a-b)\right]$

1463. Compute:

(1) $\log_{10} 2\frac{1}{2} - \log_{10} 250$; (2) $\log_3 5 \log_{25} 27$;

(3) $\log_{1/2} 28$ if $\log_7 2 = a$;

(4)* $\log_6 16$ if $\log_{12} 27 = a$.

1464. Represent the number (1) 3 in the form of a power of the number 2; (2) 10 in the form of a power of the number 5; (3) a as a power of the number b.

1465. Convert to powers:

(1) $\log x - 2\log y = \log S$; (2) $3\log_2 z + \frac{2}{3}\log_2 t = 1$;

(3) $\frac{2}{3}\log x - \log\sqrt{z} = 0$; (4) $\frac{1}{3}\log_a x - \log_a y = 2$.

1466. Simplify:

(1) $\frac{\log_a (\log_a N)^3}{\log_a \log_a N}$; (2) $\frac{\log_2 \log_2 N^2}{1 + \log_2 \log_2 N}$;

(3) $\dfrac{\log_a N}{\log_{ab} N}$;

(4) $\log_a \tan 1^\circ + \log_a \tan 2^\circ + \ldots + \log_a \tan 88^\circ + \log_a \tan 89^\circ$.

1467. May we put the sign of equality between the given expressions:

(1) $\log_a^2 x$ and $\log_a x^2$; (2) $\log_a^2 m$ and $\log_a \log_a m$;

(3) $\log_a x^4$ and $2 \log_a x^2$; (4) $\log_a x^2$ and $2 \log_a x$;

(5) $\log_2 \tan 15^\circ + \log_2 \tan 75^\circ$ and $\log_\pi \cot 1.25\,\pi$?

1468. Prove that the difference between the logarithms of two successive natural numbers decreases with an increase in these numbers ($a > 1$).

1469. The logarithms of which numbers is it sufficient to know to be able to compute the logarithms (to the same base) of all the natural numbers of the first decade?

1470. 1. Given: $\log_{10} 2 = a$, $\log_{10} 3 = b$, $\log_{10} 5 = c$. Find: (a) $\log_{25} 4$, (b) $\log_9 \sqrt[3]{2}$, (c) $\log_8 \sqrt{125}$.

2. Compute without using the tables of logarithms $\log_4 5 \log_{25} 8$.

1471*. 1*. Prove that if $\log_a x$, $\log_b x$, $\log_c x$ form an arithmetic progression, then $c^2 = (ac)^{\log_a b}$.

2. Prove that if certain numbers form a geometric progression, then their logarithms form an arithmetic progression.

3*. Find three successive terms of a geometric progression if their sum is equal to 62 and the sum of their logarithms to the base 10 equals 3.

1472*. Simplify the expression $\log_4 \dfrac{x^2}{4} - 2 \log_4 4x^4$ and then evaluate it for $x = -2$.

58. COMMON LOGARITHMS

1473. Knowing that $\log_{10} 2 = 0.3010$ and $\log_{10} 3 = 0.4771$, find: (a) $\log_{10} 6$; $\log_{10} 24$; $\log_{10} 0.36$; $\log_{10} \frac{3}{4}$, $\log_{10} \frac{1}{6}$; (b) $\log_{10} \sqrt{6}$; $\log_{10} \sqrt{18}$; $\log_{10} \sqrt[3]{12}$; $\log_{10} \sqrt[3]{36}$.

1474. Knowing that $\log_{10} 2 = 0.3010$ determine how many digits the number 2^{200} has.

1475. Transform the following negative logarithms to the artificial form:

$-0.3748,\ -0.7499,\ -0.3501,\ -2.1784,\ -1.0037.$

1476. Represent the following logarithms in negative form.

$\bar{1}.3458;\ \bar{2}.7989;\ \bar{3}.0037;\ \bar{1}.0506;\ \bar{2}.9909;\ \bar{5}.3979.$

1477. Using the Tables find the logarithms of the following numbers:

(1) 7, 13, 24, 76, 136, 748, 459, 989, 7485, 3748;

(2) 2.734, 0.7485, 18.09, 37.51, 166.7, 0.03785;

(3) 0. 083756, 1347.567, 20.003757, 17.83475.

1478. Find the numbers from their logarithms:

(1) 2.3475, 0.1723, 3.7583, 4.7129, 0.0537, 4.0089;

(2) 5.0711, 3.6772, 0.0792, $\bar{1}$.6628, $\bar{3}$.7018 $\bar{2}$.0354;

(3) $-0.1381,\ -2.3003,\ -3.1565,\ -5.8119,\ -0.0374.$

1479. Using the tables of logarithms of trigonometric functions find the common logarithms of the following quantities:

(1) sin 13°17′, sin 5°29′, cos 83°49′, cos 87°51′;

(2) sin 52°34′, sin 63°28′, cos 3°13′, cos 12°56′;

(3) tan 3°14′, tan 12°34′, cot 76°11′, cot 85°31′;

(4) tan 45°45′, tan 71°59′, cot 8°29′, cot 21°38′.

1480. Using the Tables find the angle α if:

(1) $\log_{10} \sin \alpha = \bar{1}.3142$; (2) $\log_{10} \sin \alpha = \bar{2}.7483$;

(3) $\log_{10} \cos \alpha = \bar{1}.1543$; (4) $\log_{10} \cos \alpha = \bar{3}.8345$;

(5) $\log_{10} \tan \alpha = \bar{2}.1783$; (6) $\log_{10} \cot \alpha = 0.5431$.

Carry out the operations involving logarithms:

1481. (1) $\bar{2}.3748+1.7485$; (2) $\bar{3}.3453+4.8564$;

(3) $\bar{1}.1847+\bar{3}.4958$; (4) $\bar{2}.1131+\bar{1}.7598$;

(5) $\bar{1}.1734-\bar{3}.1560$; (6) $\bar{2}.7453-\bar{1}.1154$;

(7) $\bar{3}.1835-\bar{4}.9658$; (8) $1.3458-\bar{5}.7891$;

(9) $1-\bar{3}.5492$; (10) $\bar{2}-\bar{3}.5748$.

1482. (1) $1.3452+\bar{3}.4856-\bar{2}.1454+1.7186$;

(2) $0.0537-\bar{2}.1509+\bar{1}.0039-\bar{3}.1448$;

(3) $\bar{2}.1749+\bar{1}.1949-\bar{4}.1546-0.9354$.

1483. (1) $\bar{1}.1742\times 3$, $\bar{1}.9415\times 4$, $\bar{3}.1674\times 2$;

(2) $\bar{2}.5346\times(-2)$, $\bar{1}.1349\times(-3)$, $\bar{3}.1703\times(-1)$;

(3) $\bar{3}.5346\times(-2)$, $\bar{1}.1548\times(-0.4)$, $\bar{1}.4536\times(-0.1)$.

1484. (1) $\bar{2}.1548:2$, $\bar{4}.5748:2$;

(2) $\bar{1}.1794:2$, $\bar{2}.8453:5$;

(3) $\bar{3}.1793:(-4)$, $\bar{2}.9436:(-2)$, $\bar{3}.1719:(-2)$;

(4) $\bar{3}.9378:(-0.1)$, $\bar{4}.1546:(-0.2)$, $\bar{3}.1658:(-0.4)$.

1485. (1) $\bar{2}.3748:5+1.7457-\frac{1}{3}\times\bar{5}.1548$;

(2) $\frac{1}{3}\times\bar{4}.3953+2\times(-2.1748)-0.3\times\bar{3}.7642$;

(3) $\frac{1}{5}\times\overline{10}.1748+\frac{1}{3}\times\bar{2}.7454+5\times 0.0348$;

(4) $-0.3\times\bar{2}.1854+\frac{1}{5}\times\bar{8}.7642-0.1\times\bar{8}.0348$.

1486. What will happen to a number if the characteristic of its logarithm is (a) increased by 2, (b) decreased by 3?

2. How will the difference $\log_{10} m-\log_{10} n$ change if m and n are replaced respectively by (a) m^2 and n^2, (b) $10m$ and $10n$, (c) $100m$ and $10n$, (d) $0.01m$ and $0.001\ n$, (e) $\sqrt{m}$ and $\sqrt{n}$?

(3) By how much is $\log_{10} 100m$ greater than $\log_{10}\frac{m}{100}$?

1487. Find the mistake in the following transformations. Given the identity $\log_{10}\sin\alpha=\log_{10}\sin\alpha$, $\left(0<\alpha<\frac{\pi}{2}\right)$. Let us double the left member leaving the right one unchanged. We get

$$2\log_{10}\sin\alpha>\log_{10}\sin\alpha,\ \text{or}\ \log_{10}\sin^2\alpha>\log_{10}\sin\alpha;$$

whence

$$\sin^2\alpha>\sin\alpha\ \text{or}\ \sin\alpha>1,$$

which is obviously incorrect.

Compute the following with the aid of tables of logarithms:

1488. (1) 2.736×8.749; (2) 31.15×0.87348;

(3) 0.833×0.0456; (4) 4.538×8.742;

(5) 12.754×7.347; (6) $5.454 : 0.3845$;

(7) $3721\times 0.0542\times 1.0034$;

(8) $3.734\times 0.3755 : 2.0356$; (9) 2.7037^3;

(10) $(-0.236)^5$; (11) $\sqrt[4]{76.37}$;

(12) $\sqrt[3]{-0.06431}$.

1489. (1) $x = \dfrac{8.705}{0.6984\times 24.51}$; (2) $x = \dfrac{34.83\times 0.03798}{2.127\times 0.3154}$;

(3) $x = \dfrac{6200\times 0.242^3}{0.8247}$; (4) $x = \dfrac{(-27.32)^3\, 0.9357^2}{6.174^2}$;

1490. (1) $x = \dfrac{196.5\times 0.04537}{6.75\times 23.32}$; (2) $x = \dfrac{0.0475\times (6.381)^4}{0.8356}$;

(3) $x = \dfrac{(0.3571)^4\times (1.383)^2}{(0.5463)^2}$; (4) $x = \dfrac{(1.894)^2\times (23.4)^3}{(44.16)^2\times 0.8357}$.

1491. (1) $x = 2742\sqrt[3]{0.06347}$; (2) $x = 13.56^2 : \sqrt[4]{742.3}$;

(3) $x = \dfrac{72.15^3\sqrt[3]{0.0729}}{2.173^3\sqrt[4]{534.5}}$; (4) $x = \dfrac{\sqrt{0.536}\sqrt[4]{0.0962}}{3.171^3\sqrt[3]{16.74^2}}$.

1492. (1) $x = \sqrt{\dfrac{0.84^2\sqrt{0.08365}}{3.2^2\sqrt[4]{0.0832^3}}}$;

(2) $x = \dfrac{54.532\sqrt[3]{-364\sqrt{0.836}}}{\sqrt[4]{(53.72)^3\sqrt[3]{0.00576}}}$;

(3) $x = \dfrac{6.814\sqrt[3]{0.06371^2\sqrt{0.2165^3}}}{14.07\sqrt{0.08448}\sqrt[3]{0.5605^2}}$;

(4) $x = \dfrac{0.143^2\sqrt[3]{-99.8\times 0.7504^2}}{\sqrt[4]{4058^2\sqrt{0.1}}}$.

1493. (1) $x = \dfrac{\sin^2 35°47'\sqrt{\cos 19°17'}}{\sqrt{0.5378}\tan^2 45°23'}$;

(2) $x = \dfrac{\cos 75°28' \sqrt[5]{0.5}}{\sqrt[3]{\tan^2 70°16'} \sin \dfrac{\pi}{9}}$;

(3) $x = \sqrt[3]{\dfrac{2 \cos^2 12°27' \sqrt{\tan 20°34'}}{\cot^2 37°14'}}$;

(4) $x = \left(\dfrac{3 \cos 75°46'}{\tan^2 52°43'}\right)^{\frac{1}{3}} \sqrt[4]{5 \sqrt{\cot 12°14'}}$.

1494. (1) $x = \pi \sqrt[3]{5.183} + \sqrt[4]{15.35}$;

(2) $x = \sqrt[4]{25.37^2 - \sqrt[5]{834.57}}$;

(3) $x = \dfrac{2.832^2}{\sqrt[3]{15.79}} + \dfrac{\sqrt[4]{5.824}}{1.735^2}$;

(4) $x = \sqrt[4]{\dfrac{8.732^2 - 0.2 \sqrt[3]{65.37}}{2.4539^2}}$.

1495. (1) $x = \dfrac{\sqrt[3]{17.86} - \sqrt[3]{15.37}}{5.625^2 - 2.0344^3}$;

(2) $x = \dfrac{\sqrt{4.378^2 - 2.5 \sqrt[4]{1.725^3}}}{\sqrt[4]{7} + \sqrt[3]{5}}$;

(3) $x = \sqrt[7]{\sqrt[5]{\cos 16°32'} + \sqrt[3]{\tan \dfrac{2\pi}{13}}}$;

(4) $x = \sqrt[3]{\log_{10} \sin 25°17'} + 0.75^2 \sqrt[5]{\tan^2 48°13'}$.

1496. (1) $x = \sqrt[8]{8 + \sqrt[8]{8}}$;

(2) $x = \sqrt[7]{1 - 8.378 \sqrt[3]{-3.724}}$;

(3) $x = \dfrac{(-3.85)^3 - \sqrt[3]{0.999}}{\sqrt[4]{8}}$;

(4) $x = \sqrt{\log_{10} 24.73} + \sqrt[3]{\log_{10} 0.762 + \log_{10} \sqrt{5.378}}$;

(5) $x = \left(5 + \sqrt[3]{\log_{10} 0.7}\right)^{2.734}$;

(6) $x = \left(1 + \sqrt[3]{\log_{10} 3}\right)^{\frac{2}{0.5}}$.

1497. Find x if:

(1) $\log_3 x = 2.7384$; (2) $\log_5 x = \bar{1}.7382$;

(3) $\log_{50} x = \bar{1}.0708$; (4) $\log_{0.2} x = \bar{1}.8033$;

(5) $x = \log_{0.3} 0.3783$; (6) $x = \log_{0.02} 0.002$;

(7) $x = \log_{0.5} 1000$; (8) $x = \log_{0.5} 0.001$.

Carry out the calculations with the aid of a slide rule.

1498. Find the logarithms of the given numbers and check on the results using the Tables:

(1) $\log_{10} 6$, $\log_{10} 1.5$, $\log_{10} 25.3$, $\log_{10} 374$, $\log_{10} 25.47$;

(2) $\log_{10} 27.45$, $\log_{10} 49.56$, $\log_{10} 0.507$, $\log_{10} 2.034$.

1499. Find x if:

(1) $\log_{10} x = 0.553$, $\log_{10} x = 1.754$, $\log_{10} x = 2.347$, $\log_{10} x = 2.034$;

(2) $\log_{10} x = 2.569$, $\log_{10} x = 0.074$, $\log_{10} x = 1.396$, $\log_{10} x = 0.954$;

(3) $\log_{10} x = \bar{1}.650$, $\log_{10} x = \bar{2}.056$, $\log_{10} x = \bar{3}.467$; $\log_{10} x = \bar{4}.742$.

1500. Compute:

(1) $0.356^{2.152}$; (2) $5.73^{1.28}$; (3) $3.75 \times 2.17^{0.7}$;

(4) $2.34^{3.8} \times 5.81^{0.78}$; (5) $\sqrt[1.3]{85.3}$;

(6) $\sqrt[1.75]{8.34}$; (7) $18.3\sqrt[1.12]{37.1}$;

(8) $\sqrt[4.1]{85.3}\ 2.17^{-3.15}$;

(9) $\dfrac{17.1^{2.3}}{3.74^{2.4}}\sqrt[1.2]{73.2}$.

1501. 1. Compute the length of the circumference whose radius $R = 27.48$ dm.

2. Compute the area of the circle whose radius $R = 13.75$ dm.

3. Compute the length of the circumference if the area of the circle $S = 2.835$ m^2.

1502. Compute the area of a triangle if (1) its base $a = 1.374$ m, and altitude $h = 2.003$ m;

(2) the sides equal 238.4 cm and 79.3 cm and the angle between them $\alpha = 37°23'$.

1503. 1. Compute the volume of a cylinder whose radius of the base $R = 24.16$ dm, and the altitude $h = 11.73$ dm.

2. Find the altitude of a cylinder whose volume $v = 751.2$ dm^3, and the radius of the base $R = 8.37$ dm.

3. Find the length of steel wire whose diameter $d = 2.50$ mm and mass $m = 8.3$ kg if the density of steel is equal to 7900 kg/m^3.

1504. The surface area and the volume of the sphere are computed by the formulas $S = 4\pi R^2$ and $V = \frac{4}{3}\pi R^3$, respectively. Determine the surface area and the volume of the Earth if it is assumed to be a sphere of radius $R = 6371$ km.

1505. The empirical formula for determining the number of equal circle contained in a circle, inscribed in it and touching one another is as follows: $n = 0.905 \times \left(\frac{D}{d} - 0.97\right)^2 + 3.7$. Using this formula, determine how many pieces of wire of diameter $d = 1.25$ mm can be placed inside a pipe of diameter $D = 11.2$ mm.

59. EXPONENTIAL AND LOGARITHMIC EQUATIONS

Solve the following exponential equations:

1506. (1) $3^x = 81$; (2) $2^{-x} = 32$; (3) $8^x = 16$;

(4) $9^x = 27$; (5) $\sqrt[3]{128} = 2^{3x}$; (6) $6^{\sqrt{1-x}} = 216$;

(7) $25^x = \frac{1}{5}$; (8) $3^{\sqrt{2x+1}} = 243$;

(9) $\sqrt[3]{7^x} = \sqrt[5]{343}$.

1507. (1) $\left(\frac{5}{12}\right)^{2x-3} = (2.4)^{3x-2}$;

(2) $\left(\frac{2}{3}\right)^x = \left(\frac{3}{2}\right)^3$; (3) $\left(\frac{5}{3}\right)^x = \frac{9}{25}$;

(4) $\sqrt[3]{a^x} = \sqrt{a^{3x+2}}$; (5) $(0.125)^x = 128$;

(6) $2^{x+2} = \sqrt{0.5}$; (7) $3^{\sqrt[x]{2}} = 81$; (8) $2^{3x} = \sqrt[3x]{2}$;

(9) $\sqrt{\frac{1}{32}} = \left(\frac{1}{512}\right)^{-x}$.

1508. (1) $\sqrt[x]{256} = 4^x$; (2) $2^{2\sqrt{2x}} = 16$;

(3) $\sqrt[3]{1.5^{5x}} = 3\frac{3}{8}$; (4) $3^{2x}5^x = 2025$;

(5) $\left(\frac{2}{3}\right)^x \left(\frac{9}{8}\right)^x = 2\frac{10}{27}$; (6) $3^{x-3} = 1$;

(7) $a^{(x+1)(x-2)} = 1$; (8) $(a^{x^2+x-2})^{3-x} = 1$;

(9) $7^{x^2-4x+6} = 343$; (10) $\left(\frac{4}{3}\right)^x \left(\frac{9}{8}\right)^{x-1} = \frac{32}{81}$.

1509. (1) $4^{\sqrt{2x-4}} = 64 \times 2^{\sqrt{2x-4}}$;

(2) $1000\sqrt[x]{0.1} = 100^x$;

(3) $2^{x^2-7.7x+17.5} = 16\sqrt{2}$; (4) $\sqrt{4^{x(x-1)-\frac{1}{2}}} = \sqrt[4]{2}$;

(5) $\left(\frac{4}{9}\right)^x \left(\frac{27}{8}\right)^{x-1} = \frac{2}{3}$;

(6) $\sqrt[x]{27^{2x-1}} = \sqrt{9^{2x-1}}$.

1510. (1) $8\sqrt{(0.25)^{5-\frac{x}{2}}} = 2^{\sqrt{x-1}}$;

(2) $\sqrt[3]{2^x}\sqrt[3]{3^x} = 1296$; (3) $2^x 5^x = 0.1\,(10^{x-2})^4$;

(4) $\left(1\frac{2}{3}\right)^x \left(\frac{9}{25}\right)^x = 1$;

(5) $\sqrt[x+1]{4096} = 2\sqrt[x+2]{32768}$; (6) $6^{2x+5} = 27^x 2^{x+10}$.

1511. (1) $\sqrt[x-1]{\sqrt[3]{3^{3x+1}}} - \sqrt[2x+5]{9^{x+5}} = 0$;

(2) $2(2^{\sqrt{x}+3})^{\frac{1}{2\sqrt{x}}} - \sqrt[\sqrt{x}-1]{16} = 0$;

(3) $2^{1-x^2}2^{2x} = 64^{-1}$;

(4) $5\sqrt[x+2]{3125^{x+1}} - \sqrt[x+3]{15625^{x+2}} = 0$.

1512. (1) $3^{x-1} = 5^{1-x}$; (2) $3 \times 2^x = 4 \times \sqrt[\frac{x}{2}]{3}$;

(3)* $\left(\frac{4}{3}\right)^{x-1} \times \sqrt{\frac{3}{4}} = \frac{2}{(\sqrt[4]{3})^{3x-4}}$;

(4) $5^{5\sqrt{x}} = (5^{\sqrt{x}-4})^x$.

1513. With the aid of the Tables of logarithms solve the following exponential equations:

(1) $10^x = 200$; (2) $2^x = 12$; (3) $3^x = 10$;

(4) $\left(\frac{2}{3}\right)^x = 8$; (5) $10^x = \sqrt[x]{3}$; (6) $\sqrt[x]{1.7308} = \sqrt[10]{10}$.

1514. Graph the function $y = 2^x$. Using this graph, solve the following equations:

(1) $2^x = 3$; (2) $2^x = 6$; (3) $2^x = 0.5$; (4) $2^x = 2.5$.

Solve the following equations:

1515. (1) $2^{\cos 2x} = \sqrt{2}$; (2)* $3^{|x^2-3x+2|} = 9^{x+1}$;

(3) $5^{-\frac{1}{2}\sin 2x} = 25(125)^{-\sin^2 x}$;

(4) $3\times 3^{\sin 4x+\sin^2 x} = (\sqrt{3})^{3+\cos 2x}$.

1516. (1) $5^{x+1}+5^x = 750$; (2) $2^x-2^{x-2} = 3$;

(3) $7^x-7^{x-1} = 6$; (4) $2^{x+2}-2^{x-1} = 28$;

(5) $3^{x+2}+3^x = 10$; (6) $5^{2x+1}-5^{2x-1} = 120$.

1517. (1) $5^{x+1}+5^x+5^{x-1} = 155$;

(2) $2\times 7^{x+2}+7^{x-1} = 687$;

(3) $3^{2x}-2\times 3^{2x-1}-2\times 3^{2x-2} = 1$;

(4) $5\times 3^{2x-1}-9^{x-\frac{1}{2}} = 9^x+4\times 3^{2x-2}-81$;

(5) $3^{2x-1}+3^{2x-2}-3^{2x-4} = 315$;

(6) $2^{x-1}+2^{x-2}+2^{x-3} = 896$.

1518. (1) $6^x+6^{x+1} = 2^x+2^{x+1}+2^{x+2}$;

(2) $3^x-5^{x+2} = 3^{x+4}-5^{x+3}$;

(3) $3^x+3^{x+1}+3^{x+2} = 5^x+5^{x+1}+5^{x+2}$;

(4) $7^{x-1}+7^x+7^{x+1} = 5^{x-1}+5^{x+1}+5^{x+2}$.

1519. (1) $3^{2x}-3^x = 72$; (2) $7^{2x}-6\times 7^x+5 = 0$;

(3) $4^x-2^{x+1} = 48$; (4) $5^{2x}-6\times 5^x-475 = 0$.

1520. (1) $3^{2x+5} = 3^{x+2}+2$; (2) $4^{2x-3}-3\times 4^{x-2}-1 = 0$;

(3) $3^{4\sqrt{x}}-4\times 3^{2\sqrt{x}}+3=0$;

(4) $25^{\sqrt{x-2}}-5\times 5^{\sqrt{x-2}}-500=0$.

1521. (1) $4+\frac{2}{3^x-1}=\frac{5}{3^{x-1}}$;

(2) $\left(\frac{3}{5}\right)^{x-1}+\left(\frac{3}{5}\right)^{1-x}=2$;

(3) $12\times 3^{\frac{1}{2x}}-3^{\frac{1}{x}}=27$;

(4)* $4^{x-\sqrt{x^2-5}}-12\times 2^{x-1-\sqrt{x^2-5}}+8=0$.

1522*. (1) $16^{\sin^2 x}+16^{\cos^2 x}=10$;

(2) $\left(\sqrt{3+\sqrt{8}}\right)^x+\left(\sqrt{3-\sqrt{8}}\right)^x=34$;

(3) $\left(\sqrt[4]{\sqrt{10}+3}\right)^x-\left(\sqrt[4]{\sqrt{10}-3}\right)^x=12\sqrt{10}$;

(4) $\left(\frac{2}{3}\right)^x+9(1.5)^x=10$.

1523*. (1) $4^x+6^x=9^x$; (2) $\sqrt[x]{9}-3\sqrt[x]{6}+2\sqrt[2x]{16}=0$;

(3) $2^{2x+2}-6^x-2\times 3^{2x+2}=0$;

(4) $2\times 81^x=36^x+3\times 16^x$.

Solve the following logarithmic equations:

1524. (1) $\log_x(2x^2-3x)=1$; (2) $\log_{x+2}(3x^2+4x-14)=2$;

(3) $\log_{x+2}(17x^2-6x+8)=3$;

(4) $\log_x(3x^2+10x)=3$.

1525. (1) $\log_{\sqrt{2x^2+1}}(5x^4-3x^2+7)=4$;

(2) $\log_{10}\log_{10}\log_{10}x=0$;

(3) $\log_{\sqrt{x-1}}(x^4-8x^2-2x+1)=4$;

(4) $\log_2\log_2\log_2 x=0$.

1526. (1) $\log_{\sqrt{2}}\log_2\log_4(x-15)=0$;

(2) $\log_{10}\left(\log_2\log_3\sqrt{x}+1\right)=0$;

(3) $\log_x\log_3\log_x 2x^2=0$;

(4) $\log_{10}[3+2\log_{10}(1+x)]=0$.

1527. (1) $\log_{10}(x+3) = -\log_{10} 2$;

(2) $3\log_{10}\sqrt[3]{x} = \log_{10}(3x-4)$;

(3) $\log_{10}(x-1) = \log_{10} 5$;

(4) $\log_2(x-1) = \log_2(x^2-x-16)$.

1528. (1) $\log_{10}(x+6)-2 = \frac{1}{2}\log_{10}(2x-3)-\log 25$;

(2) $\frac{\log_{10}(2x-5)}{\log_{10}(x^2-8)} = 0.5$;

(3) $\frac{\log_{10}(x-3)}{\log_{10}(x^2-21)} = \frac{1}{2}$.

1529. (1) $\frac{1}{2}\log_{10}\sqrt{x^2+x-5} = \log_{10} x + \log_{10}\frac{1}{x}$;

(2) $\log_{10}\sqrt{5x-4}-\log_{10}\sqrt{x-1} = 2+\log_{10} 0.08$;

(3) $\log_{10} 8+4\log_{10} 2 = \log_{10} 3-\log_{10} 12+$
$+\log_{10} 2^{3x^2-20x+2}$;

(4) $\frac{1}{2}\log_{10} 2+\log_{10}\sqrt{x^2+4x+5}-$
$-\frac{1}{2}\log_{10}(x^2-4x+5) = \frac{1}{2}$.

1530. (1) $\log_{10} 100+\frac{1}{3}\log_{10}\left(1125-5^{\sqrt{3x}}\right) = 3$;

(2) $\log_{10} 2+\log_{10}(4^{x-2}+9) = 1+\log_{10}(2^{x-2}+1)$;

(3) $\log_{10}\sqrt{1-x^2}-3\log_{10}\sqrt{1-x} = \log_{10}\sqrt{1+x}+2$;

(4) $2\log_{10}\sqrt{x^2-36}+\frac{1}{3}\log_{10}(x^3+3x^2+3x+1)-$
$-\log_{10}(x+6)-2\log_{10} 3 = \log_{10} 2$.

1531*. (1) $\log_{10}\frac{1}{2^x+x-1} = x(\log_{10} 5-1)$;

(2) $\log_{10}(6\times 5^x-25\times 20^x) = x+\log_{10} 25$;

(3) $\log_{10}(2^x+x-4) = x(1-\log_{10} 5)$;

(4) $\log_{10}(x^2-x-6)-x = \log_{10}(x+2)-4$.

1532. (1) $\frac{1}{12}\log_{10}^2 x = \frac{1}{3}-\frac{1}{4}\log_{10} x$;

(2) $0.25 \log_{10}^4 x+8 = 3 \log_{10}^2 x$;

(3) $\frac{1}{2} \log_{10}^2 x+2 \log_{10} x = -1.5$;

(4) $2 \log_{10}^5 x = 5 \log_{10} x-3 \log_{10}^3 x$.

1533. (1) $\log_{10}^4 (x-1)-1.25 \log_{10}^2 (x-1)^2+4 = 0$;

(2) $\log_{10}^4 (x^2+1)^2-\log_{10}^2 (x^2+1)^3-7 = 0$;

(3) $(4 \log_{10}^2 x-1)(\log_{10}^2 x^2+1) = 15$;

(4)* $\log_{10}^m x^r-\log_{10}^n x^m = 0$.

1534. (1) $x^x = x$; (2) $x^{\log_2 x+4} = 32$;

(3) $x^{\frac{\log_{10} x+7}{4}} = 10^{\log_{10} x+1}$;

(4) $(0.1)^{-[\log_{10}(x+2)+2 \log_{10} 20]} = 2(x+201)$;

(5) $x^{1+\log_{10} x} = 0.001^{-\frac{2}{3}}$;

(6) $x^{3 \log_{10}^3 x-\frac{2}{3} \log_{10} x} = 100 \sqrt[3]{10}$.

1535. (1) $\sqrt{x^{\log_{10} \sqrt{x}}} = 10$; (2) $x^{\sqrt{x}} = \sqrt{x^x}$;

(3) $x^{\log_a x} = a^2 x$; (4) $x^{2 \log_{10}^3 x-1.5 \log_{10} x} = \sqrt{10}$;

(5) $\sqrt[x]{0.125} = 3^{2-\log_3 18}$;

(6) $2x^{\log_{10} x}+3x^{-\log_{10} x} = 5$.

Using the formulas for converting the logarithm of a number to one base to a logarithm of the same number to the other base, solve the following equations:

1536. (1) $\log_2 x+\log_3 x = 1$; (2) $\log_2 x+\log_x 2 = 2$;

(3) $\log_9 x+\log_{x^2} 3 = 1$; (4) $\log_5 x+\log_x 5 = 2.5$;

(5) $\log_3 x+\log_5 x = \log_{10} 15$;

(6) $\log_{16} x+\log_4 x+\log_2 x = 7$;

(7) $\log_a x \log_b x = \log_a b$; (8) $\log_{a^2} x+\log_{x^2} a = 1$.

1537*. (1) $\log_2^2 (x-1)^2-\log_{0.5}(x-1) = 5$;

(2) $\log_7 2+\log_{49} x = \log_{\frac{1}{7}} \sqrt{3}$;

(3) $\log_a x \log_b c(1+\log_c a) = \log_b x \log_c x \log_a c$.

Solve the following systems of equations:

1538. (1) $\begin{cases} 2^{x+y} = 32, \\ 3^{3y-x} = 27; \end{cases}$ (2) $\begin{cases} 3^x - 2^{2y} = 77, \\ 3^{\frac{x}{2}} - 2^y = 7; \end{cases}$

(3) $\begin{cases} y^{x^2-7x+12} = 1, \\ x+y = 6; \end{cases}$ (4) $\begin{cases} 2^x 3^y = 24, \\ 2^y 3^x = 54; \end{cases}$

(5) $\begin{cases} 3^x 5^y = 75, \\ 3^y 5^x = 45; \end{cases}$ (6) $\begin{cases} 8^{2x+1} = 32 \times 2^{4y-1}, \\ 5 \times 5^{x-y} = \sqrt{25^{2y+1}}. \end{cases}$

1539. (1) $\begin{cases} \sqrt[x]{4^y} = 32\sqrt[y]{8^x}, \\ \sqrt[x]{3^y} = 3\sqrt[y]{9^{1-x}}; \end{cases}$ (2) $\begin{cases} \log_{10} x - \log_{10} y = 1. \\ x+y^2 = 200; \end{cases}$

(3) $\begin{cases} \log_{10}(x^2+y^2) - 1 = \log_{10} 13, \\ \log_{10}(x+y) - \log_{10}(x-y) = 3\log_{10} 2; \end{cases}$

(4) $\begin{cases} \log_{10} x + 2\log_{10} y = -3, \\ \log_{10} x^3 - \log_{10} y^2 = 7; \end{cases}$ (5) $\begin{cases} xy = 40, \\ x^{\log_{10} y} = 4; \end{cases}$

(6) $\begin{cases} x^y = 5x-4, \\ \log_x 16 = y. \end{cases}$

1540. (1) $\begin{cases} \log_x \log_2 \log_x y = 0, \\ \log_y 9 = 1; \end{cases}$ (2) $\begin{cases} \log_4 x - \log_2 y = 0, \\ x^2 - 5y^2 + 4 = 0; \end{cases}$

(3) $\begin{cases} \log_y x + \log_x y = 2, \\ x^2+y = 12; \end{cases}$ (4) $\begin{cases} 2^x 4^y = 32, \\ \log_{10}(x-y)^2 = 2\log_{10} 2; \end{cases}$

(5) $\begin{cases} 10^{3-\log_{10}(x-y)} = 250, \\ \sqrt{x-y} + \frac{1}{2}\sqrt{x+y} = \frac{26-y}{\sqrt{x-y}}; \end{cases}$ (6) $\begin{cases} 3^x 2^y = 576, \\ \log_{\sqrt{2}}(y-x) = 4. \end{cases}$

1541. Solve the following inequalities:

(1) $3^{2x-5} < 3^{5x-6}$; (2) $\left(\frac{1}{2}\right)^{x^2-3x+4} < \left(\frac{1}{2}\right)^{x+9}$;

(3) $\log_2 x > 3$; (4) $\log_2(x-4) < 1$;

(5) $\log_2(2x+4) > \log_2(5x+3)$;

(6) $\log_{1/3}(2x^2-3x+5) < \log_{1/3}(x^2+2x+1)$;

(7) $\log_2 x > 1$; (8)* $\log_{1/3}(x^2+2x) > 0$;

(9)* $\log_{1/5}(2x^2+5x+1) < 0$; (10)* $\log_x \frac{4x+5}{6-5x} < 1$.

1542. 1. For what values of n will the equation $x^2-2x+\log_5 n = 0$ have (a) real roots, (b) imaginary roots?

2. For what values of n will the quadratic trinomial $x^2+4x+ +4\log_7 n$ be a perfect square?

Solve the following inequalities:

1543. (1) $2^{\log_{10}(2x-3)} > \frac{1}{2}$;

(2) $\frac{1}{\log_{10} x} - \frac{1}{\log_{10} x-1} > 1$;

(3) $9^x < 8\times 3^x+9$; (4) $\log_{1/3}(2x-15) > 0$;

(5)* $\log_{1/2}(3x^2-5x-3) < \log_{1/2}(4x-3)$.

1544. (1) $\log_{1/2}(x^2-5x+7) < 0$;

(2)* $0.5^{\log_5 \log_{0.3}(x-0.7)} < 1$;

(3) $\log_3 \frac{x-3}{x+3} > 0$; (4) $\log_2(x^2+2x) < 3$.

1545. (1) $2\log_3(x-1)-\log_3(2x-5) > 1$;

(2)* $4^x \leqslant 3\times 2^{\sqrt{x}+x}+1^{1+\sqrt{x}}$;

(3)* $\log_{x^4}\left(\frac{4x-5}{|x-2|}\right) \geqslant \frac{1}{4}$;

(4)* $\log_5 \frac{|x^2+4x|+3}{x^2+|x-5|} \geqslant 0$.

1546*. (1) $\log_2 \frac{4x-11}{2x^2-4x-6} \leqslant -1$;

(2) $\log_2(2^x-2)\times \log_{1/2}(2^{x+1}-4) > -2$;

(3) $\log_2 \log_{1/2}(x^2-3x+2) \leqslant 1$;

(4) $\left(\tan\frac{\pi}{3}\right)^x - 6\left(\tan\frac{\pi}{3}\right)^{-x} < 1$.

CHAPTER XVI

Review Problems

1547. The cosine of one of the angles of an isosceles trapezoid is equal to $-\frac{5}{13}$. Find the values of the trigonometric functions of the angles of this trapezoid.

1548. In an isosceles trapezoid the smaller base equals the lateral side. The tangent of one of the angles of this trapezoid is equal to $-4\sqrt{5}$. Find the values of the trigonometric functions of the angle between the diagonal and the longer base.

1549. (1) Find $\sin\alpha$, $\cos\alpha$, $\tan\alpha$ if $\tan\frac{\alpha}{2}=\frac{1}{2}$;

(2) Find $\sin 2\alpha$ and $\cos 2\alpha$ if $\sin\alpha=-\frac{\sqrt{3}}{3}$, $270^\circ<\alpha<360^\circ$;

(3) Find $\sin 2\alpha$ and $\cos 2\alpha$ if $\cos\alpha=-\frac{4}{5}$, $180^\circ<\alpha<270^\circ$;

(4) Find $\sin 2\alpha$, $\cos 2\alpha$, $\tan 2\alpha$ if $|\tan\alpha|=3$.

1550. Compute without using the Tables:

(1) $\sin 157^\circ\cos 307^\circ-\sin 293^\circ\cos 143^\circ$;

(2) $\tan 130^\circ\tan 131^\circ\tan 132^\circ\ldots\tan 139^\circ\tan 140^\circ$.

1551. Simplify:

(1) $\dfrac{\cos 4\alpha-\cos 2\alpha}{\sin 3\alpha \sin \alpha}$; (2) $\dfrac{1+\cos \alpha+\cos 2\alpha+\cos 3\alpha}{\cos \alpha+2\cos^2 \alpha-1}$;

(3) $\dfrac{\tan (45^\circ+\alpha)+\tan (\alpha-45^\circ)}{\cot (45^\circ+\alpha)+\cot (45^\circ-\alpha)}$;

(4) $\dfrac{\sin^6 \alpha+\cos^6 \alpha-1}{\sin^4 \alpha+\cos^4 \alpha-1}$.

1552. Simplify:

(1) $\dfrac{4\sin^2 \alpha-\sin^2 2\alpha}{4-4\sin^2 \alpha-\sin^2 2\alpha}$; (2) $\dfrac{\tan^2 2\alpha \tan^2 \alpha-1}{\tan^2 \alpha-\tan^2 2\alpha}$;

(3) $\dfrac{\cot^2 \alpha \cot^2 \beta-1}{\tan^2 \alpha \tan^2 \beta-1}$; (4) $\dfrac{2a\cos 2\alpha+(a^2-1)\sin 2\alpha}{a\cos \alpha-\sin \alpha}$.

Prove the following identities:

1553. (1) $1-\cot^2 \alpha = -\dfrac{\cos 2\alpha}{\sin^2 \alpha}$;

(2) $\dfrac{1+\cot \alpha}{1-\cot \alpha} = \cot (\alpha-45^\circ)$;

(3) $\dfrac{\sin^2 \alpha-\sin^2 \beta}{\sin \alpha \cos \beta+\sin \beta \cos \alpha} = \sin (\alpha-\beta)$;

(4) $\dfrac{\sin^2 \alpha-\tan^2 \alpha}{\cos^2 \alpha-\cot^2 \alpha} = \tan^6 \alpha$;

(5) $\dfrac{\sin (2\alpha+\beta)}{\sin \alpha}-2\cos (\alpha+\beta) = \dfrac{\sin \beta}{\sin \alpha}$.

1554. (1) $(\sin \alpha-\sin \beta)^2+(\cos \alpha-\cos \beta)^2 = 4\sin^2 \dfrac{\alpha-\beta}{2}$;

(2) $1-\sin^2 \alpha-\sin^2 \beta = \cos (\alpha+\beta)\cos (\alpha-\beta)$;

(3) $\tan \alpha+2\sin^2 \alpha \cot 2\alpha = \sin 2\alpha$;

(4) $\dfrac{\cos \alpha-\sin \alpha}{1+\tan \alpha} = \cos \alpha \tan (45^\circ-\alpha)$;

(5) $\sin 2\alpha+\sin 8\alpha+\sin 10\alpha = 4\sin 5\alpha \cos 4\alpha \cos \alpha$.

1555. Transform the following to products:

(1) $\cos 2\alpha+\sin 2\alpha-\tan \alpha$; (2) $\sin (\alpha-\beta)-\sin \alpha-\sin \beta$;

(3) $\sin \alpha+\cos \alpha+\sin 2\alpha+\cos 2\alpha+\sin 3\alpha+\cos 3\alpha$.

1556. Compute:

(1) $\sin\left(\arcsin\frac{1}{3}+\arccos\frac{1}{3}\right)$;

(2) $\sin\left[\frac{1}{2}\arctan(-2.4)\right]$;

(3) $\cot\left[\frac{1}{2}\arcsin\left(-\frac{\sqrt{33}}{7}\right)\right]$;

(4) $\cot^2\frac{13\pi}{4}-\sin\left[\frac{7\pi}{2}-2\arctan(1-\sqrt{2})\right]$.

Solve the following equations:

1557. (1) $\cos 4x\tan 2x=0$; (2) $\csc x\tan x=0$;

(2) $\sin x+\cos x-\sin x\cos x=1$;

(3) $\cos x+\cos 2x+\cos 3x=0$;

(4) $\cos^2 x-\sin^2 x+\tan^2 x=1$;

(5) $\tan^2 x+\tan^2 3x=2\tan 3x\tan x$;

(6) $\sin 5x\cos 3x=\sin 6x\cos 2x+\sin 2x$;

(7) $\sin 6x\cos 5x=\cos 3x\sin 3x$.

1558. (1) $x=\arccos[\sin(-150^\circ)]$;

(2) $x=\arccos(\sin x)$;

(3) $\arcsin x=3\arcsin\frac{x}{2}$;

(4) $\arcsin x=2\arcsin\frac{x}{\sqrt{3}}$.

1559. The sides of a triangle form an arithmetic progression whose difference is equal to 4 cm. The area of the triangle is 24 cm^2. Determine the sides of the triangle.

1560. A plane cuts off the edges of a trihedral angle line segments whose lengths form an arithmetic progression with the difference of 2 cm. Find the lengths of the segments, if the volume of the pyramid thus obtained equals 8 cm^3.

1561. 1. Given the geometric progression 2, -6, 18, ..., find a_8.

2. Find a_1 of the geometric progression with $a_{10}=48\sqrt{2}$, $q=\sqrt{2}$.

1562. 1. Find q in the geometric progression $a_1 = 1, a_{10} = -\frac{1}{512}$.

2. Find n in the geometric progression $a_1 = 2, q = \frac{2}{3}, a_n = \frac{128}{729}$.

1563. Find the simple three-digit number whose digits form a geometric progression.

1564. Find the sum of the first seven terms of a geometric progression, in which the product of the third and fifth terms is equal to three times the seventh term, and the sum of the first and eighth terms to 387.

1565. Find the geometric progression in which

(1) $a_4 : a_6 = 4$ and $a_2 \times a_8 = \frac{1}{4}$; (2) $a_1 = 3$ and $S_n = \frac{1}{81} \times$ $\times (S_8 - S_4)$.

1566. 1. Find the geometric progression in which $a_1 = 2$ and $\frac{S_{10}}{S_5} =$ $= 33$.

2. Find the sum of the first five terms of a geometric progression if $a_n = 5^n$.

1567. Between the numbers 5 and 3645 insert geometric means so that the ratio of the last of them to the first one equals 81.

1568. Solve the following equations:

(1) $\sqrt{25^{x^2+x-0.5}} = \sqrt[4]{5}$; (2) $\left(\frac{3}{4}\right)^{x-1} \sqrt[\pi]{3\frac{13}{81}} = \frac{9}{16}$;

(3) $4^{\sqrt{x+1}} = 32 \times 2^{\sqrt{x+1}}$; (4) $25^{\cos 2x} - 2 \times 5^{\cos 2x} = 0$;

(5) $2^{3x} 5^{2x} = 64 \times 10^{12}$; (6) $5^{x-1} + 5^{x-2} + 5^{x-3} = \frac{31}{125}$.

1569. Compute:

(1) $7^{2 \log_7 3}$; (2) $2^{-\frac{1}{2} \log_2 9}$; (3) $10^{\log_{10} a + \log_{10} b}$;

(4) $10^{1 - \log_{10} 1\frac{2}{3}}$; (5) $\log_{\sqrt{11}} \sqrt{11}$;

(6) $\log_9 27 - \log_{27} 9$.

1570. Between which neighbouring integers are the logarithms of the following numbers contained?

(1) 5, 9, 25, 50, 100, 500 to the base 2;

(2) 0.02, 0.038, 0.007, 0.0042 to the base 10.

1571. Solve the following equations:

(1) $\log_5 8^{x-1} \log_2 125 = x+7$;

(2) $\sqrt{\log_{10}\left(91+3^{\sqrt{\frac{x}{2}}}\right)} = \log_{10} 10^{\sqrt{2}}$;

(3) $\sqrt{\log_9 (9x^8) \log_3 (9x)} - \log_3 x^3 = 0$;

(4) $\sqrt{1+\log_2 x} + \sqrt{4 \log_2 x - 3} = 5$.

1572. Compute:

(1) $x = \dfrac{\sin^2 47^\circ 15' \sqrt{0.753^3}}{\sqrt{\tan^2 54^\circ 49' \cos 37^\circ 25'}}$;

(2) $x = \dfrac{1.173^4 \cot^5 49^\circ 23'}{\sqrt{18.3^2 - \sqrt{\sin 35^\circ 53'}}}$;

(3) $x = \sqrt{\tan 39^\circ \sqrt{\tan 70^\circ \sqrt{\tan 50^\circ}}}$;

(4) $x = 0.384^{\log_{10} \sin^2 18^\circ 35'}$.

1573. 1. Plot the graph of the function

$$y = \begin{cases} -x^2+1 & \text{if} \quad x > 0 \\ |\sin x| & \text{if} \quad x \leqslant 0. \end{cases}$$

2. Replace the angle of 120° by a sum of two angles such that the product of their sines is the greatest possible.

3. Solve the equation $\sin x - 2 \sin 2x + \sin 3x = 0$.

4. Compute $\sin\left[\arccos \dfrac{1}{\sqrt{1+m^2}} - \arccos \dfrac{m}{\sqrt{1+m^2}}\right]$.

5. Reduce the fraction $\dfrac{a^8+a^6+a^4+a^2+1}{a^4+a^3+a^2+a+1}$.

6. Find the domains of definition of the given functions:

(a) $y = \log_{10} \sin x$; (b) $y = \log_{10} \cos x$;

(c) $y = \log_{10} \tan x$; (d) $y = \log_{10} \cot x$.

1574. 1. Graph the function $|y| = \sin x$.

2. Does an angle x exist such that

(a) $2^{\sin^2 x} = \sin x$; (b) $2^{\sin^2 x} = \cos x$; (c) $2^{\cos^2 x} = \sin x$?

3. Solve the equation $\sin x + \sin\left(x+\frac{\pi}{3}\right) + \sin\left(x+\frac{2\pi}{3}\right) = 0$.

4. Reduce the fraction $\frac{m^8+m^6n^2+m^4n^4+m^2n^6+n^8}{m^4+m^3n+m^2n^2+mn^3+n^4}$.

5. Compute $\tan\left(\arcsin\frac{n}{\sqrt{1+n^2}}+\arccos\frac{1}{\sqrt{1+n^2}}\right)$.

6. Calculate (without using the Tables) $\log_{10} 853.79 - \log_{10} 8.5379$.

1575. 1. Find the least and the greatest values of the function $y = m\cos x + n$.

2. Of all right triangles choose one such that the product of cosines of its acute angles is the greatest possible.

3*. Find the domain of definition of the function $y = \arcsin\frac{2x}{1+x}$.

4. Solve the equation $2(1-\sin 2x) = \tan\left(\frac{\pi}{4}+x\right)$.

5. Compute without using the tables of logarithms $10^{0.5-\log_{10} 0.375\sqrt{10}} - \log_{2\sqrt{2}} 0.0625$.

6. Solve the equation $27^{2\sqrt{x}} = 4\times 3^{\sqrt{9x}} - 3$.

7. Show that the terms of the progression with the general term $a_n = 2^n$ satisfy the relation $a_{n+2} - 3a_{n+1} + 2a_n = 0$.

1576. 1. Find the period, the least and the greatest values of the function $y = \sin^4 x + \cos^4 x$.

2. Simplify the expression $\sin^2(\alpha+45^\circ) - \cos(15^\circ+2\alpha)\sin 15^\circ - -\sin^2(\alpha-30^\circ)$ and then evaluate it for $\alpha = 15^\circ$.

3. Find the domain and the range of the function $y = \sqrt{\log_{10}\cos x}$.

4. Solve the inequalities: (a) $|\log_{10} x| < 1$; (b) $|\log_{10} x - 2| < 2$.

5. Solve the following equations:

(a) $\cos x - \cos 3x = \sin 3x$; (b) $\arccos x = 2\arccos x\sqrt{3}$.

6. Set up a sequence of all positive roots of the equation $\sin\frac{1}{x} = 1$. Is this sequence (a) finite, (b) bounded, (c) increasing?

1577. 1*. Find the roots of the function $y = \cos\frac{1}{x}$. For what values

of the argument x does this function take on the greatest and the least values? Analyse it for evenness and oddness. Plot the graph of this function.

2*. Prove that $\tan 9^\circ - \tan 27^\circ - \tan 63^\circ + \tan 81^\circ = 4$.

3. Let us solve the equation $2\sin x - \sqrt{1-\cos^2 x} = -3$ in the following way: since $1-\cos^2 x = \sin^2 x$, we have $2\sin x - \sqrt{\sin^2 x} = -3$; whence $\sin x = -3$ which is impossible, i.e. the equation has no roots. On the other hand, for $x = 270^\circ$, $2\sin 270^\circ - \sqrt{1-\cos^2 270^\circ} = 2\times(-1) - \sqrt{1} = -3$, i.e. $x = 270^\circ$ is the root of the equation. Find the mistake in solving the equation.

4. Simplify $\frac{2}{\sin 2\alpha} - \cot\alpha$.

5. Find the sum of 20 terms of the given series:

(a) $1-2+4-4+16-8+64-\ldots$;

(b) $6-1+3-3+\frac{3}{2}-9+\frac{3}{4}-27+\ldots$.

1578. 1. Find the greatest and the least values of the function

$$y = m\sin^2 x + n.$$

2. Knowing that $x - y = 120^\circ$, find the greatest value of the sum $\cos x + \cos y$.

3. Prove that $\tan 67^\circ 30' = 1+\sqrt{2}$.

4. Solve the equation $\cos 4x - 8\cos^4 x + 1 = 0$.

5. Simplify the expression $\cot\alpha + \frac{1}{\sin\alpha}$.

6. Transform $\frac{1}{\tan\alpha+1} + \frac{1}{\tan\alpha-1}$ into a product.

7. In a geometric progression $q = \frac{1-\sqrt{5}}{2}$.

Prove that $a_m - a_{m-2} = a_{m-1}$.

1579. 1. Find the least and the greatest values of the function $y = \tan x + \cot x$ if $0 < x < \frac{\pi}{2}$.

2. Acute angles of a right triangle are interrelated as follows: $\cos A + \sin B = \cot A$. Find these angles.

3. Find the domain of definition of the function $y = \arcsin\sqrt{x^2+0.75}$.

4. Solve the equation $\sin\left(2x-\frac{\pi}{12}\right)+\cos\left(2x-\frac{\pi}{12}\right)=$

$=\sqrt{2}\cos\left(3x\pm\frac{\pi}{6}\right)$.

5. Transform to a product the expression $\tan\alpha+\sin\alpha(1+$ $+\sin\alpha)+\cos\alpha(1+\cos\alpha)$ and then evaluate it for $\alpha=10°$.

6. Prove that if the terms of a geometric progression are interconnected by the relation $a_1+a_4=a_2+a_3$, then they are equal to one another.

7*. For what values of y is the inequality

$$x^2\log_2\frac{4(y+1)}{y}+2x\log_2\frac{2y}{y+1}+\log_2\frac{(y+1)^2}{4y^2}>0$$

fulfilled for any x.

1580. 1. Is there an angle x such that

(a) $\sin x\cos x=\sin 20°$; (b) $\sin x\cos x=\sin 30°$;

(c) $\sin x\cos x=\sin 40°$.

2. Find the greatest value of the function

$$y=5\sin x-12\cos x.$$

3. Transform to a product the expression

$$1-\sqrt{3}\sin 2\alpha-2\sin^2\alpha.$$

4. Prove that if the terms of a geometric progression satisfy the relation $a_{k-1}+a_{k+1}=3a_k$, then the ratio of the progression is an irrational number.

5. Prove that $|\sin x|+|\cos x|\geqslant 1$.

6.* Solve the equation $\tan x-2\tan 2x+\tan 4x=0$.

7. Solve the equation $2(2^{\sqrt{x}+3})^{2^{-1}x^{-0.5}}-\sqrt[\sqrt{x}-1]{4^2}=0$.

1581. 1*. For what values of α are the roots of the equation

$$x^2-2(\cos\alpha-\sin\alpha)x+1+\sin\alpha=0$$

(a) real and different, (b) real and equal.

2. Compute the product $\cos 12°\cos 24°\cos 48°\cos 96°$.

3. Prove that if $\log_n a+\log_n c=2\log_n b$, then the numbers a, b, c constitute a geometric progression.

4. The fifth term of a geometric progression equal to 4 is its middle term. Find the product of all terms of this progression.

5. Prove that $\log_{\log_3 2} \frac{1}{2} > 0$.

6. Solve the equation $\log_{\cos x} \sin x + \log_{\sin x} \cos x = 2$.

7. Find the greatest value of the function $y = \frac{5}{2^{x^2}+4}$.

1582. 1. Find the domain of definition of the function

$$y = 2^{\sin x} + 2^{\frac{1}{\cos x}}.$$

2. Find $\sin(\alpha+\beta)\cdot\sin(\alpha-\beta)$ if $\tan\alpha$ is equal to the integral solution of the inequality $4x^2-16x+15 < 0$, and $\cos\beta$ to the slope of the bisector of the first quadrant.

3. Compute $\arcsin\left[\cos \operatorname{arccot}\left(-\frac{1}{\sqrt{3}}\right)\right]$.

4. Solve the equation $1-\tan^4 x = 16\tan^2 x \cos 2x$.

5. For what values of y is the inequality $x^2+x\log_2 y^4+\log_2 y^4+3 < 0$ fulfilled for all values of x?

6*. For what values of α are the roots of the equation $x^2-2(\sin\alpha+\cos\alpha)x+1+2\cos^2\alpha = 0$ (a) real and different; (b) real and equal; (c) complex?

1583. 1 Find the domain of definition of the function

$$y = \arcsin(\arcsin x).$$

2. Solve the equation $\tan^3 x+\tan^2 x-3\tan x-3 = 0$.

3. Find the domain of definition of the function $y = \sqrt{\frac{2^x+2^{-x}}{2^x-2^{-x}}}$.

4. Solve the equation $3^{\sqrt{4x-1}} - 2^{4-\sqrt{4x+1}} = \frac{200}{4^{\sqrt{x}+0.25}}$.

5. Given the arithmetic progression $a_1, a_2, a_3, a_4, \ldots, a_n$. Prove that

$$\frac{1}{\sqrt{a_1}+\sqrt{a_2}} + \frac{1}{\sqrt{a_2}+\sqrt{a_3}} + \ldots + \frac{1}{\sqrt{a_{n-1}}+\sqrt{a_n}} = \frac{n-1}{\sqrt{a_1}+\sqrt{a_n}}.$$

6. Compute $10^{\log_{10} a - \log_{10} b}$.

7. Find $\sqrt{2(\sqrt{10}-1)\cos(\alpha-\beta)}$ if $\sin\alpha$ and $\sin\beta$ are respectively equal to the coordinates x and y of the point of intersection of the circle $9x^2+9y^2=5$ and the straight line $x+y=1$, α and β being acute angles.

1584. 1. Compute $\tan(\alpha+\beta)\cot(\alpha-\beta)$, where $\tan\alpha$ is equal to the root of the equation $\sqrt{2x+6}-\sqrt{x+2}=3$, and $\tan\beta$ to the extraneous root of the same equation.

2. Solve the equation

$$3\left[\log_2\cos\left(\frac{\pi}{2}-x\right)+\log_2(1-\cos 2x)\right]=2.$$

3. Find the least value of the expression

$$2\log_{10}x-\log_x 0.01.$$

4. Solve the equation $2+2^2+2^3+\ldots+2^x=2046$.

5. What sign has $\cos(\sin x)$?

6. Find the geometric progression if the number of its terms equals 8. The sum of the terms occupying the even places is equal to $3\frac{63}{64}$, and that of the terms occupying odd places to $-1\frac{127}{128}$.

1585. 1. Find the least positive value of the sum $(\alpha+\beta)$ if $(\sin\alpha+\cos\alpha)(\sin\beta+\cos\beta)=2\cos\alpha\cdot\cos\beta$.

2. Solve the equation $2\cos^2 2x+\cos 6x-1=0$.

3. Solve the equation $2x=\arctan(\cot x)$.

4. Solve the equation $0.5^{x^2-10x+13.5}=\frac{8}{\sqrt{2}}$.

5. Compute $\tan\left[\arccos\frac{a-b}{\sqrt{a^2+b^2}}-\arcsin\sqrt{\frac{2ab}{a^2+b^2}}\right]$.

6. Prove the identity $\log_{ab}x=\frac{\log_a x\log_b x}{\log_a x+\log_b x}$.

7. Show that the function $y=\frac{1+a^{kx}}{1-a^{kx}}$ is odd.

1586. 1. Find the least value of the function

$$y=2^x+2^{-x}.$$

2. Find the domain of definition of the function

$$y = \log_{10}(x-1)+\arcsin\frac{x}{2}$$

and construct it on the number scale.

3. Compute $2^{\log_3 4}-4^{\log_3 2}$.

4. Determine the dependence between the variables x and y and graph the function (a) $\log_y x+\log_x y = 2$; (b) $\log_y x+\log_x y = -2$.

5. Solve the system $\begin{cases}\log_n u+\log_n v+\log_n 4 = 2+\log_n 9\\ u+v = 5n.\end{cases}$

6. Prove that $\dfrac{\log_{10}^2 x+2}{\sqrt{\log_{10}^2 x+1}} \geqslant 2$.

1587. 1. Find the domain of definition of the function

$$y = \sqrt{\log_{10}\sin\pi x}\,.$$

2. Solve the equation

$$\sin x+\cos x = 4\cos^2 x\sin x+4\sin^3 x.$$

3. Evaluate $\log_{10}\sin 30^\circ\log_{10}\sin 60^\circ\log_{10}\sin 90^\circ$.

4. Without using the Tables prove that

$$\frac{1}{\log_3\pi}+\frac{1}{\log_4\pi} > 2.$$

5. Solve the equation

$$4+4\left(-\frac{1}{2}\right)+4\left(-\frac{1}{2}\right)^2+\ \ldots\ +4\left(-\frac{1}{2}\right)^x = 2\frac{5}{8}.$$

6. Compute without using the Tables

$$\cos^4\frac{\pi}{8}+\cos^4\frac{3\pi}{8}+\cos^4\frac{5\pi}{8}+\cos^4\frac{7\pi}{8}.$$

7. If the terms of one geometric progression are multiplied (divided) by the corresponding terms of another geometric progression, then will the obtained sequence be a geometric progression?

CHAPTER XVII

Complex Numbers

60. THE CONCEPT OF A COMPLEX NUMBER. GEOMETRICAL REPRESENTATION OF COMPLEX NUMBERS

1588. If m and n are natural numbers, then in the set of what numbers is each of the following equations solvable?

(1) $x-m=n$; (2) $x+m=n$; (3) $mx=n$;

(4) $\frac{x}{m}=n$; (5) $mx^2=n$.

Give numerical examples.

1589. (1) Is the equation $ax^2=b$, where a and b are arbitrary numbers, solvable in the set of real numbers?

(2) What operation always becomes performable due to introducing the number whose square is assumed to be equal to -1?

1590. Transform the given expressions using the number i:

(1) $\sqrt{-3}+4\sqrt{27}$; (2) $\sqrt{-5}+4\sqrt{-125}-5\sqrt{125}$.

1591. Verify the following equalities:

(1) $\sqrt{-4}=\pm 2i$; (2) $\sqrt{-\frac{16}{25}}=\pm\frac{4}{5}i$;

(3) $\sqrt{-0.25}=\pm 0.5i$.

Set up a quadratic equation whose roots would be the following numbers: (a) i and $-i$, (b) $2i$ and $-2i$, (c) $0.5i$ and $-0.5i$.

1592. For the given numbers give (a) opposite, (b) conjugate, and (c) reciprocal numbers:

$$0,\ -3,\ \sqrt{2},\ -4i,\ 2-3i,\ -\frac{1}{2}+i\frac{\sqrt{3}}{2}.$$

1593. 1. In the coordinate plane construct the unit vector $\overrightarrow{OA}$ corresponding to the number 1.

2. Construct the vectors corresponding to the given numbers. Find the moduli and the arguments:

(a) $3,\ -2,\ \sqrt{2},\ -4,\ 5,\ i,\ -i,\ 3i,\ -0.5i$;

(b) $-1+i,\ 2-3i,\ -4+3i,\ -3-4i$.

1594. 1. What is the location of the vectors representing (a) opposite numbers; (b) conjugate numbers?

2. Given the point representing the number $-4+3i$. What numbers are represented by the points symmetrical to the given one with respect to the (a) x-axis; (b) y-axis; (c) origin?

1595. Carry out two operations with the unit vector $\overrightarrow{OA}$ corresponding to $+1$: (1) stretch four times and turn it through an angle of 180°; (2) stretch two times and turn it through an angle of $-90°$; (3) stretch five times and turn it through an angle of 60°.

Construct the given vector $\overrightarrow{OA}$ and the newly obtained vectors on a complex plane and write the numbers corresponding to them.

1596. Given the moduli and the arguments of complex numbers. Construct (in the coordinate plane) the vectors corresponding to these numbers:

(1) $r = 3,\ \varphi = 180°$; (2) $r = 1,\ \varphi = -90°$;

(3) $r = 4,\ \varphi = 300°$; (4) $r = 2,\ \varphi = 135°$;

(5) $r = \frac{1}{2},\ \varphi = 90°$; (6) $r = 3.5,\ \varphi = 120°$.

1597. 1. Compare the moduli and the arguments of conjugate and opposite complex numbers.

2. Find (on the complex plane) the locus of points representing all the complex numbers: (a) having the modulus 3, 5; (b) having the argument $\frac{5}{6}\pi,\ -\frac{\pi}{2},\ \pi,\ \frac{2}{3}\pi$; (c) whose argument exceeds $\frac{\pi}{6}$, but is less than $\frac{2}{3}\pi$.

1598. 1. $z = x+yi$; $r = \sqrt{x^2+y^2} = \text{const}$. What is the location of the points corresponding to (a) $z+2$; (b) $z-1+i$?

2. What is the location on the plane of the points, for which (a) $|z| = 2$; (b) $1 < |z| < 2$; (c) $|z| \geqslant 3$?

3. Locate the points representing the complex numbers, for which

(1) $|z+i| \leqslant 1$; (2) $|z+2i| = |z-(-2i)| \leqslant 4$;

(3) $1 < |z+3-2i| < 2$; (4) $|z+i| = |z-2|$;

(5) $|z-1| = |z-3| = |z-i|$.

1599. 1. If $|z| = 3$, then where are the points representing the numbers (a) $2-z$; (b) $-1+3z$ located?

2. Where are the complex numbers z located if (a) $|i-z| < 1$; (b) $|2z+1| > 2$?

1600. Find the length of the segment connecting the points representing the following pairs of numbers:

(1) 0, -5; (2) 5, -3; (3) 7, 2; (4) 3, $-4i$;

(5) $-6i$, $3i$; (6) 0, $-1+i$; (7) $-1-i$, $2+3i$;

(8) $3-2i$, $3+5i$.

61. THE ALGEBRAIC AND THE TRIGONOMETRIC FORM OF A COMPLEX NUMBER

1601. Represent the following numbers in the trigonometric form:

(1) 3, -5, $6i$, $-2i$, 1, i, $1+i$, 0, 1, $-i\sqrt{3}$, $-\sqrt{3}+i$;

(2) $-2+2i$, $-2-2i\sqrt{3}$, $5+12i$, $\sqrt{2}-i\sqrt{2}$, $-3+2i$.

1602. Express the following numbers in the algebraic form:

(1) $2\left(\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}\right)$; (2) $5(\cos 0+i\sin 0)$;

(3) $3\left(\cos\frac{3}{2}\pi+i\sin\frac{3}{2}\pi\right)$; (4) $\cos\pi+i\sin\pi$;

(5) $4\left[\cos\left(-\frac{\pi}{4}\right)+i\sin\left(-\frac{\pi}{4}\right)\right]$;

(6) $10\left(\cos\frac{4}{3}\pi+i\sin\frac{4}{3}\pi\right)$.

1603. Is the given form of complex numbers trigonometric? If not, then represent them in the trigonometric form:

(1) $3[\cos(-90°)+i\sin(-90°)]$;

(2) $4\cos 120°-4i\sin 120°$;

(3) $2[\cos 80°+i\sin(-80°)]$;

(4) $2.5(\cos 300°+i\sin 30°)$;

(5) $-5(\cos 40°-i\sin 40°)$;

(6) $-3(\cos 15°+i\sin 15°)$.

62. OPERATIONS INVOLVING COMPLEX NUMBERS IN THE ALGEBRAIC FORM

1604. Carry out the operation of addition. Give the geometrical representation of the addends and the sums:

(1) $3+4$; (2) $-3+1$; (3) $-2+(-5)$;

(4) $5i+i$; (5) $-3i+4i$; (6) $-2i+(-3i)$;

(7) $-5i+2i$; (8) $5+(-4i)$; (9) $-0.6i+(-2.4i)+(-i)$;

(10) $(2+4i)+(1-2i)$; (11) $(2-5i)+(-3+i)$;

(12) $-3+(2-i)+(-4+5i)$.

1605. Construct the following sums and show the validity, for complex numbers, of the equalities:

(a) expressing the commutative law for addition:

(1) $5+4=4+5$; (2) $-2i+(-3i)=-3i+(-2i)$;

(3) $2-3i=-3i+2$;

(4) $(3+i)+(-2-3i)=(-2-3i)+(3+i)$;

(b) expressing the associative law for addition:

(1) $(-2i+3i)+4i=-2i+(3i+4i)$;

(2) $[(1+i)+(-2+2i)]+(3-4i)=(1+i)+[(-2+2i)+(3-4i)]$.

Carry out the operation of addition:

1606. (Orally).

(1) $(5-4i)+(3+7i)$; (2) $(2+4i)+(-5-i)$;

(3) $(2+3i)+(2-3i)$; (4) $(3+4i)+(-3+4i)$.

1607. (1) $(0.5-3.3i)+(1.5-0.7i)+(-4-i)$;

(2) $5+(3-4i)+2i+(-6+7i)$;

(3) $\left(1\frac{4}{5}+\frac{3}{4}i\right)+\left(1\frac{3}{4}-\frac{5}{6}i\right)+\left(-\frac{3}{4}+2i\right)$;

(4) $(0.14-1.8i)+(1.36+0.8i)+(2.5-0.5i)$.

1608. Construct and compute the difference of numbers:

(1) $(4+3i)-(2+i)$; (2) $(-3+4i)-(2-i)$;

(3) $(5+3i)-(-1+i)$; (4) $(3-4i)-(3+4i)$;

(5) $(2-3i)-(-2+3i)$; (6) $(5+2i)-(-5+2i)$.

1609. Perform the indicated operations:

(1) $(1.2-0.4i)+(0.3-1.4i)-(1.5+0.7i)-(2.3+0.6i)$;

(2) $(-2a+3bi)+(a+bi)+(-4a-2bi)-(-2a+5bi)$;

(3) $(5x-3yi)+(2x-8yi)-[(-2x+yi)-(7x-2yi)]$;

(4) $(m-mi)+(-3m+2ni)-[(m+ni)-(-5m-10ni)]$.

1610. 1. For what condition is the sum of the two complex numbers a_1+b_1i and a_2+b_2i (a) a real number, (b) an imaginary number?

2. For what condition is the difference of the two complex numbers a_1+b_1i and a_2+b_2i (a) a real number, (b) an imaginary number? Give examples.

1611. Find the real numbers x and y if:

(1) $(1-i)x+(1+i)y = 1-3i$;

(2) $(2+3i)x^2-(3-2i)y = 2x-3y+15i$;

(3) $(4x^2+3xy)+(2xy-3x^2)i = 4y^2-\frac{1}{2}x^2+(3xy-2y^2)i$.

1612. 1. Show by construction and computation that (1) the modulus of the sum of the numbers $4+3i$ and $6-8i$ is less than the sum of their moduli; (2) the modulus of the sum of the numbers $4-3i$ and $-8+6i$ is less than the sum of their moduli (is equal to the difference of their moduli); (3) the modulus of the sum of the numbers $-4+3i$ and $-8+6i$ equals the sum of their moduli.

2. Determine the modulus of the sum of complex numbers. What is the location of the vectors representing two complex numbers, if the modulus of the sum of these numbers is equal to the sum of their moduli or the difference of their moduli?

1613. 1. Show on a drawing that to multiply the number 2 by 3 means to stretch the vector representing the multiplicand 2 three times without changing its direction.

2. Interpret the multiplication of the number 2 by -3 as operations involving the vector representing the number 2; the same for multiplying $3-4i$ by 2 and $3-4i$ by -2.

1614. 1. Show geometrically that to multiply the number 3 by the number $2i$ means to stretch the vector representing the multiplicand 3 two times, and to turn it through 90°.

Multiply the following:

1615. (Orally).

(1) $2i \cdot 4i$; (2) $4i \cdot (-3i\sqrt{2})$; (3) $4i \cdot (-5i)$;

(4) $(3-5i) \cdot 4$; (5) $(1+i)(-4)$; (6) $(-2+3i) \cdot 5$.

1616. (1) $(-3-4i) \cdot 2i$; (2) $(-8+7i)(-3i)$;

(3) $(4-9i) \cdot 0$; (4) $(1+i)(1-i)$;

(5) $(a+bi)^2$; (6) $(2+3i)(-4+i)$.

1617. (1) $(3+5i)(5+3i)$; (2) $(0.5-0.2i)(2+3i)$;

(3) $(\sqrt{3}-i)(\sqrt{2}+i\sqrt{3})$; (4) $(5+i\sqrt{3})(5-i\sqrt{3})$;

(5) $(2-3i)(-1-i)(3+4i)$;

(6) $(5+4i)(-2-i)(5-4i)(-2+i)$.

1618. 1. For what condition will the product of the two numbers a_1+b_1i and a_2+b_2i be (a) a real number, (b) an imaginary number? Give examples.

2. Write two complex numbers whose sum and product are real numbers.

1619. Using complex numbers, factorize the following:

(1) a^2+b^2; (2) a^2+9b^2; (3) a^4+b^4; (4) $4m^2+25n^2$;

(5) a^2+1; (6) $5a^2+b$; (7) $a^2+\frac{b^2}{9}$; (8) $a+b$;

(9) $a+16$; (10) $5+\sqrt{3}$; (11) $\frac{a^2}{2}+\frac{b}{3}$; (12) $1+\tan^2\alpha$.

Perform the operation of division:

1620. (1) $6i : 2$; (2) $10i : (-4)$; (3) $6i : (-2i)$;

(4) $10i:2i$; (5) $9i:(-0.5i)$; (6) $\frac{5}{3i}$;

(7) $\frac{6}{1-2i}$; (8) $\frac{4}{1+2i}$; (9) $\frac{2i}{1+i}$;

(10) $\frac{4i}{2-3i}$; (11) $\frac{1+i}{1-i}$; (12) $\frac{1-i}{1+i}$.

1621. (1) $\frac{1+i\sqrt{3}}{1-i\sqrt{3}}$; (2) $\frac{4-i\sqrt{2}}{1+i\sqrt{2}}$; (3) $\frac{5-2i}{1-2i}$;

(4) $\frac{4-3i}{1+3i}$; (5) $\frac{\sqrt{5}-i}{\sqrt{5}-2i}$; (6) $\frac{-\sqrt{3}+i\sqrt{6}}{-1+i\sqrt{3}}$;

(7) $\frac{-3\sqrt{2}+i}{1+3i\sqrt{2}}$; (8) $\frac{a}{i\sqrt{a}}$; (9) $\frac{a+i\sqrt{b}}{a-i\sqrt{b}}$;

(10) $\frac{a+bi}{b-ai}$.

1622. 1. Given two numbers: $2-3i$ and $4+5i$. Find the modulus of their (a) sum, (b) difference, (c) product, (d) quotient.

2. Find the modulus of the quotient obtained as a result of dividing any non-zero number by its conjugate number.

1623. 1. Show that the modulus of the product of two numbers a_1+b_1i and a_2+b_2i is equal to the product of their moduli.

2. Show that $\left|\frac{a_1+b_1i}{a_2+b_2i}\right| = \left|\frac{a_1+b_1i}{a_2+b_2i}\right|$.

Carry out the following operations:

1624. (Orally).

(1) i^2, i^3, i^4, i^6, i^7, i^{40}, i^0;

(2) $(-i)^3$, $(-i)^{10}$, $(-i)^{25}$.

1625. (1) i^{142}; (2) i^{457}; (3) i^{1089};

(4) $1+i+i^2+i^3+i^4$; (5) $(1-i)^2$;

(6) $(4-2i)^2$; (7) $(5+3i)^2$; (8) $(2-3i)^2$;

(9) $(2+3i)^2-(2-3i)^2$.

1626. (1) $(2+ai)^2$; (2) $(a-3bi)^2$; (3) $(2c+3di)^2$;

(4) $(1+i)^4$; (5) $(1-i)^4$; (6) $\left(\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)^2$.

1627. (1) $(1+i)^3$; (2) $(2-i\sqrt{3})^3$;
(3) $(3-i\sqrt{3})^3$; (4) $(1+i)^3+(1-i)^3$;
(5) $(1+i)^3-(1-i)^3$; (6) $(a+bi)^3-(a-bi)^3$.

1628. (1) $(x-1-i)(x+1+i)$; (2) $[(1+i)(1-i)]^6$;
(3) $\left(\frac{1-i}{\sqrt{2}}\right)^4$; (4) $\left(\frac{1+i}{1-i}\right)^6$;
(5) $\left(\frac{1+i}{1-i}\right)^{4n+1}$; (6) $\left(\frac{1+i}{1-i}\right)^{4n+2}$.

1629. Show that if $m = \frac{-1-i\sqrt{3}}{2}$, $n = \frac{-1+i\sqrt{3}}{2}$, then

(1) $m^3 = 1$; (2) $n^3 = 1$; (3) $m^2 = n$; (4) $n^2 = m$.

1630. 1. Prove that $x^3+y^3-3xy = -1$ if $x = \frac{-1+3i}{2}$, $y = \frac{-1-3i}{2}$.

2. Prove that if the roots of the equation $ax^2+bx+c = 0$ are real and different, then the equation $2a^2x^2+2abx+b^2-2ac = 0$ has imaginary roots.

1631. 1. Under what condition the square of the complex number $a+bi$ is (a) a real number, (b) an imaginary number.

2. Find the condition on which the cube of the number $a+bi$ is (a) a real number, (b) an imaginary number.

1632. Compute:

(1) $(1+i)^{-1}$; (2) $(1-i)^{-2}$; (3) $(25-17i)^0$;
(4) $\left(\frac{2}{1-i\sqrt{7}}\right)^{-4}$; (5) $[(2+2i)(2-2i)]^{\frac{2}{3}}$;
(6) $[(\sqrt{3}+i)(\sqrt{3}-i)]^{-\frac{3}{2}}$;
(7) $[(\sin\alpha+i\cos\alpha)(\sin\alpha-i\cos\alpha)]^{-1}$.

Find the square root:

1633. (1) $\sqrt{5+12i}$; (2) $\sqrt{5-12i}$; (3) $\sqrt{-5-12i}$;
(4) $\sqrt{-3+4i}$; (5) $\sqrt{7-24i}$; (6) $\sqrt{1-i\sqrt{3}}$;
(7) $\sqrt{1-i}$; (8) $\sqrt{1+i\sqrt{3}}+\sqrt{1-i\sqrt{3}}$.

1634. (1) $\sqrt{i}$; (2) $\sqrt{-i}$; (3) $\sqrt{4}$; (4) $\sqrt{5}$;
(5)* $\sqrt[4]{16i}$; (6)* $\sqrt[4]{-81i}$; (7) $\sqrt[4]{4}$; (8)* $\sqrt[4]{-4}$.

1635. Solve the given quadratic equations and verify the property of the sum and the product of the roots of each of them:

(1) $x^2-6x+13=0$; (2) $x^2-4x+6=0$;

(3) $2x^2+(5-i)x+6=0$; (4) $x^2+(4-6i)x+10-20i=0$.

1636. Set up a quadratic equation given its roots:

(1) $x_1=i$, $x_2=-i$; (2) $x_1=4$, $x_2=-2i$;

(3) $x_1=2-i$, $x_2=-3+2i$; (4) $x_{1,2}=\frac{1\pm i\sqrt{3}}{2}$.

1637. 1. For what real values of a will the roots of the quadratic equation $(a+5)x^2+2ax+(a-1)=0$ be imaginary?

2. For what k will the quadratic equation $x^2+2(3+i)x+k=0$ have equal roots?

1638. Solve in the set of complex numbers the following equations:

(1) $x^3+1=0$; (2) $x^3-8=0$; (3) $27x^3+1=0$;

(4) $27x^3-125=0$; (5) $x^4-16=0$; (6) $x^4+81=0$;

(7) $x^6+2x^3+1=0$; (8) $x^6-9x^3+8=0$.

1639. Check the properties of the roots of the equation $x^3-1=0$:

(1) $x_1+x_2+x_3=0$; (2) $x_1x_2x_3=1$.

1640. Simplify:

(1) $\frac{3}{1+i}-\frac{4}{4-2i}+\frac{2}{1-i}$;

(2) $\frac{1+i}{1-i}-\frac{1-i}{1+i}$; (3) $\frac{(1-i)^3}{1-i^3}$;

(4) $\frac{\sqrt{5+12i}+\sqrt{5-12i}}{\sqrt{5+12i}-\sqrt{5-12i}}$.

1641. 1. Prove the validity of the given formulas by squaring them:

(a) $\sqrt{a+bi}+\sqrt{a-bi}=\sqrt{2(\sqrt{a^2+b^2}+a)}$;

(b) $\sqrt{a+bi}-\sqrt{a-bi}=i\sqrt{2(\sqrt{a^2+b^2}-a)}$.

2. With the aid of the above formulas transform the following expressions:

(1) $\sqrt{8+6i}+\sqrt{8-6i}$; (2) $\sqrt{8+6i}-\sqrt{8-6i}$;

(3) $\sqrt{40+9i}+\sqrt{40-9i}$; (4) $\sqrt{-15+8i}+\sqrt{-15-8i}$;

(5) $\sqrt{-15+8i}-\sqrt{-15-8i}$;

(6) $\sqrt{-35+12i}-\sqrt{-35-12i}$.

1642. Find the real values of x and y for which:

(1) $\frac{x-1}{3+i}+\frac{y-1}{3-i}=i$;

(2) $y^2+iy^2+6+i=2x+ix$;

(3) $\sqrt{x^2-2x+8}+(x+4)i=y(2+i)$;

(4) $(x-i)^2-(2y+i)^2=4(\sqrt{3}-1)i-2y^2-x$.

63. OPERATIONS INVOLVING COMPLEX NUMBERS IN THE TRIGONOMETRIC FORM

1643. Find the following products:

(1) $2(\cos 60^\circ+i\sin 60^\circ)\cdot 3(\cos 45^\circ+i\sin 45^\circ)$;

(2) $\sqrt{2}(\cos 30^\circ+i\sin 30^\circ)\cdot 2\sqrt{2}(\cos 60^\circ+i\sin 60^\circ)$;

(3) $3\left(\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}\right)\cdot 2\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right)$;

(4) $\sqrt{3}(\cos 120^\circ+i\sin 120^\circ)\cdot\frac{\sqrt{3}}{2}(\cos 150^\circ+i\sin 150^\circ)$;

(5) $(\cos 45^\circ+i\sin 45^\circ)\cdot 4(\cos 15^\circ+i\sin 15^\circ)$;

(6) $2(\cos 120^\circ+i\sin 120^\circ)\cdot\frac{\sqrt{2}}{2}(\cos 60^\circ+i\sin 60^\circ)$;

(7) $2(\cos 150^\circ+i\sin 150^\circ)\cdot\sqrt{3}(\cos 210^\circ+i\sin 210^\circ)$;

(8) $3(\cos 70^\circ+i\sin 70^\circ)\cdot 2[\cos(-20^\circ)+i\sin(-20^\circ)]$.

Using De Moivre's formula compute the following:

1644. (1) $[3(\cos 45^\circ + i \sin 45^\circ)]^6$;

(2) $[2(\cos 15^\circ + i \sin 15^\circ)]^4$;

(3) $(\cos 30^\circ - i \sin 30^\circ)^8$; (4) $\left(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4}\right)^{10}$;

(5) $[2(\cos 60^\circ + i \sin 60^\circ)]^6$; (6) $\left(-\cos \frac{\pi}{4} + i \sin \frac{\pi}{4}\right)^8$.

1645. (1) $(\cos 150^\circ + i \sin 150^\circ)^{10}$;

(2) $(\cos 30^\circ + i \sin 30^\circ)^5$; (3) $(1+i)^{10}$;

(4) $(3 - i\sqrt{3})^5$; (5) $(-2-2i)^8$;

(6) $\left(\frac{1}{2} - i\frac{\sqrt{3}}{2}\right)^9$; (7) $\left(-\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)^{12}$;

(8) $(1-i)^{10}$.

1646. Verify the following equalities:

(1) $\left(\frac{-1+i\sqrt{3}}{2}\right)^6 + \left(\frac{-1-i\sqrt{3}}{2}\right)^6 = 2$;

(2) $\left(\frac{-1+i\sqrt{3}}{2}\right)^5 + \left(\frac{-1-i\sqrt{3}}{2}\right)^5 = -1$;

(3) $\left(\frac{\sqrt{3}+i}{2}\right)^6 + \left(\frac{i-\sqrt{3}}{2}\right)^6 = -2$.

1647. Prove:

(1) $(\cos 60^\circ + i \sin 60^\circ)^6 = 1$;

(2) $[2(\cos 45^\circ + i \sin 45^\circ)]^4 = -16$;

(3) $[\sqrt{3}(\cos 45^\circ + i \sin 45^\circ)]^6 = -27$;

(4) $[\sqrt{2}(\cos 56^\circ 45' + i \sin 56^\circ 15')]^8 = 16i$.

1648. Find the quotient:

(1) $4(\cos 75^\circ + i \sin 75^\circ) : 0.4(\cos 30^\circ + i \sin 30^\circ)$;

(2) $3(\cos 45^\circ + i \sin 45^\circ) : 1.5(\cos 135^\circ + i \sin 135^\circ)$;

(3) $4(\cos 240^\circ + i \sin 240^\circ) : 2(\cos 60^\circ + i \sin 60^\circ)$.

1649. Compute all the values of the given radicals and represent them geometrically:

(1) $\sqrt[3]{\cos 120^\circ + i \sin 120^\circ}$;

(2) $\sqrt[4]{16(\cos 240^\circ + i \sin 240^\circ)}$;

(3) $\sqrt[5]{\cos 250^\circ + i \sin 250^\circ}$; (4) $\sqrt[6]{\cos 60^\circ + i \sin 60^\circ}$.

1650. Compute:

(1) $(\cos 120^\circ + i \sin 120^\circ)^{\frac{1}{2}}$;

(2) $[8(\cos 60^\circ + i \sin 60^\circ)]^{\frac{2}{3}}$;

(3) $[4(\cos 300^\circ + i \sin 300^\circ)]^{-\frac{1}{2}}$;

(4) $[3(\cos 150^\circ + i \sin 150^\circ)]^{-1}$.

1651. Find all the values of the given root:

(1) $\sqrt[3]{1}$; (2) $\sqrt[3]{-1}$; (3) $\sqrt[3]{8}$; (4) $\sqrt[3]{-27}$; (5) $\sqrt[4]{1}$;

(6) $\sqrt[4]{-1}$; (7) $\sqrt[4]{-625}$; (8) $\sqrt[4]{i}$; (9) $\sqrt[6]{1}$; (10) $\sqrt[6]{-1}$;

(11) $\sqrt[3]{2-2i}$; (12) $\sqrt[4]{-24-8\sqrt{3}i}$.

1652. Here we cite the famous Euler formula $e^{xi} = \cos x + i \sin x$.

(1) With the aid of the Euler formula obtain the basic identity of trigonometry, $\sin^2 x + \cos^2 x = 1$, replacing x by $-x$ and multiplying e^{xi} by e^{-xi}.

(2) With the aid of the equality $e^{xi} \cdot e^{yi} = e^{(x+y)i}$ and the Euler formula derive the addition formulas for sine and cosine.

(3) With the aid of the equality $(e^{xi})^2 = e^{2xi}$ and the Euler formula derive the formulas $\sin 2x = 2 \sin x \cos x$ and $\cos 2x = \cos^2 x - \sin^2 x$.

(4) With the aid of the equality $(e^{xi})^2 = e^{2xi}$ and the Euler formula derive the De Moivre formula.

CHAPTER XVIII

Compound Interest, Combinatorics, and the Binomial Theorem

64. COMPOUND INTEREST

1653. Two hundred roubles is deposited in a savings bank for a five-year period. What will the final sum be if the bank pays annually 2% compound interest?

1654. What sum will accumulate from 500 roubles placed on a deposit account paying 3% interest in a savings bank for 10 years?

1655. What initial sum must be placed on a deposit account for a period of 10 years in order to obtain 3000 roubles?

1656. What initial sum must be placed on a current account paying 2% interest in order to obtain a final sum of 2000 roubles in 15 years?

1657. In how many years will m roubles placed on a deposit account double?

1658. A sum of 500 roubles is placed on a deposit account. In how many years will the depositor accumulate a sum of 600 roubles?

1659. One depositor put 900 roubles on a deposit account, and another 1000 roubles on a current account. What will each have in the bank at the end of 20 years?

1660. A forest has an annual growth rate of 2%. At the present time a section of the forest has about 20,000 cubic metres of timber. How much timber will there be in that section in 15 years?

1661. Several years ago there were 20,000 trees in a forest, and now there are 32,000. How many years did this increase take if the number of trees increases by 0.1 of their number at the start of each year?

1662. A city has a population of 200,000. What will the population be in 10 years if the annual growth rate is an average of 3%?

1663. How many years will it take a city to increase its population 1.5 times if the annual growth rate is an average of 3%?

1664. What was the population of a city 10 years ago if the present population is 300,000 and the annual growth rate is 3.5%?

1665. The population of a country increases annually by 1/80. How many years will it take the population to double?

1666. Certain equipment depreciates by $p\%$ per year. What will the cost of the equipment be 7 years hence if the original cost was 10,000 roubles and the annual depreciation rate is 2%?

1667. Certain equipment costs 100,000 roubles. In 5 years, due to depreciation, the cost will be 60,000 roubles. Find the annual depreciation rate of the equipment.

1668. The USSR produced 66,300 million kilowatt-hours of electricity in 1948 and 412,400 million kilowatt-hours in 1963. Find the mean annual growth rate in the generation of electric energy in the USSR during these years.

1669. In 1952 the housing facilities of the USSR in the cities and towns totalled 557 million square metres; in 1964 they came to 1182 million square metres. Find the average annual growth rate in floor space (housing) during the indicated years.

1670. A person deposited 400 roubles on a current account and after four years took out half the sum on the account and placed the other half on a deposit account. What will the depositor have in the bank 10 years after this operation?

1671. A total of 1200 roubles is deposited in a savings bank. One year later 240 roubles is withdrawn. A year after this operation the account reads 1071 roubles. What is the bank's interest rate?

1672. A person deposits a certain sum in savings bank and in one year obtains 10 roubles in accumulated interest. He then adds 50 roubles and in another year the deposit plus the interest comes to 561 roubles. What was the original deposit and what is the interest rate?

1673. If a person places 200 roubles on a deposit account at the beginning of each year, what sum will accumulate in 15 years?

1674. How much money should be deposited in a savings bank

at the start of each year in order to obtain 3000 roubles at the end of a 10-year period with an interest rate of 3%?

1675. How long will it take to accumulate 2500 roubles on a deposit account if 200 roubles is deposited at the beginning of each year?

1676. A person places 300 roubles on a deposit account in a savings bank and then at the beginning of each succeeding year deposits 150 roubles. What sum of money will he have on his account at the end of 10 years?

1677. A deposit of 200 roubles is made in a savings bank on a current account. Five years later 150 roubles is withdrawn. Then, beginning with the seventh year, the depositor deposits 100 roubles at the beginning of each year on a current account. Find the sum he has in the bank at the expiration of 15 years.

1678. In the course of 7 years a person placed 100 roubles at the beginning of each year on a deposit account. The following three years no deposits were made. What sum of money did he have on his account 10 years after his first deposit?

1679. A loan of 1000 roubles is to be paid off in 5 years at 3% compound interest. What sums must be paid at the end of each year?

1680. A loan of 2000 roubles at 2% compound interest is to be paid back in 5 years. What are the yearly payments and what is the debt at the end of three years?

65. COMBINATORICS

1681. 1. Find all the permutations of four elements a, b, c, d taken two at a time, A_4^2, taken three at a time, A_4^3.

2. Find all the permutations of four elements a, b, c, d.

3. Find all the combinations of four elements taken three at a time.

1682. Compute:

(1) A_5^2; (2) A_7^3; (3) $A_8^4 - A_8^3$; (4) $\dfrac{A_6^5 - A_6^4}{A_5^4 - A_5^3}$;

(5) $0.2\,\dfrac{A_5^4 + A_5^3 + A_5^2}{A_4^3 + A_4^2 + A_4^1}$.

1683. Reduce the following fractions:

(1) $\dfrac{5!+4!}{6!}$; (2) $\dfrac{7!-5!}{4!4!}$; (3) $\dfrac{n!}{(n+1)!}$; (4) $\dfrac{(n+1)!}{(n-1)!}$;

(5) $\frac{(n+2)!}{(n-2)!}$; (6) $\frac{(2n)!}{n!}$; (7) $\frac{(3n-1)!}{(2n+1)!}$; (8) $\frac{(n+k)!}{(n+k-2)!}$.

1684. Compute:

(1) P_5; (2) P_6-P_4; (3) $\frac{P_6+P_4}{P_5}$; (4) $\frac{P_7-P_5}{P_6}$;

(5) $\frac{P_5+P_6}{P_7}$; (6) $\frac{A_7^3 \cdot P_4}{6!}$; (7) $\frac{P_8+A_7^6}{P_7}$; (8) $\frac{7!A_{12}^5}{P_{13}}$.

1685. Compute:

(1) C_7^3; (2) C_4^3; (3) C_8^6; (4) C_{10}^7; (5) $\frac{C_6P_3}{A_6^2}$;

(6) $C_8^3+\frac{A_9^4}{P_4}$; (7) $\frac{A_{10}^6+P_{10}}{C_9^6 7!}$.

1686. How many straight lines can be drawn through six points of which no three points lie on one and the same straight line?

1687. Determine the maximum possible number of points at which five straight lines intersect.

1688. How many triangles can be constructed by connecting the vertices of an octagon three at a time.

1689. Determine the number of all the diagonals of a regular (a) pentagon, (b) octagon, (c) dodecagon.

1690. How many terms are there in a squared polynomial consisting of n terms?

1691. How many (a) three-digit, (b) four-digit numbers can be set up using the digits 1, 2, 3, 4, 5?

1692. How many five-digit numbers can be written using the digits 2, 3, 4, 5, 6, and what is the sum of all the digits entering all these numbers equal to?

1693. How many (a) five-, (b) six-digit numbers can be set up using the digits 0, 1, 2, 3, 4, 5?

1694. How many even five-digit numbers can be set up using the digits 2, 3, 4, 5 and 9?

1695. How many five-digit numbers, multiple of five, can be represented by the digits 0, 1, 2, 3, and 5?

1696. Four scouts have to be chosen from a group of 12. In how many different ways can this be done?

1697. In how many different ways can four passengers be seated in a railway-coach compartment for four?

1698. At a graduation party 20 students of the technical school exchanged photographs. How many photos changed hands?

1699. Twelve persons meet and shake hands. How many handshakes were there in all?

1700. A trade-union committee consists of 9 persons and has to elect a chairman, a vice-chairman, and a treasurer. How many various combinations can there be?

1701. Twelve lorries are needed to help in harvesting operations at a kolkhoz. In how many ways can they be chosen from a pool of 30 vehicles?

1702. A railway station has 6 sidings. In how many ways can four trains be positioned on them?

1703. The dial of a telephone has 10 digits. Each telephone of an automatic station has a five-digit number, the first digit being the same in all numbers. What is the largest number of subscribers with four distinct digits at the end of each number?

1704. How many distinct four-digit numbers can be written down using each of the following digits once: 1, 3, 5, 7, 9, 0?

1705. A group of 16 students are to set up two construction teams with 10 and 6 students in each. In how many ways can this be done?

1706. Fourty-five games were played at a chess tournament, with each contestant playing once against each of the others. How many participants were there?

1707. Determine the number of permutations of the letters in the word *rocket*. How many permutations can be made so that (a) the letter *r* is the first letter, (b) the letter *r* is the first letter and the digraph *et* forms the last two letters?

1708. In how many ways can one officer and 5 soldiers be chosen for guard duty if there are 40 soldiers and 3 officers?

1709. In how many ways can a football team be chosen consisting of three forwards, three half-backs, four backs and a goalkeeper if in all there are 6 forwards, three half-backs, 6 backs and one goalkeeper to choose from?

1710. All possible two-factor products are formed from the numbers 1, 2, 3, ..., 100. How many numbers out of the total obtained are multiples of 3?

1711. Find the number of permutations:

(1) of $m+1$ elements taken 3 at a time; (2) of $m-2$ elements taken 4 at a time; (3) of $m+n$ elements taken $m-n$ at a time; (4) of $m+n+2$ elements taken $m+n$ at a time.

1712. Find the number of permutations:
(1) of $m+2$ elements; (2) of $m+k$ elements;
(3) of $m+2n$ elements; (4) of $3m+1$ elements.

1713. Find the number of combinations:
(1) of $m+1$ elements taken $m-1$ at a time;
(2) of $m+n$ elements taken $n-1$ at a time;
(3) of $m+2$ elements taken $m-3$ at a time;
(4) of $m+n$ elements taken $m-n$ at a time.

1714. Find:
(1) $\dfrac{A_n^3+A_n^5}{A_n^4}$; (2) $\dfrac{A_n^{n-1}-P_n}{C_m^{m-2}}$; (3) $\dfrac{A_{n-1}^{k-1}P_{n-k}}{P_{n-1}}$;
(4) $\dfrac{C_{m+n}^n P_n - A_{m+n}^n}{P_n}$; (5) $\dfrac{A_{m+3}^m}{(m+2)\,A_{m+1}^{m-2}}$;
(6) $\dfrac{A_{m+2}^2 P_m - 2P_{m+1}}{P_{m+1}}$.

1715. 1. Compute $C_8^8+C_9^8+C_{10}^8+C_{11}^8$.
2. Compute C_m^6 if $C_m^{18} = C_{18}^{m+2}$.
3. Which is greater $A_{10}^n P_{10-n}$ or P_9 and how many times?

1716. Check the correctness of the following equalities:
(1) $\dfrac{A_m^6+A_m^5}{A_m^4} = (m-4)^2$; (2) $C_m^n+C_m^{n-1} = C_{m+1}^n$;
(3) $A_m^{m-3}P_3 = P_m$.

1717. Solve the following equations:
(1) $A_x^2 = 12$; (2) $P_x = 24$; (3) $A_{x+2}^3 = 42x$;
(4) $C_x^3 = 2x$; (5) $C_{x+2}^4 = x^2-1$; (6) $30P_x = P_{x+2}$;
(7) $5C_x^3 - C_{x+2}^4 = 0$; (8) $C_x^7 - C_x^5 = 0$; (9) $A_7^x - xA_7^{x-1} = 0$;
(10) $P_{x-3}A_x^3 = 20P_{x-2}$; (11) $C_{x-1}^2+C_x^3 = 16$;
(12) $P_{x+2} = 182A_x^{11}P_{x-11}$.

1718. Solve the given systems of equations:

(1) $\begin{cases} A_x^2 = 272 \\ C_x^y = 136; \end{cases}$ (2) $\begin{cases} A_x^y : A_{x+1}^y = \dfrac{2}{5} \\ C_x^y : C_x^{y-1} = \dfrac{2}{3}; \end{cases}$ (3) $\begin{cases} C_x^y = C_x^{y+2} \\ C_x^2 = 153; \end{cases}$

(4) $C_{x+1}^{y+1} : C_{x+1}^{y} : C_{x+1}^{y-1} = 5 : 5 : 3$; (5) $\begin{cases} A_{2x}^{y-2} : A_{2x}^{y-3} = 8 \\ C_{2x}^{y-2} : C_{2x}^{y-3} = 8 : 3. \end{cases}$

1719. 1. How many elements should be taken so that the number of their permutations taken 4 at a time is 6 times greater than those taken 2 elements at a time?

2. The number of permutations of n elements taken 2 at a time is 6 times greater than the number of permutations of $(n-5)$ elements taken 2 at a time; find n.

3. Find m from the condition $C_{m-2}^{2}+C_{m-3}^{2}+C_{m-4}^{2} = 20$.

4. Determine the number of elements (n) if it is known that C_n^2 constitute 16.(6)% of C_{n+2}^{4}.

66. MATHEMATICAL INDUCTION

1720. (1) Prove that the sum of the first n odd numbers is equal to the squared number of addends, i.e. $1+3+5\ldots+(2n-1) = n^2$.

(2) Prove that for any natural n

$$1+2+3+4+\ldots+n = \frac{n(n+1)}{2}.$$

1721. Prove that the square of a polynomial is equal to the sum of the squares of all its terms and all possible doubled two-factor products of its terms, i.e.

$$(a_1+a_2+\ldots+a_n)^2 = a_1^2+a_2^2+\ldots+a_n^2+2(a_1a_2+a_1a_3+$$
$$+\ldots+a_1a_n+a_2a_3+a_2a_4+\ldots+a_2a_n+\ldots+a_{n-1}a_n).$$

1722. Using the method of mathematical induction prove that for any natural n the following equalities are satisfied:

(1) $1^3+2^3+\ldots+n^3 = \left[\frac{n(n+1)}{2}\right]^2$;

(2) $\frac{1}{1\times2}+\frac{1}{2\times3}+\frac{1}{3\times4}+\ldots+\frac{1}{n(n+1)} = \frac{n}{n+1}$;

(3) $1\times2+2\times3+3\times4+\ldots+n(n+1) = \frac{n(n+1)(n+2)}{3}$;

(4) $\frac{1}{1\times3}+\frac{1}{3\times5}+\ldots+\frac{1}{(2n-1)(2n+1)} = \frac{n}{2n+1}$;

(5) $\cos\alpha\cos2\alpha\cos4\alpha\cos8\alpha\ldots\cos2^n\alpha = \frac{\sin2^{n+1}\alpha}{2^{n+1}\sin\alpha}$.

1723. 1. Prove that for any integral n the expression n^3+11n is divisible by 6 without remainder.

2*. Prove that n^5-n is divisible by 30 without remainder for all integral n.

3*. Prove that the number $7\times5^{2n-1}+2^{3n+1}$ is exactly divisible by 17 for any natural n.

1724. Prove the validity of the formulas for the sum of n first terms of:

(a) the arithmetic progression $S_n = \frac{a_1+a_n}{2}n = \frac{2a_1+(n-1)d}{2}n$;

(b) the geometric progression $S_n = \frac{a_1q^n-a_1}{q-1}$.

1725. Prove that de Moivre's formula $(\cos\alpha+i\sin\alpha)^n = \cos n\alpha+i\sin n\alpha$ holds true for any natural n.

1726. Prove that a difference between equal powers of two numbers is always divisible by the difference between these numbers, i.e. a^n-b^n is always divisible by $a-b$ for any natural n.

1727. Prove that for any natural n

$$|a_1+a_2+\ldots+a_n| \leqslant |a_1|+|a_2|+\ldots+|a_n|.$$

1728. Prove the inequality

$$(1+a)^n > 1+na$$

where $a > -1$ and $a \neq 0$, n natural number exceeding unity.

1729. For what natural values of n is the inequality $2^n > n$ true?

1730. Prove that $|\sin nx| \leqslant n|\sin x|$ if n is natural.

67. THE BINOMIAL THEOREM

1731. Find the product of the given binomials:

(1) $(x+1)(x+2)(x+3)(x+4)$;

(2) $(x-1)(x-2)(x-3)(x-4)$;

(3) $(y+1)(y+3)(y-4)$;

(4) $(b-1)(b-2)(b-3)(b-4)(b-5)$;

(5) $(a-b)(a+2b)(a-3b)(a+4b)$;

(6) $(a+1)(a+2)(a+3)(a+4)(a+5)$;

(7) $(m+1)(m-2)(m+3)(m-4)(m+5)$;

(8) $(a+i)(a+2i)(a+3i)(a+4i)$.

1732. 1. Find the coefficient of m in the product

$$(m-2)(m-3)(m-4).$$

2. Find the coefficient of x^3 in the product

$$(x-i)(x-2i)(x-3i)(x-4i).$$

Find the expansions for the given binomials:

1733. (1) $(x+m)^5$; (2) $(x+1)^6$; (3) $(x-2)^8$;
(4) $(m^2+1)^7$; (5) $(b^2-a)^5$; (6) $(x^2-y^2)^6$.

1734. (1) $(2a+3b)^5$; (2) $(1-3x)^6$; (3) $(2m-3n)^6$;
(4) $(x^2-2y)^7$; (5) $(3x^2+y^2)^4$; (6) $\left(\frac{1}{2}a+2b^2\right)^5$.

1735. (1) $\left(\sqrt[3]{m}-\sqrt[3]{n}\right)^7$; (2) $\left(\sqrt{x}-\sqrt[4]{x}\right)^6$;
(3) $\left(\sqrt[3]{x}-\sqrt{x}\right)^4$; (4) $\left(\sqrt{x}+y\right)^5$;
(5) $\left(\sqrt{x}+\sqrt{y}\right)^6$; (6) $\left(\sqrt[3]{a}+\sqrt[3]{b}\right)^8$.

1736. (1) $\left(\frac{1}{\sqrt[3]{a}}-\frac{1}{\sqrt[3]{b}}\right)^5$; (2) $\left(\frac{m}{\sqrt{n}}+\frac{n}{\sqrt{m}}\right)^6$;
(3) $\left(\sqrt{a+b}-\sqrt{a-b}\right)^4$; (4) $\left(a-\sqrt{1+a}\right)^6$;
(5) $\left(\sqrt{x^2-1}+1\right)^4+\left(1-\sqrt{x^2-1}\right)^4$.

1737. (1) $(1+i)^6$; (2) $(1-i)^6$; (3) $\left(\sqrt{3}-3i\right)^4$;
(4) $(x+iy)^4+(x-iy)^4$;
(5) $\left(2-\sqrt{2i}\right)^6+\left(2+\sqrt{2i}\right)^6$.

1738. Find:

(1) the eighth term of the binomial expansion of $(x+2)^{12}$;

(2) the fourth term of the binomial expansion of $\left(x+\sqrt{2}\right)^9$;

(3) the middle term of the binomial expansion of $\left(\frac{1}{x}-\sqrt{x}\right)^{16}$;

(4) two middle terms in the expansion of the binomial:

(a) $(\sqrt{x}-\sqrt[4]{y})^{15}$; (b) $(m^{\frac{1}{3}}-2n^{\frac{3}{2}})^{17}$.

1739. Find:

(1) the term of the expansion of $(x+1)^{12}$, containing x^6;

(2) the term of the expansion of $(\sqrt{x}+\sqrt{2})^{15}$, containing x^5;

(3) the term of the expansion of $(\sqrt{x}+\sqrt{y})^{18}$, containing x^8;

(4) the term of the expansion of $(m^{\frac{2}{3}}-m^{\frac{4}{3}})^{12}$, containing $m^{\frac{32}{3}}$;

(5) the term of the expansion of $(x^{\frac{1}{3}}y^{-1}+x^{-\frac{1}{4}}y)^{18}$, containing x^{-1}.

1740. Find:

(1) the term of the expansion of $\left(\sqrt{x}+\frac{1}{\sqrt[4]{x}}\right)^9$, not containing x;

(2) the term of the expansion of $\left(\sqrt[3]{a^2}-\frac{1}{a}\right)^{15}$, not containing a;

(3) the term of the expansion of $\left(\frac{m}{\sqrt[3]{m}}+m^{-\frac{2}{7}}\right)^{10}$, not containing m;

(4) the term of the expansion of $(\sqrt[3]{x}+\sqrt{x^{-1}})^{15}$ not containing x;

(5) the term of the expansion of $(z^5+z^{-20})^{1000}$ not containing z.

1741. Find the exponent of the binomial if

(1) in the expansion of $(1+x)^n$ the coefficient of the fifth term is equal to the coefficient of the ninth term;

(2) in the expansion of $(ax^{-\frac{1}{2}}+xa^{-\frac{1}{2}})^n$ the ratio of the coefficient of the third term to that of the second is equal to $\frac{11}{2}$;

(3) in the expansion of $\left(9x-\frac{x^{-\frac{1}{2}}}{\sqrt{3}}\right)^n$ the binomial coefficient of the third term is equal to 105;

(4) the sum of the coefficients of the first, second and third terms of the expansion of $(x^{-1}+x^2)^n$ equals 46.

1742. In the expansion of the binomial $(1+a)^n$ the coefficients of the fifth, sixth and seventh terms form an arithmetic progression. Find n.

1743. 1. In a geometric progression $a_1 = i^{-1}$, $q = 1+i$; find a_6.

2. In a geometric progression $a_1 = i$, $q = 1+i^{-1}$; find a_7.

1744. 1. In the expansion of $(x^2+x^{-1})^n$ the coefficients of the

fourth and thirteenth terms are equal to each other. Find the term independent of x.

2. In the expansion of $\left(\frac{x\sqrt[3]{x}}{6}+x^{-\frac{28}{15}}\right)^n$ the sum of the coefficients of the last three terms equals 79. Find the term independent of x.

1745. In the expansion of the binomial $\left(z^{-\frac{3}{2}}-z^{\frac{1}{3}}\right)^n$ ther atio of the third binomial coefficient to the fifth one is equal to $\frac{2}{7}$. Find the term containing $z^{-\frac{5}{2}}$.

1746. In the expansion of the binomial $\left(a^{\frac{3}{2}}+a^{-\frac{1}{3}}\right)^n$ the sum of all the binomial coefficients equals 128. Find the term containing a^5.

1747. Find the rational terms in the expansion of the binomials:

(1) $\left(3^{\frac{1}{3}}+2^{\frac{1}{2}}\right)^5$; (2) $\left(3^{\frac{1}{5}}+2^{\frac{1}{7}}\right)^{24}$; (3) $(\sqrt{5}-\sqrt{2})^{10}$.

1748*. Find x, y and z if the second, third and fourth terms of the expansion of $(x+2y)^z$ are equal to 240, 720 and 1080, respectively.

1749. 1. Find the total number of terms in the expansion of $(x+y)^{2n}+(x-y)^{2n}$ after all possible simplifications are carried out.

2. Without expanding the binomial $\left(\sqrt[5]{x}+\sqrt[10]{y}\right)^{55}$, find the number of the terms in its expansion which are free of radicals.

1750. Prove that the sum of the coefficients in the expansion of the binomial $(5a-4b)^n$ is independent of n and is equal to unity for all values of n.

1751. Carry out approximate computations:

(1) $(0.997)^8$; (2) $(2.003)^{10}$; (3) $\sqrt[5]{0.998}$;

(4) $(0.9998)^5$; (5) $\sqrt{1.003}$; (6) $\sqrt[6]{1.002}$.

CHAPTER XIX

Limits

68. THE FUNCTION AND ITS PROPERTIES

1752. Determine and construct on the number axis $Ox(Oy)$ the domain of variation of the variables $x(y)$ specified by the inequalities:

(1) $|x| < 3$; (2) $x^2 \leqslant 16$; (3) $|x| \geqslant 2$; (4) $x^2 > 9$;

(5) $|x-2| < 3$; (6) $2 < x^2 < 9$; (7) $|y+3| > 4$;

(8) $|2y-5| \geqslant 3$.

1753. Evaluate the given functions:

(1) $f(x) = 2x^3+3x-5$ for $x = 0, x = 2, x = -1, x = 1+h$;

(2) $f(x) = \sqrt{x^2-10x+9}$ for $x = 0, x = 9, x = 10, x = -1$;

(3) $f(x) = 2 \arcsin x - \arctan 2x$ for $x = -\frac{1}{2}, x = \frac{1}{2}, x = \frac{\sqrt{3}}{2}$;

(4) $f(x) = x^2-2x+1$ for $x = 1, x = \frac{1}{a}, x = a$.

1754. Find the roots of the function $f(x) = x^2-20x+64$ and evaluate it for x equal to the (a) arithmetic mean, (b) geometric mean of these roots.

1755. 1. Given the function $f(x) = x^2-3x+\frac{1}{x^2}-\frac{3}{x}$. Show that $f\left(\frac{1}{x}\right) = f(x)$.

2. Given the function $f(x) = x^4+5x^2+1$. Show that $f\left(\frac{1}{x}\right) = \frac{f(x)}{x^4}$.

1756. $f(x) = x^2-3x-1$. Evaluate $f(0)$, $f(3)$, $f(-1)$, $f(a+1)$, $f(a)+1, f(a^2)$.

1757. $f(x) = \frac{2x-3}{x^2+4}$. Find $f(0), f(-1), f(2), f\left(\frac{1}{x}\right)$, $\frac{1}{f(x)}$.

1758. Find (1) $\frac{f(b)-f(a)}{b-a}$; (2) $\frac{f(a+h)-f(a-h)}{2h}$ if $f(x) = x^2$.

1759. Which of the given functions are even and which are odd?

(1) $y = 3x^4$; (2) $y = 2x-3\sqrt[5]{x}$; (3) $y = x^3-2x-1$;

(4) $y = \sin 2x$; (5) $y = 3\sin^2 2x$; (6) $y = |\cos x|$;

(7) $y = \tan^2 x-3$; (8) $y = 5x \sin 3x$; (9) $y = |x|-x^3$;

(10) $y = 2^x$; (11) $y = 2^{x^2}$; (12) $y = \frac{x^2-3}{x^2+3}$;

(13) $y = \frac{a^{2x}-1}{a^x}$.

Find the domains of definition of the given functions:

1760. (1) $y = x^3-2x^2+3x-1$; (2) $y = \sqrt{3x-x^2}$;

(3) $y = \frac{2}{4-x^2}$; (4) $y = \sqrt{x^2-3}$;

(5) $y = \frac{\sqrt{x+1}}{x^2-2}$;

(6) $y = \frac{x+2}{x^2+5x+6}-\sqrt[3]{3x+2}$;

(7) $y = \arcsin\frac{2x-1}{3}$.

1761. (1) $y = \frac{\sin x}{x}$; (2) $y = \log_2 (x^2 - 16)$;

(3) $y = \sqrt{1+x} - 3\sqrt[4]{4-x}$; (4) $y = \frac{3-\sqrt[3]{x}}{\sqrt{2\cos 2x}}$;

(5) $y = \log_{10}(x-1) - \arcsin \frac{x}{3}$;

(6) $y = \frac{2^x + 2^{-x}}{2^x - 2^{-x}}$.

69. THE LIMIT AND CONTINUITY OF A FUNCTION

1762. 1. The vertex of a triangle moves from its initial position to infinity along a straight line parallel to the base. How does each of the sides of the triangle change? each of its angles? its altitude? its perimeter? its midline? its area? the sum of its interior angles? Which of these quantities remain unchanged and which increase without bound? Which quantities have limits?

2. Answer the same questions if the vertex of a triangle moves to infinity in a straight line perpendicular to the base travelling away from it.

1763. Making a direct use of the definition of the limit of a number sequence, find the limit of the number sequence whose general term

(1) $x_n = \frac{n+1}{n}$; (2) $x_n = \frac{1}{n}$; (3) $x_n = \frac{(-1)^n}{n}$;

(4) $x_n = \left(-\frac{1}{2}\right)^n$; (5) $x_n = \frac{1+(-1)^n}{n}$;

(6) $x_n = (-1)^n \frac{n}{n+1}$; (7) $x_n = \frac{2}{\cos \frac{1}{n}}$;

(8) $x_n = -3 + \frac{5}{n}$.

1764. 1. Why is it possible to assert that the following sequences of numbers have limits?

(a) $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \ldots, \frac{n}{n+1}, \ldots$; (b) 2.9, 2.99, 2.999, ...;

(c) $\frac{1}{2}, \frac{1}{3}, \frac{1}{5}, \frac{1}{7}, \frac{1}{11}, \ldots, \frac{1}{p}$, where p is a simple number;

(d) $a_n = \sin\frac{1}{n} - 1$.

2. Give examples of sequences which have no limit.

3. Do the following sequences have limits?

(a) $\frac{1}{2}, -\frac{3}{4}, \frac{7}{8}, -\frac{15}{16}, \frac{31}{32}, \ldots$;

(b) $\frac{1}{2}, \frac{3}{4}, \frac{7}{8}, \frac{15}{16}, \frac{31}{32}, \ldots$.

1765. 1. Give examples of sequences of rational numbers whose limits are the numbers: $\sqrt{2}$, $\sqrt{5}$, 3.

2. Why is it possible to assert that the following sequences have no limits:

(a) $\frac{1}{2}, \frac{2}{3}, \frac{1}{3}, \frac{3}{4}, \frac{1}{4}, \frac{4}{5}, \frac{1}{5}, \frac{5}{6}, \frac{1}{6}, \ldots$;

(b) $2, -2, 2, -2, \ldots$.

1766. 1. Show that the values of the function $f(x) = 5 - 2x^2$ differ from unity arbitrarily little if x is sufficiently close to $\sqrt{2}$ $(x \neq \sqrt{2})$.

2. Show that the values of the function $f(x) = \frac{x^2+x-2}{x-1}$ differ from 4 arbitrarily little if x is sufficiently close to 2 $(x \neq 2)$.

For what variation of the argument x will the following functions be infinitesimals:

1767. (1) $y = x - 1$; (2) $y = x^2$; (3) $y = x^2 - 4$;
(4) $y = x^2 + 1$; (5) $y = x^2 - 3x + 2$;
(6) $y = 2x^2 - 3x - 5$.

1768. (1) $y = \frac{x^2-1}{x+2}$; (2) $y = \frac{2}{x}$; (3) $y = \frac{1}{\sqrt{2x^2-4x}}$;
(4) $y = 2^x$; (5)* $y = 3^{\frac{1}{x+1}}$; (6) $y = \frac{2^x - 2^{-x}}{2^x + 2^{-x}}$.

1769. (1) $y = x \sin\frac{1}{x}$; (2) $y = \cos x$; (3) $y = \frac{1+x}{\tan x}$;
(4) $y = 1 - \cos x$; (5) $y = \sin x \cos x$;
(6) $y = \sin x + \cos x$.

1770. (1) $y = \sin 2x + \sin 4x$; (2) $y = \sin 2x - \sin 6x$;
(3) $y = \cos x + 1$; (4) $y = \cos x + \cos 2x$;
(5) $y = \cos x - \cos 3x$; (6) $y = \tan x + \tan 2x$.

1771. (1) $y = \log_2 (x-1)$; (2) $y = \log_{1/2} (2x^2 - x)$;
(3) $y = \log_{10} |\sin x|$; (4) $y = \arcsin (\cos x)$;
(5)* $y = \arccos \dfrac{1-x^2}{x}$.

For what variation of the argument x will the following functions be infinitely large:

1772. (1) $y = x+1$; (2) $y = \frac{1}{2} x^2$; (3) $y = x^2 - 4$;
(4) $y = x^2 + 3x + 2$; (5) $y = \dfrac{x^2-4}{x^2-1}$; (6) $y = \dfrac{2}{x}$;
(7) $y = \dfrac{3}{2-x^2}$; (8) $y = \dfrac{x-1}{2x^2-3x-5}$.

1773. (1)* $y = \dfrac{1}{\sqrt{2x^2+4x}}$; (2) $y = 2^x$; (3) $y = 3^{-x}$;
(4)* $y = 10^{\left|\frac{1}{\sin x}\right|}$; (5) $y = 2^x + 2^{-x}$;
(6) $y = \dfrac{3^x + 3^{-x}}{3^x - 3^{-x}}$; (7) $y = 10^{|\cot 2x|}$.

1774. (1)* $y = \log_2 (x+2)$; (2)* $y = \log_{1/2} (2x^2 + x)$;
(3) $y = \tan (\arcsin x)$; (4) $y = \dfrac{1}{1-\sin x}$;
(5)* $y = \log_{10} \sin x$; (6)* $y = \log_{10} \tan x$.

Compute the limits of the given functions:

1775. (1) $\lim\limits_{x \to 2} (4x+3)$; (2) $\lim\limits_{x \to -1} (x^2 - 3)$;
(3) $\lim\limits_{x \to \frac{\pi}{4}} \sin 2x$; (4) $\lim\limits_{x \to 0} (x^3 + 2x^2 - 2)$;
(5) $\lim\limits_{x \to -2} (3x^4 - 3x^2 + 5x - 1)$;
(6) $\lim\limits_{x \to -\frac{\pi}{3}} \cos x$; (7) $\lim\limits_{x \to \frac{\pi}{2}} \tan \dfrac{x}{2}$;
(8) $\lim\limits_{x \to 0} \cot x$.

1776. (1) $\lim\limits_{x\to -2}\dfrac{x^2-4}{x}$; (2) $\lim\limits_{x\to -1}\dfrac{x}{x^2+1}$;

(3) $\lim\limits_{x\to \pi}(\sin 2x-\cos x)$; (4) $\lim\limits_{x\to -1}\dfrac{x-2}{x}$;

(5) $\lim\limits_{x\to 0}\dfrac{x^2+1}{x-1}$; (6) $\lim\limits_{x\to 2}\dfrac{2x^2+3}{5-x^2}$;

(7) $\lim\limits_{x\to 1}\dfrac{x^2-2x+1}{x^2+1}$.

1777. (1) $\lim\limits_{x\to 2}\left(5x^2-2x+\dfrac{4}{x}\right)$; (2) $\lim\limits_{x\to 1}\dfrac{x^2+4x+3}{x^2-3x+4}$;

(3) $\lim\limits_{x\to 3}\dfrac{x^2+6}{\sqrt{x^3-2}}$; (4) $\lim\limits_{x\to 0}\dfrac{x^2-3x+2}{2x^2-x+1}$;

(5) $\lim\limits_{x\to -3}\left[\dfrac{x}{1-x}-2(x-1)\right]$.

1778. (1) $\lim\limits_{x\to 1}\dfrac{x^2+2^x}{\sqrt{x+3}}$; (2) $\lim\limits_{x\to 6}\log_{10}\left(x+\sqrt{x^2-20}\right)$;

(3) $\lim\limits_{x\to -1}(x^3-3^{x+1}+8)$; (4) $\lim\limits_{x\to \frac{\pi}{4}}(\sin x\sin 2x\sin 3x)$.

1779. (1) $\lim\limits_{x\to -1.5}\arcsin\dfrac{x+3}{3}$;

(2) $\lim\limits_{x\to \sqrt{2}}\arccos\left(-\sqrt{\dfrac{x^2+1}{2x^2}}\right)$;

(3) $\lim\limits_{x\to +\infty}\arctan x$; (4) $\lim\limits_{x\to 0}\operatorname{arccot}\dfrac{x-1}{x+1}$.

1780. (1) $\lim\limits_{x\to \infty}\left(2-\dfrac{1}{x}\right)$; (2) $\lim\limits_{x\to 0}\left(3+\dfrac{1}{x}\right)$;

(3) $\lim\limits_{x\to \infty}\left(\dfrac{4}{x^2}-\dfrac{3}{x}+2\right)$; (4) $\lim\limits_{x\to \infty}\dfrac{5}{x^4+x^2}$;

(5) $\lim\limits_{x\to -0}\dfrac{5}{x}$; (6) $\lim\limits_{x\to +0}\dfrac{5}{x}$;

(7) $\lim\limits_{x\to \infty}\dfrac{5}{x}$.

1781. (1) $\lim\limits_{x\to+\infty} 2^x$; (2) $\lim\limits_{x\to+\infty} \left(\frac{1}{2}\right)^x$;

(3) $\lim\limits_{x\to 0} 2^x$; (4) $\lim\limits_{x\to+\infty} \log_2 x$;

(5) $\lim\limits_{x\to+0} \log_2 x$; (6) $\lim\limits_{x\to+\infty} \log_{\frac{1}{2}} x$;

(7) $\lim\limits_{x\to+0} \log_{\frac{1}{2}} x$; (8) $\lim\limits_{x\to\infty} (2^x+2^{-x})$;

(9) $\lim\limits_{x\to 0} (2^x-2^{-x})$.

1782. (1) $\lim\limits_{x\to 2+0} \dfrac{x-2-\sqrt{x-2}}{2\sqrt{x-2}}$;

(2) $\lim\limits_{x\to 1+0} \arcsin \dfrac{\sqrt{x-1}+\sqrt{x+2}}{2}$;

(3) $\lim\limits_{x\to -1+0} \dfrac{|x+1|}{x+1}$; (4) $\lim\limits_{x\to -1-0} \dfrac{|x+1|}{x+1}$;

(5) $\lim\limits_{x\to+0} 5^{\frac{1}{x}}$; (6) $\lim\limits_{x\to-0} 5^{\frac{1}{x}}$;

(7) $\lim\limits_{x\to 3+0} \log_{10}(x-3)$; (8) $\lim\limits_{x\to 2+0} \log_{10}(x^2+5x+6)$.

1783. (1) $\lim\limits_{x\to 0} \dfrac{x+4}{x^2+x}$; (2) $\lim\limits_{x\to 0} \dfrac{x+2}{x-1}$;

(3) $\lim\limits_{x\to -2} \dfrac{x^2-4}{x+2}$; (4) $\lim\limits_{x\to 0} \dfrac{2+x+x^2}{x+2x^2+x^3}$;

(5) $\lim\limits_{x\to 0} \dfrac{x^2-4x}{x^2-2x}$; (6) $\lim\limits_{x\to -2} \dfrac{x^2+5x+6}{x+2}$.

1784. (1) $\lim\limits_{x\to 5} \dfrac{2x^2-11x+5}{4x^2-16x-20}$; (2) $\lim\limits_{x\to -3} \dfrac{x^3+27}{2x^2-3x-27}$;

(3) $\lim\limits_{x\to -2} \dfrac{x^5+2x^4+x^2+3x+2}{x^4+2x^3+3x^2-5x-22}$;

(4) $\lim\limits_{x\to \frac{3}{2}\pi} \dfrac{\cos^2 x}{1+\sin^3 x}$; (5) $\lim\limits_{x\to 3} \dfrac{x^2-6x+9}{2x^2+x-21}$;

(6) $\lim\limits_{x\to 2} \dfrac{x^3-5x^2+8x-4}{x^3-3x^2+4}$.

1785. (1) $\lim\limits_{x\to 1}\dfrac{x^3+x^2-x-1}{x^3-x^2-x+1}$;

(2) $\lim\limits_{x\to 4}\left(\dfrac{2x^2-4x-24}{x^2-16}-\dfrac{1}{4-x}\right)$;

(3) $\lim\limits_{x\to -2}\left(\dfrac{2}{x+2}+\dfrac{1}{x^2-2x+4}-\dfrac{24}{x^3+8}\right)$;

(4) $\lim\limits_{x\to \frac{\pi}{4}}\dfrac{\sin x-\cos x}{\cos 2x}$;

(5) $\lim\limits_{x\to 3}\left(\dfrac{1}{x-3}-\dfrac{6}{x^2-9}\right)$;

(6) $\lim\limits_{x\to +\infty}\left(x-\sqrt{x^2+4x}\right)$.

1786. (1) $\lim\limits_{x\to +\infty}\dfrac{1}{x(\sqrt{4x^2+2}-2x)}$;

(2) $\lim\limits_{x\to +\infty}\left[x\left(\sqrt{x^2+2}-x\right)\right]$;

(3) $\lim\limits_{x\to 1}\dfrac{1-\sqrt{x}}{1-x^2}$; (4) $\lim\limits_{x\to -1}\dfrac{x+1}{2-\sqrt{4+x+x^2}}$;

(5) $\lim\limits_{x\to 4}\dfrac{\sqrt{x}-2}{3-\sqrt{2x+1}}$; (6) $\lim\limits_{x\to 0}\dfrac{\sin x}{1-\sqrt{1+\sin x}}$.

1787. (1) $\lim\limits_{x\to 2}\dfrac{x-2}{\sqrt{x+7}-3}$; (2) $\lim\limits_{x\to 0}\dfrac{\sqrt{4+x}-\sqrt{4-x}}{4x}$;

(3) $\lim\limits_{x\to 0}\dfrac{\sqrt{a+2x}-\sqrt{a-2x}}{x}$;

(4) $\lim\limits_{x\to \infty}\dfrac{\sqrt{1+4x^2}-\sqrt{x^2-1}}{\sqrt{1+4x^2}+\sqrt{x^2-1}}$;

(5) $\lim\limits_{x\to 0}\dfrac{x^2}{1-\sqrt[3]{1+x^2}}$;

(6) $\lim\limits_{x\to \frac{3\pi}{2}}\dfrac{\cos x}{\sin x+\sqrt{\sin^2 x+\cos x}}$.

1788. (1) $\lim\limits_{x\to -\infty}\dfrac{\sqrt{x^2+2}}{x-2}$; (2) $\lim\limits_{n\to \infty}\dfrac{3x^2+2}{x^4+1}$;

(3) $\lim\limits_{x\to\infty}\dfrac{2(x-1)^3}{x^3+1}$;

(4) $\lim\limits_{x\to\infty}\dfrac{2x^2-4x+1}{4x^2+6x-5}$; (5) $\lim\limits_{x\to\infty}\dfrac{2x^3+3x^2-1}{x^2-1}$;

(6) $\lim\limits_{x\to\infty}\dfrac{\sqrt{1+x^2}-\sqrt{4+x^2}}{x}$;

(7) $\lim\limits_{x\to\infty}\dfrac{\sqrt{1+9x^2}+\sqrt{x^2-1}}{\sqrt{1+9x^2}-\sqrt{x^2-1}}$.

1789. (1) $\lim\limits_{x\to0}\dfrac{\sin x}{x}$; (2) $\lim\limits_{x\to0}\dfrac{x}{\sin x}$; (3) $\lim\limits_{x\to0}\dfrac{\tan x}{x}$;

(4) $\lim\limits_{x\to0}\dfrac{\sin 2x}{x}$; (5) $\lim\limits_{x\to0}\dfrac{\sin\frac{x}{2}}{x}$; (6) $\lim\limits_{x\to0}\dfrac{x}{\sin\frac{x}{2}\cos\frac{x}{2}}$;

(7) $\lim\limits_{x\to0}\dfrac{\sin 4x}{\sin 3x}$; (8) $\lim\limits_{x\to0}\dfrac{2x}{\cos x}$; (9) $\lim\limits_{x\to0}\dfrac{\sin 2x}{5x}$.

1790. (1) $\lim\limits_{x\to0}\dfrac{\sin^2 2x}{x^2}$; (2) $\lim\limits_{x\to0}\dfrac{\sin^2 x}{x^3}$;

(3) $\lim\limits_{x\to0}\dfrac{\cos x-\cos 3x}{x^2}$;

(4) $\lim\limits_{x\to0}\dfrac{\sin 3x+\sin 4x}{x}$;

(5) $\lim\limits_{x\to1}(x-1)\tan\dfrac{\pi x}{2}$;

(6) $\lim\limits_{x\to0}\dfrac{1-\cos x}{2x^2}$;

(7) $\lim\limits_{x\to-1}\dfrac{1+x}{\cos\frac{\pi}{2}x}$.

1791*. (1) $\lim\limits_{x\to-3}\dfrac{x^2-9}{\arctan(x+3)}$;

(2) $\lim\limits_{x\to0}\dfrac{\sin 4x}{1-\sqrt{1-x}}$;

(3) $\lim\limits_{x\to\frac{\pi}{4}} \dfrac{\cos 2x}{\pi-4x}$; (4) $\lim\limits_{x\to 0} (x \cot 2x)$;

(5) $\lim\limits_{x\to 0} \dfrac{x}{\arcsin 2x}$; (6) $\lim\limits_{x\to 0} \dfrac{\sin x-\tan x}{x^3}$.

1792*. (1) $\lim\limits_{x\to\frac{\pi}{4}} \left[\cos 2x \tan\left(x+\dfrac{\pi}{4}\right)\right]$;

(2) $\lim\limits_{x\to\frac{\pi}{2}} \dfrac{\cos x+\cot x}{\pi-2x}$;

(3) $\lim\limits_{x\to 0} \dfrac{\sin 2x}{3-\sqrt{2x+9}}$;

(4) $\lim\limits_{x\to\frac{\pi}{4}} \dfrac{\cos 2x}{x(\tan x-1)}$.

1793*. (1) $\lim\limits_{n\to\infty} \dfrac{5^n-2}{5^{n+1}+3}$; (2) $\lim\limits_{n\to\infty} \dfrac{2+4+6+\ldots+2n}{1+3+5+\ldots+(2n+1)}$;

(3) $\lim\limits_{n\to+\infty} \dfrac{1+2+3+\ldots+n}{n^2-4}$;

(4) $\lim\limits_{n\to\infty} \left(n \sin \dfrac{\alpha}{n}\right)$.

1794. (1) $\lim\limits_{n\to\infty} \left(1+\dfrac{1}{n}\right)^n$; (2) $\lim\limits_{x\to 0} (1+x)^{\frac{1}{x}}$;

(3) $\lim\limits_{x\to-\infty} \left(1+\dfrac{1}{x}\right)^x$; (4) $\lim\limits_{n\to\infty} \left(1+\dfrac{2}{n}\right)^n$;

(5) $\lim\limits_{x\to 0} \sqrt[x]{1+3x}$; (6) $\lim\limits_{x\to\infty} \left(\dfrac{x-2}{x+2}\right)^{x+2}$.

Find the increments of the given functions:

1795. (1) $y = 2x-1$ for $x = 2$ and $\Delta x = 0.5$;

(2) $y = 2x^2$ for $x = 1$ and $\Delta x = 0.1$;

(3) $y = x^3$ for $x = -2$ and $\Delta x = 0.5$;

(4) $y = x^2-2x$ for $x = 1$ and $\Delta x = -0.2$.

1796. (1) $y = \frac{4}{x}$ for $x = 2$ and $\Delta x = -0.4$;

(2) $y = \sqrt{x}$ for $x = 1.44$ and $\Delta x = 0.25$;

(3) $y = \cos x$ for $x = \frac{\pi}{3}$ and $\Delta x = \frac{\pi}{15}$;

(4) $y = \frac{2.4}{\sqrt{3x}}$ for $x = 0.48$ and $\Delta x = 0.27$.

1797. 1. Find the increment of the function $y = 2x^2+4x$ for $x = -1.1$ and $\Delta x = 0.2$. Give a graphical explanation of the answer.

2. Same for the function $y = \sin 2x$ for $x = 0.7837$ and $\Delta x = 0.0034$.

1798. 1. Find the increment ΔS of the area of a square whose side $a = 12$ cm for $\Delta a = 0.4$ cm.

2. Find the increment ΔS of the area of a circle whose radius $R = 10$ cm for $\Delta R = 0.6$ cm.

3. Find the increment of the volume of a cube with the edge $a = 4$ dm if each of its edges is increased by 2 cm.

1799. Find the increments (in the general form) of the given functions:

(1) $y = x-1$; (2) $y = x^2+3x-4$; (3) $y = x^3-3x-4$;

(4) $y = \frac{x^2+1}{x-1}$; (5) $y = \log_{10} x$; (6) $y = \cos x$;

(7) $y = x \sin x$.

1800. 1. Given the function $y = f(x)$. Find $\frac{\Delta y}{\Delta x}$ if

(a) $y = 2x-3$ for $x = -3$ and $\Delta x = 0.5$;

(b) $y = x^2+1$ for $x = 2$ and $\Delta x = 0.2$;

(c) $y = x^2-2x$ for $x = 0.8$ and $\Delta x = 0.4$;

(d) $y = 4^x$ for $x = 0$ and $\Delta x = 0.5$.

2. What is the mechanical and the geometrical meaning of the ratio $\frac{\Delta y}{\Delta x}$?

1801. Find the ratio $\frac{\Delta y}{\Delta x}$ in the general form for the functions:

(1) $y = 0.5x+1$; (2) $y = x^2-1$; (3) $y = \frac{x+1}{x-1}$;

(4) $y = \sin x$; (5) $y = \log_{10} x$; (6) $y = 5$;

(7) $y = x^3 + 3x$.

1802. 1. Taken on the parabola $y = x^2$ are the points $A(1, y_1)$ and $B(3, y_2)$. Find the slope of the chord AB.

2. Given the curve $y = 2^x$ and points A and B on it whose abscissas are equal to -1 and 1, respectively. Find the slope of the chord AB.

1803. Prove that the following functions are continuous:

(1) $y = x$ at the point $x = -2$; (2) $y = x^2$ at the point $x = 0$;

(3) $y = \sin x$ at the point $x = \frac{\pi}{6}$;

(4) $y = \sqrt{25 - x^2}$ at the point $x = 3$;

(5) $y = 2^x$ at the point $x = 2$;

(6) $y = \frac{2x}{1+x^2}$ at the point $x = -1$.

1804. Prove that the following functions are continuous for any permissible real values of x:

(1) $y = x^3$; (2) $y = x^2 - x$; (3) $y = \cos 2x$;

(4) $y = e^x$; (5) $y = \sqrt{2x^2 + 16}$; (6) $y = \frac{x}{x^2+1}$;

(7) $y = \log_{10} x$.

1805. 1. Prove that the function $y = \frac{3x}{x^2-1}$ is continuous at all points except for $x = \pm 1$. Plot the graph of this function.

2. Same for the functions: (a) $y = \frac{2x-1}{x}$, $x \neq 0$; (b) $y = \frac{\sin x}{x}$, $x \neq 0$.

1806. Determine the points of discontinuity of the given functions:

(1) $y = \frac{2}{x-1}$; (2) $y = x^2 + \frac{3}{x+2}$; (3) $y = \frac{x^2+4}{x^2-4}$;

(4) $y = \tan x$; (5) $y = \frac{1}{2^x + 2^{-x}}$; (6) $y = \frac{\sin x}{x}$;

(7) $y = \log_{10} \sin x$; (8) $y = \frac{1}{\cos x}$.

70. THE SUM OF AN INFINITELY DECREASING GEOMETRIC PROGRESSION

Find the sums of the infinitely decreasing geometric progressions:

1807. (1) $1, \frac{1}{2}, \frac{1}{4}, \ldots$; (2) $32, 8, 2, \ldots$;

(3) $3\frac{1}{3}, \frac{2}{3}, \frac{2}{15}, \ldots$; (4) $1\frac{1}{2}, -\frac{3}{4}, \frac{3}{8}, \ldots$.

1808. (1) $9\sqrt{2}, 3\sqrt{2}, \sqrt{2}, \ldots$; (2) $-\frac{8}{9}, \frac{2}{3}, -\frac{1}{2}, \ldots$;

(3) $\sqrt{2}, 1, \frac{1}{\sqrt{2}}, \ldots$;

(4) $\sqrt{\frac{4}{3}}, -\sqrt{\frac{3}{4}}, \sqrt{\frac{27}{64}}, \ldots$.

1809. (1) $\sqrt{2}, \sqrt{\frac{1}{2}}, \frac{1}{4}\sqrt{2}, \ldots$;

(2) $\frac{\sqrt{3}+1}{\sqrt{3}-1}, \frac{\sqrt{3}-1}{\sqrt{3}+1}, \ldots$;

(3) $1, x, x^2, \ldots$ for $|x| < 1$;

(4) $\frac{\sqrt{2}+1}{\sqrt{2}-1}, -1, \frac{\sqrt{2}-1}{\sqrt{2}+1}, \ldots$;

(5) $1, \frac{1}{1,2}, \frac{1}{(1,2)^2}, \ldots$;

(6) $3, -1, \frac{1}{3}, \ldots$.

1810. 1. Find the ratio of an infinitely decreasing geometric progression, whose first term is equal to 35, and the sum of the progression to 21.

2. Find the sixth term of an infinitely decreasing geometric progression whose ratio is equal to 1/3, and its sum to 243.

1811. Find the first term of an infinitely decreasing geometric progression in which (1) the sum of the first five terms is equal to $31\frac{31}{32}$, and the sum of its all terms to 32;

(2) the sum of the progression is equal to $60\frac{3}{4}$, and its second term to -27;

(3) the sum of the progression is equal to 5.4, and its ratio to 2/9.

1812. 1. Find the sum of an infinitely decreasing geometric progression whose third term is equal to 4, and its ratio to 2/3.

2. The sum of an infinitely decreasing geometric progression is equal to 96, and the difference between the first and the second term to 24. Find this progression.

1813. The sum of an infinitely decreasing geometric progression is equal to 28, and the sum of its squared terms to 112. Find this progression.

1814*. Find the ratio of an infinitely decreasing geometric progression if the sum of its first six terms is seven times greater than the sum of its remaining terms.

1815*. 1. Form an infinitely decreasing geometric progression such that each of its terms is equal to 1/4 of the sum of all the terms standing after it.

2. Form an infinitely decreasing geometric progression such that each of its terms is three times the sum of all the terms standing after it.

1816. Represent the periodic decimal fraction in the form of the sum of all terms of an infinitely decreasing geometric progression and determine this sum: (1) 0.555...; (2) 0.161616...; (3) 0.123123...; (4) 13.777.... Formulate the rule for changing a pure periodic fraction to common fraction.

1817. Represent a mixed periodic fraction in the form of two parts so that one of them represents the sum of all terms of an infinitely decreasing geometric progression, and compute the sum of both parts:

(1) 0.4777...; (2) 0.34545...; (3) 0.17333...; (4) 2.5666....

Formulate the rule for changing a mixed periodic decimal fraction to common fraction.

1818. Find the sums of the following series:

(1) $\frac{2}{3}+\frac{4}{3^2}+\frac{2}{3^3}+\frac{4}{3^4}+\ldots$;

(2) $\frac{2}{5}+1+\frac{4}{25}-\frac{1}{2}+\frac{8}{125}+\frac{1}{4}+\frac{16}{625}-\frac{1}{8}+\ldots$.

1819. Solve the equations:

(1) $1+\log_2 \sin x+\log_2^2 \sin x+\log_2^3 \sin x+\ldots=\frac{2}{3}$;

(2) $2^{x+x^2+\cdots}=2\sqrt{2}$.

1820*. The line segment AB 4 cm long is bisected at point C, then AC is bisected at point D, CD is bisected at point E, DE is bisected at F, and so on. Determine the limit distance between the point of division and the point A.

1821. The line segment AB 5 cm long is bisected at point C, then BC is bisected at point D, CD is bisected at E, DE is bisected at F, and so on. Determine the limit distance of the division point from A.

1822. Inscribe a square into the square with the side a by connecting the midpoints of its sides with straight lines; in the square thus obtained a new square is inscribed in the same way, and so on without bound. Find the limit of the sum of the perimeters of these squares and the limit of the sum of their areas.

1823. In a regular triangle with the side of 10 cm a new triangle is inscribed by connecting the midpoints of the sides with straight lines. Then another triangle is inscribed in the obtained triangle using the same method, and so on without bound. Find the limit of the sum of the perimeters of these triangles and the limit of the sum of their areas.

1824. Given a regular triangle whose side is equal to a. The medians of this triangle form a new regular triangle, whose medians form another triangle, and so on without bound. Determine the limit of the sum of the areas of the triangles thus obtained, and the limit of the sum of their perimeters.

1825. The diagonal of a square is 80 cm long. The side of this square is taken as the diagonal of a new square, and the side of the latter as the diagonal of another square, and so on without bound. Determine the limit of the sum of the areas of all these squares.

1826. In a square with the side a a circle is inscribed; a square is inscribed in the circle, another circle is inscribed in this square and so on to infinity. Determine the limit of the sum of the areas of all the circles and the limit of the sum of the areas of all the squares.

1827. Inscribed in a regular triangle with the side a is a circle, a regular triangle in the circle, a circle in the triangle, and so on without bound. Determine the limit of the sum of the areas of all the circles and the limit of the sum of the lengths of all their circumferences.

1828. A point is taken on the side of an angle of 45° at a distance a from the vertex. A perpendicular is dropped from this point to

the second side; from the foot of this perpendicular a new perpendicular is drawn to the first side and so on without bound. Find the limit of the sum of the lengths of the perpendiculars.

1829. A point is taken on the side of an angle of 30° at a distance a from the vertex. A perpendicular is dropped from this point to the second side, from the foot of this perpendicular a new perpendicular is drawn to the first side and so on without bound. Find the limit of the sum of the lengths of the perpendiculars.

CHAPTER XX

The Derivative and Its Applications

71. THE DERIVATIVE DEFINED

1830. 1. Find the average velocity of a body moving according to the law $s = \frac{1}{2}t^2$ for the following intervals of time: (a) from $t_1 = 2$ to $t_2 = 4$; (b) from $t_1 = 4$ to $t_2 = 10$.

2. The equation of motion is $s = t^2 - 4t$. Find the average velocity for the following intervals of time:

(a) from $t_1 = 0$ to $t_2 = 2$; (b) from $t_1 = 1$ to $t_2 = 3$;

(c) from $t_1 = 2$ to $t_2 = 4$; (d) from $t_1 = 4$ to $t_2 = 8$.

1831. 1. Find the average velocity of a point moving according to the law $s = 4t + 2$ for the following intervals of time:

(a) from $t_1 = 1$ to $t_2 = 3$; (b) from $t_1 = 0.2$ to $t_2 = 0.3$.

2. Find the average rate of change of the function as x varies from $x_1 = 0$ to $x_2 = 4$:

(a) $y = x$; (b) $y = -\frac{1}{2}x + 2$; (c) $y = kx + b$.

1832. Find the average rate of change of the function:

(1) $y = 2x^2$ as x varies from $x_1 = -2$ to $x_2 = 0$;

(2) $y = x^3$ as x varies from $x_1 = -1$ to $x_2 = 1$;

(3) $y = x^2+x$ as x varies from $x_1 = 0$ to $x_2 = 3$;

(4) $y = x^2-3x+2$ as x varies from $x_1 = -4$ to $x_2 = -2$;

(5) $y = \sin x$ as x varies from $x_1 = \frac{\pi}{6}$ to $x_2 = \frac{\pi}{2}$.

1833. A point moves along a certain path from a fixed point so that the distance covered by it increases proportionally to the squared time. Knowing that at the time $t = 3$ the distance $s = 4.5$, determine the average velocity of the point over the interval from $t = 4$ to $t = 9$.

1834. 1. The law of motion is specified by the formula $s = \frac{1}{4}t^4$. Find: (a) the mean velocity of the motion within the time interval from $t_1 = 2$ to $t_2 = 6$; (b) the velocity at the moments $t_1 = 2$, $t_2 = 4$, $t_3 = 6$.

2. Same for the motion $s = \frac{1}{2}t^2$.

1835. Determine the rate of variation of the given functions at the indicated points:

(1) $y = 3x-1$ at point $x = 2$;

(2) $y = 2x^3$ at point $x = 0$;

(3) $y = x^2-3x$ at point $x = 3$;

(4) $y = \frac{1}{2}x^2+3x-1$ at point $x = 5$;

(5) $y = \frac{2}{x}$ at point $x = \frac{1}{2}$;

(6) $y = \sin x$ at point $x = \frac{\pi}{3}$;

(7) $y = \frac{x+1}{x-1}$ at point $x = -1$;

(8) $y = \sqrt{x}$ at point $x = 9$.

1836. 1. A point moves according to the law $s = t^2-4t+4$. At what time is its velocity equal to zero? to 6?

2. A point moves according to the law $s = t^3+2$. Find the formula for its velocity expressed as a function of time.

1837. The quantity of electric charge measured in coulombs changes with time according to the law $Q = 0.5t^2$, where t is time measured in seconds. Find the current after five seconds had elapsed.

1838. Computing $\lim\limits_{\Delta x \to 0} \dfrac{\Delta y}{\Delta x}$, find the derivatives of the following functions:

(1) $y = x^3$; (2) $y = x^4 + 2x$; (3) $y = \dfrac{2}{x}$; (4) $y = 2\sqrt{x}$;

(5) $y = 5$; (6) $y = \dfrac{2}{3}x^3 - 2x$; (7) $y = \sin 2x$;

(8) $y = \dfrac{x+1}{x-1}$.

Computing $\lim\limits_{\Delta x \to 0} \dfrac{\Delta y}{\Delta x}$, solve the following problems:

1839. 1. Given the function $f(x) = 3x^2$. Find $f'(0), f'(1), f'(-1), f'(3)$.

2. Given the function $F(x) = x^2 - 2x$. Find $F'(0)$, $F'(-1)$, $F'(3)$.

3. Given the function $y = 3x^2 - 2x + 5$. Find y' at the point $x = 2$.

1840. 1. Given the function $f(x) = x^3$. At what point $f(x) = f'(x)$?

2. Show that the function $f(x) = 3x^2$ satisfies the relation $f'(a+b) = f'(a) + f'(b)$.

3. Find the slope of the tangent to the curve $y = -\dfrac{1}{3}x^3$ at point $x = 1$.

1841. 1. A point moves in a straight line and for t seconds covers s metres, where $s = t^3 + 3t^2$. Find its velocity at the time $t = 4$ s.

2. A body performs rectilinear motion according to the formula $s = \dfrac{2}{t} + 3t^2$. Derive the formulas for the velocity and acceleration of this motion.

3. Find the velocity and acceleration at the given time for a point moving in a straight line according to the law expressed by the formula:

(a) $s = t^3 - 0.5t^2$, $t = 2$; (b) $s = t^2 - 2t + 1$, $t = 3$;

(c) $s = \cos 2t$, $t = \dfrac{\pi}{4}$; (d) $s = 2 \sin t$, $t = \dfrac{\pi}{6}$.

72. DIFFERENTIATING FUNCTIONS

Find the derivatives of the following functions:

1842. (1) $y = 3$; (2) $y = x$; (3) $y = 2x$; (4) $y = -x^2$;

(5) $y = x^5$; (6) $y = \frac{1}{4}x^4 - \frac{1}{3}x^3 + \frac{1}{2}x^2 + 3$;

(7) $y = \frac{3}{4}ax^8$.

1843. (1) $y = 2x^3 - 3x$; (2) $y = x(x+1)$;

(3) $y = 2x^{-4} + 5$; (4) $s = \frac{1}{4}t^4 - \frac{1}{2}t^2 + t$;

(5) $y = \frac{1}{3}x^{3n} - \frac{1}{2}x^{2n}$.

1844. (1) $y = \frac{x^4}{a} - \frac{x^2}{b}$; (2) $s = v_0 t + \frac{at^2}{2}$;

(3) $s = t^2(3t+2)$; (4) $s = t(t+1)(t+2)$;

(5) $y = \frac{3}{x^2} - \frac{4}{x}$; (6) $y = \frac{1}{x^n}$.

1845. (1) $y = \frac{1}{x}$; (2) $y = \frac{a}{x^5} + \frac{b}{x^3}$;

(3) $y = \frac{1}{x^3} + \frac{1}{x^2} + \frac{1}{x}$;

(4) $y = \frac{x^5 - 2x^3 + x^2 - 2x + 1}{x^3}$;

(5) $y = 2x^{\frac{3}{2}} - 3x^{\frac{2}{3}} + 6x^{\frac{1}{3}}$.

1846. (1) $y = 6x^{\frac{4}{3}} + 5x^{\frac{6}{5}} + 2x^{\frac{3}{2}}$;

(2) $y = 2x^{-\frac{1}{2}} - 3x^{-\frac{2}{3}}$;

(3) $y = 10x^{-\frac{1}{2}} - 5x^{-0.4} + 10x^{-0.2}$;

(4) $y = 4x^{-\frac{1}{2}} + 3x^{-\frac{2}{3}}$.

1847. (1) $y = 2\sqrt{x} - 3\sqrt[3]{x}$; (2) $y = \sqrt[3]{x^2} + 2\sqrt{x} - 4$;

(3) $f(x) = x\sqrt{x} + 3x^2\sqrt[3]{x^2}$; (4) $f(x) = \frac{2}{\sqrt[3]{x^2}} - \frac{5}{2\sqrt{x}}$.

1848. (1) $y = \frac{ax^3}{\sqrt{x}} - \frac{b}{x\sqrt{x}} + \frac{c}{\sqrt{x}}$;

(2) $y = x\sqrt{x}\sqrt[3]{x}$; (3) $s = \frac{6t\sqrt[3]{t}}{\sqrt{t}}$;

(4) $s = t(\sqrt{t}+1)$; (5) $y = -\frac{\sqrt{x}}{\sqrt[3]{x^2}}$.

1849. 1. Given the function $f(x) = x^5 - 2x^3 + 3x - 4$. Show that $f'(x)$ is an even function.

2. Given the function $f(x) = x^4 - 2x^2 + 1$. Show that $f'(x)$ is an odd function.

1850. 1. For what values of x does the derivative of the function $y = x^3 + 3x^2 - 24x$ vanish?

2. A tangent line, parallel to the x-axis, is drawn to the curve $y = 2x^2 - 8x + 5$. Find the coordinates of the point of tangency.

1851. 1. Prove that the curves $y = x^2$ and $y = 4\sqrt{x} - 3$ have a common tangent at the point (1, 1).

2. In the equation of the parabola $y = x^2 + bx + c$ determine b and c if the parabola touches the straight line $y = -x$ at point with the abscissa $x = -2$.

1852*. 1. On the curve $y = 2x^3 - 3x^2 + 3x - 2$ find the points at which the tangents to the curve are parallel to the straight line $3x - y + 2 = 0$.

2. Find the angles at which the straight line $x - y - 1 = 0$ intersects the parabola $y = x^2 - 2x + 1$.

1853. 1. A point performs rectilinear motion according to the law $s = 2\sqrt{t}$. Find its velocity at the time $t = 6.25$.

2*. A point moves in a straight line according to the law $s = \frac{1}{4}t^4 - 4t^3 + t^2$.

(a) At what times was the point at the initial position?

(b) At what times was its velocity equal to zero?

(c) How do you interpret the positive (negative) value of velocity?

(d) Give the characteristic of motion performed by the point, using the instants when $s = 0$ and when $v = 0$; the intervals of time when $s > 0$, $s < 0$, $v > 0$, $v < 0$.

1854. The distance s, in metres, covered by a body performing rectilinear motion during t seconds is specified by the equation $s = \frac{1}{3}t^3 - 2t^2 + t$. Determine the velocity and the acceleration at

$t = 3$s. Explain the meaning of the negative velocity and positive acceleration at the given time.

1855. The velocity of a point moving in a straight line is determined by the formula $v = 2t^2 - 3t$, where t is time in seconds and s is distance in metres. Determine the acceleration of the point at $t = 0$, $t = 2$, $t = 3$. At what time is the acceleration equal to zero? How do you interpret the positive (negative) value of acceleration?

1856. A bullet is fired from a pistol upwards with a velocity of 360 m/s. Find the velocity of the bullet at $t = 10$ s, and determine the time during which it flies in the upward direction. Air friction may be neglected. The equation of motion is $h = v_0 t - 4.9t^2$.

1857. From a hill 50 m high a ball is thrown upwards in the vertical direction with a velocity of 30 m/s. Find (a) its velocity after 2 seconds of climb, (b) the time at which it starts falling, (c) the maximum altitude reached with respect to the Earth's surface.

1858. A body whose mass $m = 5$ kg moves in a straight line according to the law $s = 1 - t + t^2$, where s is in metres, and t in seconds. Find the kinetic energy $\frac{mv^2}{2}$ of the body in ten seconds after its start.

1859. The angle φ through which the pulley turns is determined by the formula $\varphi = t^2 + 3t$. Find (a) the mean angular velocity within the time interval from $t = 2$ to $t = 5$, (b) the angular velocity at $t = 5$.

1860. A wheel rotates so that the angle through which it turns is proportional to the cube of time. The first revolution was completed by the wheel during two seconds. Determine the angular velocity of the wheel in 4 seconds after it started rotating.

Find the derivatives of the following functions.

1861. (1) $y = 5(x^3 + 2x)$; (2) $y = (4x - 2)(x^2 + 3x - 2)$;
(3) $y = (4x^2 + 4x + 1)(x^2 - 4x)$;
(4) $y = x^3(x^2 + 4)(x - 2)$.

1862. (1) $y = (x^2 + a^2)(x^2 - a^2)(x^2 - 2)$;
(2) $y = \sin x \cos x$;
(3) $y = (x^2 - 1)(x^2 - 2)(x^2 - 3)(x^2 - 4)$;
(4) $y = e^x \ln x$.

1863. (1) $y = x^2 \ln x$; (2) $y = x^4 2^x$; (3) $y = 3^x(x+1)$;
(4) $y = a^x e^x$; (5) $y = (x^2-4x+1)\tan x$.

1864. (1) $y = \frac{x-1}{2}$; (2) $y = \frac{5}{x+1}$; (3) $y = \frac{x+1}{x}$;
(4) $y = \frac{x^2+1}{x^2-1}$; (5) $y = \frac{x+a}{x-a}$; (6) $y = \frac{x^3}{x+2}$.

1865. (1) $y = \frac{2x^2-3x+1}{x+1}$; (2) $y = \frac{3x^2+2x-1}{2x+1}$;
(3) $y = \frac{x^2-x+2}{x^3-1}$; (4) $y = -\frac{1+3x}{x(1+x)}$.

1866. (1) $y = \frac{2x}{1-x^2} + \frac{1}{x}$; (2) $y = \frac{1-\sqrt{x}}{1+\sqrt{x}}$;
(3) $y = \frac{\ln x+2}{x}$; (4) $y = \frac{4\ln x}{1+\ln x}$.

1867. (1) $y = \frac{\ln x}{e^x}$; (2) $y = \frac{2^x}{1-\ln x}$; (3) $y = \frac{e^x}{e^x-2}$;
(4) $y = \frac{5^x}{\ln x}$; (5) $y = \frac{\sin x}{x}$; (6) $y = \frac{x}{\tan x}$.

1868. (1) $y = \frac{1-\sin x}{1+\sin x}$; (2) $y = \frac{x+e^x}{x-2^x}$;
(3) $y = \frac{x}{\cos x}$; (4) $y = \frac{e^x 3^x}{2+\ln x}$.

1869. 1. Given the function $f(x) = \frac{2a-x}{2a+x}$. Find $f'(a)$, $f'(0)$.
(2) Given the function $f(x) = \frac{e^x+1}{e^x-1}$. Find $f'(1)$, $f'(-1)$, $f'(0)$.

Find the derivatives of the following functions:

1870. (1) $y = (x+1)^4$; (2) $y = (3x-2)^3$;
(3) $y = (1-x^2)^5$; (4) $y = (x^2+2x)^4$;
(5) $s = (t^2-t+1)^4$.

1871. (1) $y = \frac{1}{(x^2+1)^3}$; (2) $y = \frac{5}{(2x-3)^5}$;
(3) $y = \frac{2}{(x^2-2x^3)^7}$; (4) $y = \sqrt{x^2+1}$;
(5) $y = \sqrt[3]{2x-1}$; (6) $y = \sqrt{1-2x-x^2}$.

1872. (1) $s = \sqrt[3]{t^3-1}$; (2) $y = \sqrt{(2+x)(3-2x)}$;

(3) $y = \sqrt[3]{x^3-3x^2}$; (4) $y = \sqrt{\frac{3}{x}}$.

1873. (1) $y = (2x-1)^2(x+1)^3$; (2) $s = t^2\sqrt{2t-3}$;

(3) $y = x\sqrt{x^2-1}$; (4) $y = (x-1)\sqrt{x^2+1}$.

1874. (1) $y = (2x+1)^2\sqrt{x-1}$; (2) $y = x^2\sqrt[3]{x^3+1}$;

(3) $y = \frac{1}{\sqrt{2x}}$; (4) $y = \sqrt{3x} - \frac{1}{\sqrt{3x}}$.

1875. (1) $y = \sqrt[4]{(1+x^2)^3}$; (2) $y = \sqrt{x+\sqrt{x}}$;

(3) $y = \left(1+\sqrt[3]{x^2}\right)^3$; (4) $y = \sqrt[3]{1+\sqrt{x}}$.

1876. (1) $y = \frac{4x^3}{(3x-2)^3}$; (2) $y = \sqrt{\frac{3}{2x^2-1}}$;

(3) $y = \frac{2x}{\sqrt{x+1}}$; (4) $y = \frac{4x-2}{\sqrt{x^2+1}}$.

1877. (1) $y = \frac{x}{\sqrt{a^2+x^2}}$; (2) $y = \frac{\sqrt{a^2+x^2}}{x}$;

(3) $y = \frac{\sqrt{x+1}}{\sqrt{x-1}}$; (4) $y = \sqrt{\frac{x^2+1}{x^2-1}}$.

1878. (1) $y = \frac{x^2+a^2}{\sqrt{x^2-a^2}}$; (2) $y = \frac{x}{x-\sqrt{a^2+x^2}}$;

(3) $y = \frac{\sqrt{a+x}-\sqrt{a-x}}{\sqrt{a+x}+\sqrt{a-x}}$; (4) $y = \frac{1}{\sqrt{a^2+x^2}-x}$.

1879. (1) $y = \ln(x+1)$; (2) $y = \ln(2x+1)$;

(3) $y = \ln(3-2x)$; (4) $y = \ln(x^2-2x+1)$;

(5) $y = \ln(3+2x)$.

1880. (1) $y = \log_{10} 4x$; (2) $y = \log_{10}(4x^2+1)$;

(3) $y = \ln\frac{1}{x}$; (4) $y = \ln^2 x$; (5) $y = \ln x^2$;

(6) $y = \ln(1+\ln x)$.

1881. (1) $y = \ln\frac{1-x}{1+x}$; (2) $y = \ln\frac{1+x^2}{1-x^2}$;

(3) $y = x \ln x$; (4) $y = \ln(x-\sqrt{x})$;

(5) $y = \ln\ln x$; (6) $y = \ln\frac{x^2}{1+x^2}$.

1882. (1) $y = \ln\sin x$; (2) $y = \ln\cos x$; (3) $y = \ln\tan x$;

(4) $y = \ln\cot x$; (5) $y = \ln\sin 2x$;

(6) $y = \ln\cos 5x$.

1883. (1) $y = \ln^2 x^2$; (2) $y = \ln\sqrt{x^2+3x+1}$;

(3) $y = \ln(x+\sqrt{x^2-1})$;

(4) $y = \ln(x^2+\sqrt{x^4-a^4})$.

1884. (1) $y = \frac{\ln x}{x^2}$; (2) $y = \ln\frac{x+\sqrt{1+x^2}}{x}$;

(3) $y = \ln(1+x+\sqrt{2x+x^2})$;

(4) $y = \ln\frac{1-\sqrt{x}}{1+\sqrt{x}}$.

1885. (1) $y = \frac{1}{2}\ln\sqrt{\frac{1+x}{1-x}}$; (2) $y = \ln\frac{\sqrt{x^2+1}+x}{\sqrt{x^2+1}-x}$;

(3) $y = \ln\sqrt{x^2+a^2} + \frac{2}{x^2+a^2}$;

(4) $y = \ln\left(1+\frac{1}{x}\right)$.

1886. (1) $y = x+\sin x$; (2) $y = \sin 3x$;

(3) $y = \sin^3 x$; (4) $y = x\sin x$;

(5) $y = \cos\frac{1}{x}$; (6) $y = \tan(2x+1)$.

1887. (1) $y = \cot x+x$; (2) $y = (1+\cos x)\sin x$;

(3) $y = \tan\sqrt{x+1}$; (4) $y = \sin\sqrt{1-x^2}$;

(5) $y = \cot\frac{x}{3}$.

1888. (1) $y = \cos^3 2x$; (2) $y = \cot^2 3x$; (3) $s = a \sin \frac{\pi}{2} t$;

(4) $y = \cos \frac{t}{a}$; (5) $y = \sin 2x \cos 3x$;

(6) $y = x \cos 2x$.

1889. (1) $y = \sin^3 2x \cos^2 2x$; (2) $y = 4 \tan 2\sqrt{x}$;

(3) $y = \sqrt{\cos 2x}$; (4) $y = \frac{1}{\sqrt{\sin x}}$;

(5) $y = \frac{1}{3} \tan^3 \frac{1}{x}$.

1890. (1) $y = \frac{1+\cos 2x}{\cos 3x}$; (2) $y = \frac{\tan x - 1}{\cos^2 x}$;

(3) $y = \frac{\sin^2 2x}{\cos 3x}$; (4) $y = \sqrt{\frac{1+\sin x}{1-\sin x}}$.

1891. (1) $y = \frac{1}{3} \cos^3 x - \cos x$;

(2) $y = \frac{1}{3} \tan^3 x - \tan x + x$;

(3) $y = \frac{1}{5} \cot^5 x - \frac{1}{3} \cot^3 x + \cot x + x$.

1892. 1. Given the function $f(x) = \tan^4 x$. Find $f'(0)$, $f'\left(\frac{\pi}{4}\right)$.

2. Given the function $f(x) = \frac{1+\sin 2x}{1-\sin 2x}$. Find $f'(0)$, $f'\left(\frac{\pi}{6}\right)$.

1893. A point performs rectilinear motion according to the law $s = a \cos \frac{\pi}{2} t$.

Find: (1) the formula for the velocity of motion; (2) the instants at which the velocity is equal to zero; (3) the acceleration of the point at the time $t = 2$.

73. INCREASE AND DECREASE OF A FUNCTION

Determine the intervals of increase and the intervals of decrease for the given functions:

1894. (1) $y = 2x^2 + 3$; (2) $y = 3x^2 - 6x + 1$;

(3) $y = 2 - 3x - 5x^2$; (4) $y = -x^2 + 4x - 1$;

(5) $y = 10x^2-20x+5$; (6) $y = 6-12x-18x^2$.

1895. (1) $y = 2x^3+3x^2-12x+5$;

(2) $y = 24x^3+3x^2-3x+7$;

(3) $y = \frac{x^3}{3}+\frac{7x^2}{2}+12x+6$;

(4) $y = -\frac{1}{3}x^3+5x^2+12$;

(5) $y = 3x^4+8x^3-18x^2+1$;

(6) $y = 3x^4-4x^3+6x^2-12x+12$.

1896*. (1) $y = \sqrt{x^2-3x+1}$; (2) $y = \sqrt[3]{x^3+6x^2-5}$;

(3) $y = \sin x-\frac{1}{2}x$; (4) $y = \cos x+\frac{x}{\sqrt{2}}$;

(5) $y = \ln(x^2+3x-2)$; (6) $y = \ln(7-4x-3x^2)$.

1897. (1)* $y = \frac{x^2+1}{x^2-1}$; (2)* $y = \frac{x^2-1}{x}$;

(3) $y = \frac{1-x+x^2}{1+x+x^2}$; (4)* $y = \frac{10}{4x^3-9x^2+6x}$;

(5) $y = x-e^x$; (6)* $y = x-\tan x$.

1898. Show that the given functions are monotone throughout the x-axis. Which of them are increasing functions and which are decreasing ones?

(1) $y = \frac{1}{3}x^3-x^2+5x-3$; (2) $y = x^3+x^2+x+5$;

(3) $y = x^3+x+15$; (4) $y = 6-6x+3x^2-2x^3$;

(5) $y = \cos x+\frac{\pi}{2}x$; (6) $y = \sin x-\frac{\pi}{2}x$.

1899. 1. Analyse the behaviour of the function $y = \sin x+\cos x$ at the points:

(a) $x = 0$; (b) $x = -\frac{\pi}{3}$; (c) $x = \frac{\pi}{3}$.

2. Analyse the behaviour of the function $y = \sin 2x+\cos^2 x$ at the points:

(a) $x = -\frac{\pi}{2}$; (b) $x = 0$; (c) $x = \frac{\pi}{12}$.

3. Prove that the function $y = x(1+2\sqrt{x})$ increases throughout the domain of definition.

4. Find out the behaviour of the function $y = e^{kx}$ depending on the sign of k.

74. FINDING MAXIMA AND MINIMA OF FUNCTIONS

Using the first derivative find the extrema of the given functions:

1900. (1) $y = x^2-5x+3$; (2) $y = 3x^2+x-1$;

(3) $y = \frac{1}{2}x^2+3x$; (4) $y = x-x^2$;

(5) $y = \frac{1}{3}x^3+2$; (6) $y = 3+12x-2x^2$.

1901. (1) $y = 2x^3-3x^2$; (2) $y = 2x^3-6x^2-18x+2$;

(3) $y = x^3-3x^2+3x+2$;

(4) $y = 2x^3+3x^2-12x+5$;

(5) $y = 3x^4-16x^3+24x^2-9$;

(6) $y = (x-1)^2(x-2)^2$.

1902. (1) $y = \frac{3x^2+4x+4}{x^2+x+1}$; (2) $y = \frac{x^2+6x+3}{3x+4}$;

(3) $y = \frac{x^2-5}{x-3}$; (4) $y = \frac{x^2-5x+9}{x-5}$;

(5) $y = \frac{4}{x-3}-\frac{16}{x-7}$; (6) $y = \frac{25}{7-x}-\frac{9}{3-x}$.

1903. (1) $y = x^2\sqrt{x^2+2}$; (2)* $y = \sin x+\cos x$;

(3) $y = x \ln x$; (4) $y = xe^x$;

(5)* $y = \sin 2x-x$; (6) $y = 3-2\sqrt[3]{x^2+1}$.

1904. Find the second derivatives of the following functions:

(1) $y = x^7+x^3+2x+5$; (2) $y = x(x+1)^3$;

(3) $y = (x+1)^2\sqrt{x}$; (4) $y = x^2 \sin x$;

(5) $y = \frac{x+1}{x-1}$; (6) $y = \frac{1}{1+x^3}$.

(7) $y = \sin^2 x$; (8) $y = x \ln x$;

(9) $y = e^{\sqrt{x}}$; (10) $y = \ln \sqrt{x^2+1}$.

1905. 1. Find $f''\left(\frac{\pi}{4}\right)$ if $f(x) = \ln \sin x$.

2. Find $f''(0)$ if $f(x) = \sin\left(x^2 - \frac{\pi}{3}\right)$.

3. Show that the function $y = e^x \sin x$ satisfies the equation $y''-2y'+2y = 0$.

4. Show that the function $y = \frac{1}{6}x^3 - \sin x + 2x - 3$ satisfies the equation $y'' - x - \sin x = 0$.

1906. A material point performs motion in a straight line according to the law $s = \frac{2}{3}t^3 - 2t + 3$. Find the acceleration of the point after four seconds of travel (s is expressed in metres, t in seconds).

1907. Find the velocity, the acceleration and the distance covered by a point moving in a straight line according to the indicated law (s is expressed in metres, t in seconds):

(1) $s = \frac{1}{4}t^4 + \frac{1}{2}t^2 + 5$ for $t = 2$;

(2) $s = e^{3t} - e^{-3t}$ for $t = 0$;

(3) $s = 4 \sin \frac{\pi t}{2} + 6$ for $t = 5$.

1908. A point moves in a straight line according to the law $s = \sqrt{t}$. Show that the motion is decelerated.

1909. A point performs rectilinear oscillatory motion according to the law $s = A \sin \omega t$. Determine the velocity and the acceleration of the motion at time $t = \frac{2\pi}{\omega}$. Show that the acceleration of the motion is proportional to the deflection of s.

1910. With the aid of the second derivative analyse for extremum the following functions:

(1) $y = 2x^2 - 3x + 3$; (2) $y = -3x^2 + x - 1$;

(3) $y = x^3 + 2x^2 - 4x + 1$; (4) $y = x^4 + 3x^3 + 5x^2 + 2$;

(5) $y = \frac{1}{4}x^4 + 3$; (6) $y = \frac{1}{5}x^5 - x^3 + 2x + 3$;

(7)* $y = x \sin x + \cos x$; (8) $y = x - \ln(1+x^2)$;

(9)* $y = e^x \sin x$; (10) $y = x + \frac{1}{x}$.

1911. Find the greatest and the least values of the following functions on the given interval:

(1) $y = -3x^4 + 6x^2 - 1$, $[-2, 2]$;

(2) $y = x^5 - 5x^4 + 5x^3 + 1$, $[-1, 2]$;

(3) $y = x^3 - 3x^2 + 6x - 2$, $[-1, 1]$;

(4) $y = \sqrt{100 - x^2}$, $[-6, 8]$;

(5) $y = \sin 2x - x$, $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$;

(6) $y = 2 \sin x + \sin 2x$, $\left[0, \frac{3\pi}{2}\right]$;

(7) $y = 2 \sin x + \cos^2 x$, $\left[0, \frac{\pi}{2}\right]$.

1912. What positive number added to its reciprocal yields the smallest sum?

1913. Divide the number 12 into two parts so that (1) their product is a maximum; (2) the sum of twice the first part and the square of the second part is a minimum; (3) the sum of the squares of the two parts is a minimum.

1914. Identical squares are to be cut out of the corners of an 18×18 cm square of sheet of tin so as (by bending the edges of the sheet) to obtain a box of maximum capacity. What is the side of the square required to be cut?

1915. A rectangular sheet of tin has the dimensions 48×30 cm. Obtain a maximum-capacity box by cutting out equal squares at the corners of the sheet and then bending the edges.

1916. Demonstrate that of all rectangles that can be inscribed in a circle of radius R the figure of largest area is that of a square.

1917. Find the dimensions of a rectangular plot of land of 36 square metres such that it has the smallest possible perimeter.

1918. Show that of all rectangles of a given perimeter the square has the smallest diagonal.

1919. Determine the largest area of a rectangle, one side of which lies on the base a of a given triangle and two vertices lie on the two lateral sides of the triangle, if the altitude of the triangle is h.

1920. What must the depth of a conic-shaped funnel with generatrix 20 decimeters be for the volume to be a maximum?

1921. It is required to fence in a rectangular plot of land, one side of which adjoins a house. The plot must have a definite area of S square metres. What must the ratio of the sides be if we want a fence of shortest length?

1922. A window has the shape of a rectangle with a semicircle on top. The perimeter of the window is equal to P. What must the ratio be between the width and height for the area of the window to be a maximum?

1923. A reservoir with square bottom and open top has to be coated with lead. What must the ratio be between the side of the bottom and the height of the reservoir so as to require a minimum amount of lead?

1924. What are the smallest dimensions of a cylindrical tank (covered at top and bottom) for a given capacity V?

1925. An antiaircraft gun is fired vertically upwards and the shell has an initial velocity of $v_0 = 200$ metres per second. How long will it take to reach its maximum altitude?

1926. The strength of a beam of rectangular cross section is known to be proportional to the width and the square of the thickness. Find the width of a rod of greatest strength that can be cut out of a log 16 cm in diameter.

1927. A material point is in motion in a straight line by the law $s = 6t^2 - t^3$. What is the highest speed of the point?

75. CONSTRUCTING GRAPHS OF FUNCTIONS

In problems 1928 to 1930 it is required to investigate the function and construct its graph. To do this it is advisable to do the following.

1. Determine the intervals over which the function exists and is continuous. Find the points of discontinuity of the function, if such occur, and determine the behaviour of the function in the neighbourhood of each point of discontinuity.

2. Determine whether the function is even or odd, that is, whether the graph is symmetric or not about the axis of ordinates or the origin.

3. Find the points of intersection of the graph of the function and the axes and also the intervals of constant sign.

4. Determine the behaviour of the function when $x \to +\infty$ and when $x \to -\infty$.

5. Find the points at which the function reaches its extremum and compute the extremum values of the function. Determine the intervals of increase and decrease of the function.

6. Construct the graph of the function using all the information gathered. Would it turn out that the available information is still not sufficient to plot the graph, it is necessary to find some more points belonging to it.

1928. (1) $y = x^3 - 3x^2$; (2) $y = x^2 - \frac{1}{4}x^4$;

(3) $y = (x-2)^2(x+1)$; (4) $y = (x+3)^2x^2$.

1929. (1) $y = \frac{1}{2+x}$; (2) $y = \frac{2}{x^2-4}$; (3) $y = \frac{2}{x^2+4}$,

(4) $y = x^2 + \frac{1}{x}$.

1930. (1) $y = x\sqrt{3-x}$; (2) $y = \frac{1}{\sqrt{2-x}}$;

(3) $y = \sin x + \cos x$; (4)* $y = \cos x - \cos^2 x$.

1931. Find the approximate value (accurate to 0.01) of the roots of the following equations:

(1) $x^2 - 2 = 0$; (2) $x^3 - 2 = 0$;

(3) $x^3 + x - 3 = 0$; (4) $x^3 - 2x - 5 = 0$;

(5) $x^3 + 3x + 1 = 0$; (6) $x^3 + 2x - 11 = 0$;

(7) $x^3 + x - 1 = 0$; (8)* $x^3 - 6x + 2 = 0$.

76. DIFFERENTIAL OF A FUNCTION

1932. Compute the increments and the differentials of the given functions:

(1) $y = x^2 + x$ for $x = 1$ and $\Delta x = 0.04$;

(2) $y = x^3 - 3x$ for $x = 1$ and $\Delta x = 0.01$;

(3) $y = x^3 + 2x$ for $x = -2$ and $\Delta x = 0.02$.

1933. Compute the increments and the differentials of the given functions. Find the absolute and the relative errors due to replacing the increment by the differential:

(1) $y = x^3-2x^2+5$ for $x = 3$ and $\Delta x = 0.01$;

(2) $y = \sqrt{x}$ for $x = 9$ and $\Delta x = 0.61$;

(3) $y = \dfrac{x}{\sqrt{x^2+9}}$ for $x = 4$ and $\Delta x = 0.02$.

1934. When heated, a cube, whose edge $x = 10$ cm, increases each of its edges by 0.02 cm. By what magnitude is its volume increased (give an approximate solution)?

Find the differentials of the following functions:

1935. (1) $y = 3x^4$; (2) $y = x^3-2x^2+3x-4$;

(3) $y = (x^2+a^2)^3$; (4) $y = \sqrt[3]{x^2}$;

(5) $y = \sqrt[3]{x}+\sqrt[4]{x}$.

1936. (1) $y = \sqrt{x^2-a^2}$; (2) $y = (x+1)\sqrt{1-x}$;

(3) $y = \dfrac{x+2}{x-2}$; (4) $y = \dfrac{\sqrt{1+x^2}}{x}$.

1937. (1) $y = \ln(x^2+2)$; (2) $y = x^2 \ln x$;

(3) $y = \sin 3x^2$; (4) $y = \cos^2 3x$;

(5) $y = x-\dfrac{1}{2}\sin 2x$.

1938. (1) $y = \tan(1-x^2)$; (2) $y = \ln(\sin x-\cos x)$;

(3) $y = \dfrac{1+\cos 2x}{\sin 2x}$; (4) $y = \ln\sqrt{4-x^2}$.

1939. Compute approximately the increment of the function $y = x^3-4x^2+60$ as the argument varies from $x = 4$ to $x = 4.1$.

1940. The side of a square is 8 cm long. Find the approximate increment of its area due to an increase in the length of the side equal to 0.02 cm.

1941. Find the approximate increment of the area of a circle if its radius changes from 20 cm to 20.1 cm.

1942. The edge of a cube equal to 1 m becomes 5 cm longer. What is the corresponding increase in the volume of the cube?

1943. When heated, a cube with the edge $a = 20$ cm increased its volume by 3 per cent. Find the elongation of its edge.

1944. A sphere of radius $R = 10$ cm was heated and its volume increased by 32.4π cm^3. Find the elongation of its radius.

1945. The volume of a cube increased by 6%. Find the elongation (in per cent) of its edge.

1946. Find the error (in per cent) of computing the area of a circle, if its radius was measured with an error of 1%?

1947. Evaluate approximately the following functions:

(1) $y = x^2 - x$ for $x = 2.01$;

(2) $y = x^3 - x^2 + 2x$ for $x = 3.03$;

(3) $y = x^3 - 4x^2 - 1$ for $x = -4.03$;

(4) $y = \dfrac{x}{\sqrt{x^2+9}}$ for $x = 4.1$.

CHAPTER XXI

Review Problems

1948. 1. Two cyclists start simultaneously, to meet each other, from points A and B which are 50 km apart. After two hours of cycling they met and, without making a stop, continued their way. The first cyclist arrived in B an hour and 40 minutes earlier than the second cyclist reached A. Find the speed of each cyclist, and the distance covered by each of them by the instant they met.

2. Prove the inequality $\sqrt{\frac{a^2}{b}}+\sqrt{\frac{b^2}{a}} \geqslant \sqrt{a}+\sqrt{b}$, where $a > 0$, $b > 0$.

3. Simplify the expression $\left[\frac{(a^{\frac{3}{2}}-1)(\sqrt{a}+1)}{a+\sqrt{a}+1}\right]^2+\sqrt{(a^2-1)^2+4a^2}$.

4. Solve the equations:

(a) $|x| = x$; (b) $|x| = -x$;

(c) $\frac{x}{2(x+2a)}-\frac{5a^2}{2(x^2-4a^2)}=\frac{x}{2a-x}$.

5. Find the modulus and the argument of the number z if

(a) $z = -2i(1-i)^2(1+i\sqrt{3})^3$; (b) $z=\frac{(1+i\sqrt{3})^2}{4i(1-i\sqrt{3})}$.

1949. 1. Find the four numbers of which the first three form a geometric progression, and the last three an arithmetic progression.

The sum of the first three numbers is equal to 21, and that of the last three to 36.

2. Solve the following inequalities:

(a) $\frac{2x-1}{4x+3} > 0$; (b) $\frac{2x-1}{4x+3} < 1$;

(c) $|x+4| < 3$; (d) $x^2 > x^4$; (e) $|\tan x| > \sqrt{3}$;

(f) $|\sin 2x| < \frac{\sqrt{2}}{2}$; (g) $\log_2 \frac{1-2x}{1-x} < 0$;

(h) $0.3^{\log_{\frac{1}{2}} \frac{2x-1}{3x+2}} > 1$.

3. Find the real roots of the following equations:

(a) $\sqrt{x+1}+\sqrt{x-1} = 2$; (b) $\sqrt{x-2}-\sqrt{x+2} = 2$;

(c) $5\sqrt{2x-2}-\sqrt{5x+1} = 3\sqrt{x+1}$;

(d) $\sqrt{4-x}-\sqrt{x-5} = 1$;

(e) $x^2-\sqrt{2x^2-3x+2} = 1.5(x-1)$;

(f) $\sqrt{4x^2-5x+10}-\sqrt{4x^2-5x-1} = 1$;

(g) $\sqrt{\frac{x+2}{x-2}}-3\sqrt{\frac{x-2}{x+2}} = -3\frac{5}{6}$;

(h) $\sqrt[3]{x+3} = \sqrt[3]{x-16}+1$.

4. Solve the following equations:

(a) $\tan x - \frac{1}{\cos^2 x} = 3$; (b) $\sin 4x - \sin x = 0$;

(c) $\sin 3x+\sin 4x+\sin 5x = 0$;

(d) $\cos x - \sin x = \cos 2x$.

5. Find the modulus and the argument of the number (a) opposite to the number $1-i\sqrt{3}$, (b) conjugate to this number, (c) reciprocal to it.

1950. 1. Four positive numbers form a geometric progression. The product of the first number by the fourth one equals the greater root of the equation $x^{2+\log_{10} x} = (0.001)^{-2\frac{2}{3}}$, and the sum of the squared second and the squared third number is equal to 250. Find these numbers.

2. Solve the following equations:

(a) $\frac{3}{1+x}+\frac{6}{x^2-1}=1-\frac{2}{1-x}$;

(b) $\frac{x^2+2x-3}{x-1}=3x+1$.

3. Find the domain within which the variable x changes if

(a) $(x-2)^2 \leqslant 9$; (b) $-2 < x+1 \leqslant 4$;

(c) $3|x|+2 > 8$.

4. Prove: (a) $1+2\sin 18^\circ = 2\sin 54^\circ$;

(b) $\cos 36^\circ \cos 108^\circ = -\frac{1}{4}$.

5. Find the real values of x and y if

(a) $(x^4+2xi)-(3x^2+yi)=(3-5i)+(1+2yi)$;

(b) $\sqrt{x}(i+\sqrt{y})-15=(8-\sqrt{y})i$.

1951. 1. Two teams of workers constructed a highway of s kilometres between the towns A and B. The first team started from A and the second from B. Both teams constructed equal lengths of the highway, but the daily output of the first team was a km less as compared to the second team. How many kilometers of the highway were constructed by each team per day if the first team worked t days longer than the second one?

2. Solve the given systems of equations:

(a) $\begin{cases} \begin{vmatrix} x-2y & 3y-6 \\ y & x \end{vmatrix} = 7 \\ \begin{vmatrix} x & -3 \\ y & 2 \end{vmatrix} = 5; \end{cases}$ (b) $\begin{cases} \begin{vmatrix} 2x & x-2y \\ y & x+3y \end{vmatrix} = 0 \\ x^2-xy-3x+4y+8=0. \end{cases}$

3. Solve the below systems of inequalities:

(a) $\begin{cases} x^2-x-12 > 0 \\ -\frac{x}{4} > 3x-\frac{67-2x}{2}; \end{cases}$ (b) $\begin{cases} \log_{10}(2x+6) > 1 \\ 11x-x^2-28 > 0. \end{cases}$

4. Compute

(a) $\arcsin\left(-\frac{1}{2}\right)+2\arccos\left(-\frac{\sqrt{3}}{2}\right)-\arctan\sqrt{3}$;

(b) $\cos\left[\arcsin\frac{8}{17}+2\arccos\left(-\frac{4}{5}\right)\right]$.

5. What is wrong in the following reasoning?

(a) $\frac{-1}{1}=\frac{1}{-1}$; $\frac{\sqrt{-1}}{\sqrt{1}}=\frac{\sqrt{1}}{\sqrt{-1}}$; $\frac{i}{1}=\frac{1}{i}$; $i^2=1$;

(b) $i^2=\sqrt{-1}\cdot\sqrt{-1}=\sqrt{(-1)(-1)}=\sqrt{1}=1$;

(c) $4=\sqrt{16}=\sqrt{(-16)(-1)}=\sqrt{-16}\cdot\sqrt{-1}=4i\cdot i=4i^2=-4$.

1952. 1. The sum of the first and the fourth term of an arithmetic progression is equal to $a+11$ and that of the second and the fifth term of the same progression to $b-6$. The magnitude of a equals to the term of the expansion $\left(\frac{\sqrt[3]{x}}{2}+\sqrt[-5]{x}\right)^8$, containing no x; and b is determined from the equation $\log_{\sqrt{8}} b=3\frac{1}{3}$. Find the progression.

2. Find the domain of definition of the given function and construct it on the number scale:

(a) $y=\frac{2}{x^2-4}$; (b) $y=\frac{\sqrt{x-2}}{x^2+1}$; (c) $y=\frac{\sqrt{x+3}}{x^2-16}$;

(d) $y=\sqrt{1+x}-2\sqrt[3]{5-x}$;

(e) $y=\sqrt{2+x}-3\sqrt[4]{5-x}$; (f) $y=\log_{10}(x^2-3x-4)$;

(g) $y=\sqrt{\sin 2x}$; (h) $y=\sqrt{\sin x}+\sqrt{-\cos x}$;

(i) $y=\log_{10}(x-1)-\arcsin\frac{x}{3}$.

3. Rationalize the denominators of the given fractions:

(a) $\frac{\sqrt{2}-1}{\sqrt{2}}$; (b) $\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$; (c) $\frac{2\sqrt{3}-3\sqrt{2}}{2\sqrt{3}+5\sqrt{2}}$;

(d) $\frac{2a-\sqrt{b}}{a+\sqrt{b}}$; (e) $\frac{a-b-\sqrt{a^2-b^2}}{a-b+\sqrt{a^2-b^2}}$;

(f) $\frac{a-3}{\sqrt[3]{a^2-9}}$; (g) $\frac{12}{3+\sqrt{2}-\sqrt{5}}$;

(h) $\frac{\sqrt{2\sqrt{3}-\sqrt{2}}}{\sqrt{2\sqrt{3}+\sqrt{2}}}$; (i) $\frac{3}{\sqrt[3]{4}-1}$.

4. Write the formula of the general term of the infinite number sequence $\frac{1}{5}, \frac{4}{8}, \frac{7}{11}, \frac{10}{14}, \ldots$ and prove that its limit is equal to unity.

5. What is the location of the vectors depicting complex numbers, if the moduli of these numbers are equal to one another, and their arguments are as follows:

(a) $\varphi_1 = 120°, \varphi_2 = 240°$; (b) $\varphi_1 = 120°, \varphi_2 = -60°$;

(c) $\varphi_1 = 90°, \varphi_2 = 270°$; (d) $\varphi_1 = 0°, \varphi_2 = 180°$.

Write these numbers in the trigonometric and algebraic forms, putting $r = 2$.

1953. 1. Two harvesting combines working together did a certain job in 6 days. If 0.6 of the harvest were handled by a single combine and the remainder by the second machine, then the whole job would take 12 days. How many days would it take each combine to do the job separately?

2. Find the least positive integer n for which (a) $(1-i)^n$ is a positive number, (b) $\left(\frac{1+i}{1-i}\right)^n = 1$.

3. Solve the following equations:

(a) $1+\cos 2x = \cot x$; (b) $\sin 2x+\cos 2x = 1+\sin 4x$;

(c) $2\sin^2 x-\sin^2 2x = 0$;

(d) $\cos(45°+x)-\cos(45°-x) = -0.5$.

4. For what real value of k is the sum of squared roots of the equation $x^2-kx+k-3 = 0$ the least possible?

5. The vector $\boldsymbol{a}$ forms an angle of 30° with the x-axis, and the vector $\boldsymbol{b}$ an angle of $-60°$ with the same axis. Compute $\mathrm{pr}_x(\boldsymbol{a}+\boldsymbol{b})$ and $\mathrm{pr}_x(\boldsymbol{a}-\boldsymbol{b})$ if $|\boldsymbol{a}| = 12$, $|\boldsymbol{b}| = 8$.

1954. 1. The first pipe, operating separately, takes 3 hours less time than the second pipe to fill up the pool. Both pipes were opened simultaneously, but in 10 hours the first pipe was cut off, and it took the second pipe another 5 hours and 45 minutes to completely fill up the pool. How much time does it take each pipe to fill up the pool separately?

2. On a line segment AB as on the diameter a semicircle is constructed. Then the segment AB is divided into two equal parts and on one of them a semicircle is constructed. Now the segment

AB is divided into four parts and on one of them another semicircle is constructed, and so on. Find the limit of the sum of the lengths of the semicircles thus obtained if $AB = d$.

3. Solve the following equations:

(a) $\left(\frac{1}{0.125}\right)^x = 128$; (b) $3^{2x}2^x = 324$;

(c) $5^{(x^2-x-2)(4-x)} = 1$; (d) $5^{x-1} = 7^{1-x}$;

(e) $3\times 2^x + 5\times 2^{x-2} = 17$;

(f) $2^x + 2^{x+1} + 2^{x+2} = 3^x + 3^{x+1} + 3^{x+2}$;

(g) $9^{x+1} + 3^{x+2} - 810 = 0$; (h) $2^{2\log_{10} x}\, 5^{\log_{10} x} = 8000$;

(i) $2^{x^2-2.8(3)x} = \sqrt{2}$.

4. For what real values of x and y will the numbers $3+ix^2y$ and x^2+y+4i be conjugate?

5. (1) For what values of α and β is the equality $\sin(\alpha+\beta) = \cos\alpha+\cos\beta$ correct?

(2) Solve the inequality $\cos x \cos 3x > \sin x \sin 3x$.

(3) Solve the inequality $\sin x - \sqrt{3}\cos x < 0$.

(4) Find the greatest and the least values of the function

$$y = \sin x \cos\left(x + \frac{\pi}{4}\right).$$

1955. 1. Find the sum of an infinitely decreasing geometric progression whose first term is equal to $b+2$ and the ratio to $2c^{-1}$, if b is the least value of the product of the roots of the equation $(a^2+1)x^2-3x+(a^2+1)^2 = 0$, and c is the greatest value of the sum of its roots.

2. Simplify:

(a) $\sin 2023^\circ + 2\cos 845^\circ + \sin(-2383^\circ) - \cos(-125^\circ)$;

(b) $3[\sin^4(90^\circ+\alpha)+\sin^4(360^\circ+\alpha)] - 2[\sin^6(270^\circ-\alpha)+\sin^6(180^\circ-\alpha)]$.

3. Simplify and compute the expression $6\sin(450^\circ-\alpha)+5\sin(\alpha-900^\circ)$ if $\cot(\alpha-180^\circ) = \frac{3}{4}$ and α is an acute angle.

4. Locate the points corresponding to the complex numbers $z = x+yi$, for which

(a) $|z| < 2$; (b) $2 < |z| < 4$;

(c) $-\pi < \arg z < \pi$; (d) $\frac{\pi}{6} < \arg z < \frac{2\pi}{3}$;

(e) $|z| = 2$; (f) $|z| \geqslant 5$;

(g) $|z+3| \leqslant 4$; (h) $|z-(2-i)| < 2$.

5. Compare with the unit power

$$\left(\frac{3}{4}\right)^{\frac{2}{3}}, \left(\frac{4}{3}\right)^{-\frac{5}{6}}, (1.1)^{-\frac{3}{4}}, (0.9)^{-0.8},$$

$$(1.36)^0, (\log_{10} 5)^2, (\log_{10} 15)^{-2}.$$

6. What conclusion can be drawn with respect to the numbers m and n if

(a) $\left(\frac{2}{3}\right)^m < \left(\frac{2}{3}\right)^n$; (b) $2.5^m < 2.5^n$;

(c) $(0.2)^m > (0.2)^n$; (d) $|\cos x|^m > |\cos x|^n$;

(e) $(\log_3 5)^m < (\log_3 5)^n$; (f) $(\log_5 3)^m < (\log_5 3)^n$?

1956. 1. The quadratic equation $x^2+px+q = 0$ is specified by two numbers p and q which are the coordinates of some point on a plane.

(a) What equation corresponds to the origin?

(b) Indicate on the plane a set of points corresponding to the equations the sum of whose roots equals zero; the product of whose roots equals zero; the sum of the roots equals the product of the roots.

2. Find the domains of definition of the given functions:

(a) $y = 2^{\frac{2}{x}}$; (b) $y = 3^{\sqrt{x}}$; (c) $y = a^{\frac{4}{2-x}}$; (d) $y = \left(\frac{1}{2}\right)^{\sin x}$.

3. Simplify the expressions:

(a) $\frac{a^{-2}-b^{-2}}{a^{-1}-b^{-1}} \cdot \left(\frac{a+b}{ab}\right)^{-1} : \frac{1}{2(a^2+b^2)}$;

(b) $\frac{\sqrt{2}(x-a)}{2x-a} - \left[\left(\frac{\sqrt{2x}-\sqrt{a}}{\sqrt{x}}\right)^{-2} + \frac{2\sqrt{a}}{\sqrt{2x}-\sqrt{a}}\right]^{\frac{1}{2}}$.

4. Are the given pairs of equations equivalent?

(a) $(x+2)^2 = 4$ and $x+2 = 2$; (b) $f(x) = 0$ and $f(x)a^{\varphi(x)} = 0$.

1957. 1. Two pulleys are connected with a belt transmission. During a certain interval of time the driving pulley makes by 1000 revolutions more than the driven one. Were the circumference of the driving pulley 1.5 times longer, it would make by 200 revolutions more than the driven pulley during the same time. What are the lengths of the circumferences of the pulleys if the circumference of the driven pulley is 1.5 m longer than that of the driving pulley?

2. Carry out the operations:

$$\left\{\left[2^{-2}\left(\frac{1+a}{1-a}\right)^{-0.25}\right] : \left(\frac{1-a}{1+a}\right)^{-0.75} \times 2(1-a)^{-2} - 2^{-1} : (1+a^2)\right\} \cdot (1-a^4)$$

3. Describe the properties of the functions

(a) x, x^3, x^{-1}, x^{-3} in the interval $-2 \leqslant x \leqslant 2$;

(b) x^2, x^4, x^{-2}, x^{-4} in the interval $-2 \leqslant x \leqslant 2$.

4. Find the integral solutions of the system of equations

$$\begin{cases} \dfrac{x+2}{8} - \dfrac{1}{2} < \dfrac{3x+5}{6} \\ \dfrac{3x-1}{4} + \dfrac{2x+1}{3} < \dfrac{5x-2}{8}. \end{cases}$$

1958. 1. Two lorries transported 250 tons of load. The first lorry transported 160 tons; the second lorry transported only 90 tons, since it worked one day less than the first one. If the first lorry worked as much as the second one did, and the second lorry worked as much as the first did, then they would transport equal amounts of load, but a portion of the load would remain non-transported. How many tons of the load would remain non-transported in this case?

2. Perform the indicated operations:

$$\left\{\left[x+\frac{2}{(x-1)^{-0.5}}\right]^{-0.5} + \left[x-\frac{2}{(x-1)^{-0.5}}\right]^{-0.5}\right\}^2 : (2-x)^{-2}.$$

3. Describe similar and different properties of the functions

(a) x^2 and x^{-2}; (b) x^3 and x^{-3}; (c) $x^{\frac{1}{2}}$ and $x^{-\frac{1}{2}}$.

4. Solve the inequality $\dfrac{3}{3-2x-x^2} < 1$.

1959. 1. Taking into account the rate of fuel consumption, the available fuel supply will be sufficient for two lorries and a car during ten days. How many days will three lorries run on this supply, if it is known that a lorry would spend it by 36 days earlier than a car?

2. Simplify the expression

$$\sqrt{\frac{a+b}{a-b}-\frac{2a\sqrt{a^2-b^2}}{b^2(ab^{-1}+1)^2}}\cdot\frac{1}{1+\frac{1-ba^{-1}}{1+ba^{-1}}}.$$

3. Describe the properties of the given functions:

$$x^2-2,\ x^{-2}-1,\ x^{-1}+2,\ x^3-2,\ x^{\frac{1}{2}}-2.$$

For what values of the argument are the values of the given functions equal to zero?

4. For what values of m do the straight lines $x-2y=m$ and $3x+y=8$ intersect in the fourth quadrant?

1960. 1. Here we formulate the theorem proved by Euclid in a pure geometric way: "If a regular pentagon, hexagon and decagon are inscribed in one and the same circle, then the area of the square constructed on the side of the pentagon is equal to the sum of the areas of the squares constructed on the sides of the hexagon and decagon". Prove this theorem algebraically if

$$a_5=\frac{R\sqrt{10-2\sqrt{5}}}{2},\quad a_6=R,\quad a_{10}=\frac{R(\sqrt{5}-1)}{2}.$$

2. Solve the equation $x^2-\sqrt{2x^2-8x+12}=4x+6$.

3. Solve the system of equations $\begin{cases}\sqrt[4]{x}+\sqrt[4]{y}=a\\ \sqrt[4]{xy}=b.\end{cases}$

4. Construct the graph of the function $y=0.5x^2-x-4$ and with its aid determine for what value of x the function attains the least value, and what it is equal to; for what values of x the function takes on positive values.

1961. 1. The cost of operating a motor at a speed of v km/h comes out to $(90+0.4v^2)$ roubles per hour. At what speed will the operating cost over a one-kilometre distance be the least?

2. Are the following pairs of inequalities equivalent?

(a) $3x < 0$ and $3x+\frac{5}{x} < \frac{5}{x}$;

(b) $x-2 \geqslant 0$ and $x-2+\frac{2}{3-x} \geqslant \frac{2}{3-x}$;

(c) $2x > 5$ and $\frac{2x}{x-2} > \frac{5}{x-2}$;

(d) $2x > 5$ and $2x^3 > 5x^2$;

(e) $2x > 5$ and $\frac{2x}{\sqrt{x}} > \frac{5}{\sqrt{x}}$.

3. When is the formula $\sqrt{\frac{mn^6}{p^2}} = \frac{n^3}{p}\sqrt{m}$ correct?

4. Solve the equation $\sqrt{(x-4)^2}+\sqrt{(1-x)^2} = 7$.

5. Find x if $(\cos 3x \cos 7x - \cos 4x \cos 6x)i = \sin 2x - \sin 6x$.

6. Given $f(x) = \sin x$. Show that $f(x+2m)-f(x) = 2\cos(x+m)\sin m$.

1962. 1. A line segment of length l is divided into n equal parts. On each segment thus obtained (except for the last one) a semicircle is constructed. Find the limit of the length of the wavy line if $n \to \infty$.

2. Construct the graphs of the functions $y = 2|x|$ and $y = \frac{1}{2}|x-2|$ if the graph of the function $y = |x|$ is given.

3. Prove the inequalities:

(a) $(a+b)\left(\frac{1}{a}+\frac{1}{b}\right) \geqslant 4$, $a > 0$, $b > 0$;

(b) $(a+b+c+d)\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}+\frac{1}{d}\right) \geqslant 16$, $a > 0$, $b > 0$, $c > 0$, $d > 0$;

(c) $a+b+c \geqslant \sqrt{ab}+\sqrt{bc}+\sqrt{ac}$, $a \geqslant 0$, $c \geqslant 0$.

4. Solve the equations:

(a) $1+|x|+x^2+|x^3|+\ldots = 5$ if $|x| < 1$;

(b) $8^{1+|\cos x|+\cos^2 x+\ldots} = 64$.

5. Simplify the expression

$\cos(37^\circ+\alpha)\cos(53^\circ-\alpha)-\sin(37^\circ+\alpha)\sin(53^\circ-\alpha)$.

1963. 1. Two bodies are moving along two mutually perpendicular straight lines towards the point of their intersection. Their velocities remain unchanged and are equal to 2 m/s and 4 m/s, respectively. Before they started the first body was 20 m and the second body 60 m distant from the vertex of the angle. In how many seconds after the start will the distance between the bodies be the least?

2. (a) Prove that if the sum of the positive numbers a and b equals unity, then $ab \leqslant \frac{1}{4}$, $a^2+b^2 \geqslant \frac{1}{2}$.

(b) Prove that if the sum of two positive numbers a and b is equal to 4, then $a^3+b^3 \geqslant 16$.

3. Solve the equations:

(a)* $x^2-4|x|+3 = 0$ and graph the function $y = x^2-4|x|+3$;

(b) $4^{\cos(\alpha+x)} = \sqrt[\cos x]{2^{\cos\alpha}}$;

(c) $(\tan x+2)+(\tan x+5)+(\tan x+8)+\ldots+(\tan x+23) = 116$.

4. Prove that $\cos\frac{\pi}{7}-\cos\frac{2\pi}{7}+\cos\frac{3\pi}{7} = \frac{1}{2}$.

1964. 1. A material point performs rectilinear motion according to the law $s = (6t^2-t^3)$m. At what time t will the point return to the initial position, and what distance will it cover during this time?

2. Can the equation $ax = b$ have: (a) one root,

(b) two different roots, (c) an infinite set of different roots?

3. Prove that each of the given systems of equations cannot have an infinite set of solutions:

(a) $\begin{cases}(a+2)x+3y = 1\\ 3x+(a^2-2a+4)y = 2;\end{cases}$ (b) $\begin{cases}\frac{a-2}{x}+\frac{a}{y} = 2\\ \frac{a}{x}+\frac{a+2}{y} = 3.\end{cases}$

4. Solve the following equations:

(a) $\log_{10} x+\log_{10}\sqrt{x}+\log_{10}\sqrt[4]{x}+\log_{10}\sqrt[8]{x}+\ldots = 6$;

(b) $\frac{1+3+5+\ldots+(2x-1)}{4+7+10+\ldots+(3x+1)} = \frac{40}{7x}$.

5. Verify the validity of the equality $7778^2-2223^2 = 55{,}555{,}555$ using the most rational method.

1965. 1. A vessel is filled with 20 litres of alcohol. Then one litre of alcohol is poured out and water is added instead, now one litre of mixture is removed and water is added once again. This operation is repeated for the third, fourth, fifth time, and so on. How much alcohol will remain in the vessel after eight such operations?

2. The period T of oscillation of a pendulum is calculated by the formula $T = \pi\sqrt{\frac{l}{g}}$, where l is the length of the pendulum. (a) Find T if $l = 25$ cm, $g \approx 9.81$ m/s. How many times does the period of oscillation increase if (b) l increases m times?

3. Prove that each of the given systems of equations is either inconsistent or has an infinite set of solutions:

(a) $\begin{cases} 3x-2y = a \\ 6x-4y = 3; \end{cases}$ (b) $\begin{cases} x+(2-a)y = a \\ 2x-(2a-4)y = 3a. \end{cases}$

4. Solve the equations:

(a) $5^2\times5^4\times5^6\times \ldots \times5^{2x} = 0.04^{-28}$;

(b) $x^2+x^3+x^4+ \ldots = \cos^2\frac{\pi}{4}$, $(|x| < 1)$.

5. One of the roots of the equation $x^4+2x^3-2x^2+2x-3 = 0$ is equal to i. Find the remaining roots.

1966. 1. A vessel contains 50 litres of 80°-alcohol. Determine how many litres of pure alcohol will be found in this vessel if two litres of solution is poured out of the vessel 20 times, and two litres of water is added instead each time.

2. For what values of a do the straight lines $2x-5y = a$ and $3x+ay = 4$ intersect in the third quadrant?

3. Find the numbers A and B from the identity $\frac{4x}{x^2-1} = \frac{A}{x-1}+\frac{B}{x+1}$.

4. Solve the following equations:

(a) $2\log_{10}x+\log_{10}(x-4)^2 = k^{\log_k(\log_{10}m^2)}$,

where m is the sum of the infinitely decreasing geometric progression 3, 1.2, 0.48, ...;

(b) $|z|-z = 1+2i$ where $z = x+iy$;

(c) $3\cos^2(45°+x)+\sin^2 210° = 1$;

(d) $\sin x+\cos x = \dfrac{1}{\cos x}$;

(e) $\arcsin \dfrac{x}{\sqrt{3}}+\arcsin x = \dfrac{\pi}{2}$.

5. Given $\sin\alpha-\sin\beta = m$, $\cos\alpha-\cos\beta = n$. Find $\cos(\alpha-\beta)$.

1967. 1. Each stroke of the piston of a pump removes 0.1 of the air contained in the vessel. Determine the air pressure after the tenth stroke of the piston, if the initial pressure was equal to 760 mm.

2. (a) For what values of a and b are the straight lines $2x-ay = b$ and $2x+5y = 4$ parallel?

(b) For what values of a and b do the straight lines $x-ay = 2b$ and $2x+3y = -5$ coincide?

3. Compute $\left[2\sqrt{5}(\sqrt{3}-1)+\sqrt{2}(2-\sqrt{3})+\dfrac{3\sqrt{3}-5}{2\sqrt{5}}+\ldots\right]\times$ $\times(\sqrt{40}-\sqrt{3}+1)$, first showing that the sequence in square brackets is an infinitely decreasing geometric progression.

4. The equation $3x^6-6x^5+8x^4+14x^3-33x^2-4x+10 = 0$ has the root $1-2i$. Find the remaining roots.

5. In a geometric progression $a_1 = q = \dfrac{1}{2}(\sqrt{3}+i)$.

Find the absolute value of the 20th term of the progression.

6. A segment 50 cm long is divided into n equal parts and on each of them, except for the extreme ones, regular triangles are constructed. Find the limits of the area S_n and the perimeter P_n of the toothed figure thus obtained as $n \to \infty$.

7. Graph the following functions:

(a) $y = ||x|-2|$; (b) $y = |x|x$;

(c) $y = |x^3| : x^3-x^2$.

1968. 1. A body performs rectilinear motion according to the law $s = \dfrac{1}{4}t^4-4t^3+16t^2$, where t is in seconds and s in metres. At the time $t = 0$ the body was found at point A. Starting from the point A the body moves to a stop and then immediately comes back (why?). At what point will the body stop once again and

how much time will pass from the start till the second stop? If the body keeps moving according to the mentioned law then will it have some more stops and change the direction of motion?

2. Given $f(x) = \log_{10} \frac{1-x}{1+x}$. Show that $f(a)+f(b) = f\left(\frac{a+b}{1+ab}\right)$.

3. How many different solutions has each of the following systems of equations?

(a) $\begin{cases} 3x-y = 2 \\ 3x+2y = 0; \end{cases}$ (b) $\begin{cases} 2x+y = 0 \\ x-y = 0; \end{cases}$ (c) $\begin{cases} x+2y = 0, \\ 0.5x+y = 0; \end{cases}$

(d) $\begin{cases} x-y = 3 \\ 2x-2y = 5; \end{cases}$ (e) $\begin{cases} 3x-6y = 5 \\ -x+2y = -\frac{5}{3}; \end{cases}$

(f) $\begin{cases} ax+4y = 2 \\ 3x-2y = 1. \end{cases}$

4. The polynomial $x^{\frac{5}{2}} - x^2 + 9x^{\frac{1}{2}} - 9$ has the root $x = 3i$ and is divisible by $\sqrt{x}-1$. Show this.

5. (a) Find the sum of n numbers of the form $0.9+0.99+ +0.999+\ldots$.

(b) Given $x+x^{-1} = \frac{3\sqrt{2}}{2}$. Compute x^3+x^{-3}.

6. In a triangle ABC the sum of the sides AB and AC equals m, the angle $A = \alpha$. What values must the mentioned sides and the angle α have to yield the greatest possible area of the triangle?

7. At what angle to the horizon should a projectile be launched so that, having the initial velocity $v_0 = 360$ m/s, it falls onto the ground at a distance $a = 11.4$ km from the launching point? (Air resistance should be neglected.)

8. Mention the names of the prominent Russian and Soviet mathematicians.

ANSWERS

3. (2) $64m^6n^6-1$; (3) $2^{16m}-2^{8m+12n+1}+24^{24n}$;
(4) $1+4x+6x^2+4x^3+x^4-y^4$. **5.** (4) $m^2+n^2+p^2+mp-2mn-np$.
12. (1) $8x^2+9y^4$; (2) $\frac{2+xy}{xy}$; (3) 6; (4) $\frac{4}{9}x^2$. **14.** (1) 16;
(2) 1000; (3) 18; (4) -3. **15.** (2) $(x-3)(3x-1)$;
(3) $(m-1)(2m-1)$; (4) $(b+1)(b-1)(a+b)(a-b)$.
16. (1) $6q(p-q)$; (2) $(a-b)(a+b-1)(a^2+ab+b^2)$;
(3) $x(x-1)(3x+1)(x^2-x+1)$. **19.** (1) $(a+2)(a-2)(a-5)$;
(3) $(a+3)(a^2+1)$. **20.** (2) $(a-x)(4x-3y^2)$; (3) $2(x-6y)(2a^2-3)$.
21. (1) $2(a+2)(a+3)$; (2) $(a+2)(a-2)(a^2+5)$;
(3) $(x-3y)(x-4y)$; (4) $(x-1)^2(x+1)$.
23. (1) $(a+b)(a-b)(a+1)(a-1)$; (2) $(a+2)^2(a-2)(a^2-2a+4)$;
(3) $(a+2x)(a-x+1)$; (4) $(a^2+2a+2)(a^2-2a+2)$;
(5) $(x^2+3x+4)(x^2-3x+4)$; (6) $(x^2+2x+5)(x^2-2x+5)$.

25. (1) $\frac{x+4}{x}$; (2) x^2+3. **28.** (1) 0, -7; (2) 0, -5, 5.

31. (1) $\frac{m+6}{m+3}$; (2) $\frac{a-b}{2(2a-b)}$. **32.** $-\frac{1+3z}{6}$. **33.** (1) 1;
(2) $\frac{x^2+3x+9}{x-a}$. **36.** (1) 1; (2) $-1\frac{8}{11}$. **38.** 3^{18} times. **39.** $0.5a$ kg.
40. 2.4. **63.** (1) The third one. **64.** 0.1%, 4.555 m, 4.465 m.
65. (1) $\Delta = 0.1$; (2) $\Delta = 0.2$; (3) $\Delta = 0.5$; (4) $\Delta = 0.2$.
71. (1) ≈ 6.1; (2) ≈ 10.66. **72.** (1) ≈ 13.29; (2) ≈ 82.8;
(3) ≈ 15.73. **73.** (1) ≈ 1.18; (2) ≈ 2.08. **76.** (1) ≈ 22;
(2) ≈ 0.184; (3) ≈ 6.79; **77.** ≈ 1700 N. **78.** ≈ 160 N.

102. (1) Yes; (2) no; (3) no; (4) yes; (5) no; (6) no; (7) yes;
(8) no; (9) yes; (10) yes; (11) no. **104.** (1) -1; (2) -30.
105. (1) 1.2; (2) -2; (3) $-\frac{1}{3}$; (4) any number. **106.** (1) 5;
(2) 0.5; (3) no roots; (4) $\frac{1}{6}$. **107.** (1) $-\frac{2}{11}$; (2) $-\frac{5}{13}$; (3) 8.
109. (1) $-a$; (2) 4; (3) $\frac{bd(a-c)}{ac(d-b)}$. **110.** (1) $b-a$ if $a \neq b$, and
any number if $a = b$; (2) a^2+1; (3) $\frac{a}{a+1}$. **112.** 40° C.
114. (1) ± 6; (2) ± 3; (3) 0, -8; (4) -1, 4; (5) ± 2, ± 8;
(6) -3.5; 6.5; (7) -4, 2; (8) -2, 2; (9) $x = -7$ and $x \geqslant 1$.
121. (1) $2a+b$; (2) $3a+2$. **122.** (1) 2; (2) $-\frac{1}{7}$.

123. (1) $2\frac{3}{4}$; (2) 1. **124. 1.** (a) $-\frac{9}{14}$; (b) 1.5. **2.** $\frac{1}{6}$.

130. 1. (a) -1; (b) a any number. **2.** -2. **132.** (1) $\frac{1}{2}$, -2; (2) $-\frac{2}{3}$, 0. **133.** (1) 13, 11; (2) (2.2, 0.4); $(-1, 2)$; (1, -2); (6, 5.5); (3) $\left(\frac{3}{47}, -\frac{42}{47}\right)$. **134.** (1) $(-2, 3)$; (2) (2, 3).

135. (1) $\left(\frac{3a+8}{a^2+6}, \frac{9-4a}{a^2+6}\right)$, a any real number; (2) $\left(\frac{n-am}{1-a}, \frac{m-n}{a-a^2}\right)$, $a \neq 1$, $a \neq 0$; (3) (a, b), $a \neq 0$, $b \neq 0$. **136.** (2) (4, 3, 7); (2) (2, -3, 0.5); (3) $(-1, 2, 0)$; (4) $\left(\frac{1}{2}, -\frac{1}{3}, \frac{1}{4}\right)$.

137. (1) $\left(\frac{kmp}{amp+bnp+cnq}, \frac{knp}{amp+bnp+cnq}, \frac{knq}{amp+bnp+cnq}\right)$; (2) $(-2, -5, -1)$. **138** (1) (5, -3); (2) no solutions; (3) x any number, $y = -\frac{3}{2}x$, $z = -2x$. **139.** 48. **140.** $\frac{7}{12}$.

141. Subtract 29. **142.** 35. **143.** 83.6 kgf, 508.3 kgf, 1327 kgf. **144.** 10%, 20%, 38%. **145.** 210 km, 42 km/h. **146.** 2 h 48 min. **147.** 48 km/h, 72 km/h. **148.** Increase by 20 mm. **149.** 6600. **150.** 105. **151.** 6 h. **152.** 14 h, $18\frac{2}{3}$ h. **153.** 1440 and 768. **154.** 12 and 8. **155.** 75 and 50. **156.** 2450 and 2550. **157.** 3.75 l. **158.** 62.5°. **159.** 4 kgf. **160.** 8. **162.** $\frac{a+(b-c)t}{2}$, $\frac{a+(c-b)t}{2}$. **163.** $\frac{tv(v+d)}{d}$ km.

169. (d) $|a| < |b|$. **170.** 2. m any number, $n < 0$. 3. $|x| > |y|$, $|x| < |y|$. **171.** (1) Yes; (2) yes; (3) no; (4) no; (5) yes. **172.** (1) $x > 4$; (2) $x > 4$; (3) $x > -\frac{1}{4}$; (4) $x < 7$.

173. 2. (a) $-3 < m < -1\frac{1}{3}$; (b) $m < -1.6$.

174. (1) $x > 3\frac{4}{11}$; (2) $x > \frac{1}{4}$; (3) $x > -2$; (4) $x < 2\frac{1}{4}$.

175. (1) $x < \frac{1}{2}$; (2) $x > -4$; (3) $x < 5$; (4) $x \leqslant 2$.

176. (1) $x < \frac{1}{3}$; (2) $x < 0.1$; (3) $y \leqslant 5$; (4) $t > 7$.

177. (3) $-4 < x < 4$; (4) $x < -2$ and $x > 8$; (5) $-10 < x < 4$; (6) $x < -1$ and $x > 3\frac{2}{3}$; (7) $1\frac{3}{4} \leqslant x \leqslant 2\frac{1}{4}$; (8) $-3 \leqslant x < -2$, $2 \leqslant x \leqslant 3$; (9) $-\frac{2}{3} < x < 4$.

178. (1) $x < \frac{2a+1}{1-a}$ if $a < 1$, and $x < \frac{2a+1}{a-1}$ if $a > 1$;
(2) $x > \frac{m^2+n^2}{4}$ if $|m| > |n|$ and $x < \frac{m^2+n^2}{4}$ if $|m| < |n|$;
(3) $y > \frac{ab-k}{2}$ if $a > 0$ and $y < \frac{ab-k}{2}$ if $a < 0$;
(4) $z < \frac{m(p-n)}{3}$ if $m > 0$ and $z > \frac{m(p-n)}{3}$ if $m < 0$;
(5) $x > -m$ if $m > 0$ and $x < -m$ if $m < 0$.

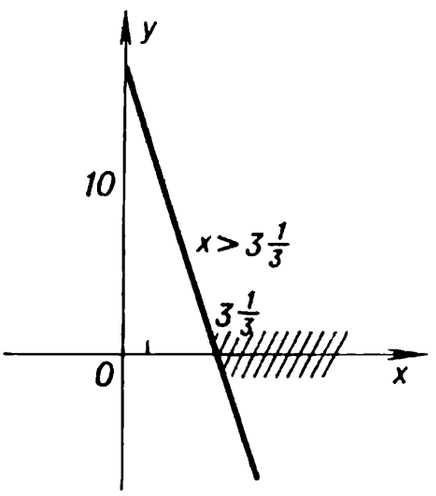
179. (1) $x > \frac{2-2a}{3(1-2a)}$ if $0 < a < \frac{1}{2}$ and $x < \frac{2-2a}{3(1-2a)}$ if $a > \frac{1}{2}$ and $a < 0$; (2) $x < \frac{apq}{p^2-q^2}$ if $pq > 0$ and $|p| < |q|$ if $pq < 0$, $|p| > |q|$; (3) $x > \frac{2n}{3m}$ if $m > 0$ and $x < \frac{2n}{3m}$ if $m < 0$;
(4) $x > \frac{2}{(a+1)^2}$ if $a > -1$ and $x < \frac{2}{(a+1)^2}$ if $a < -1$;
(5) $x > \frac{5b-10}{2b+20}$ if $-10 < b < 2$ and $x < \frac{5b-10}{2b+20}$ if $b < -10$, $b > 2$; x any number if $b = -10$. **180.** (1) 3; (2) 3. **181.** (1) 2, 3, 16, 11; (2) $-2\frac{1}{8} < a < 0$. **182.** (1) $x < 2$, $3 < x < 5$ and $x > 8$;
(2) $x < -3$, $-1 < x < 4$, $4 < x < 5$; (3) $x \leqslant -2.5$, $2.5 \leqslant x \leqslant 3$, $x = 5$; (4) $x < -\frac{1}{2}$, $x > 3$; (5) $x \leqslant -1$, $2 < x < 4$, $4 < x \leqslant 5$;
(6) $-5 < x < 11$. **185.** (1) $2 < x < 5$; (2) $x > \frac{1}{2}$; (3) $x < -3$;
(4) no solutions; (5) $-7 < x < 7$; (6) no solutions.

186. (1) $-1 < x < 3\frac{1}{3}$; (2) $-3.4 < x < 3.25$;
(3) $-10\frac{59}{95} < x < 8$; (4) $x < 5$. **187.** (1) $1.5 \leqslant x \leqslant 2$;
(2) $0 < x < 1$; (3) $2 < x < 3.5$; (4) $-4.25 < a < 4$;
(5) $-3 < x < -2$ and $2 < x < 4$. **188.** (1) 1, 2;
(2) 8, 7, 6, 5, 4, 3, 2, 1, 0, -1, ...

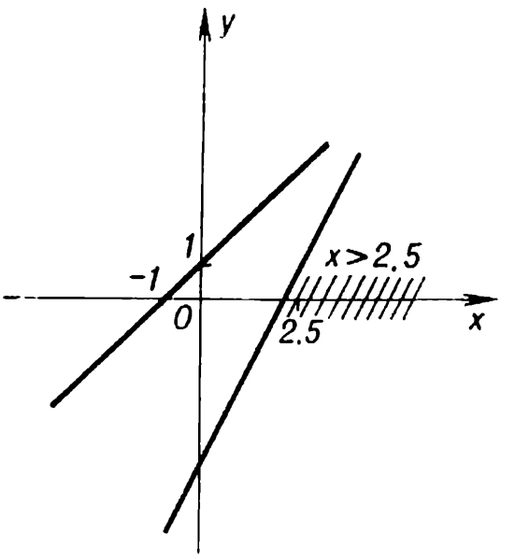
189. (1) No solutions; (2) $18 < x < 29$; (3) $-1 < x < 1$ and $3 < x < 5$; (4) $x > 4\frac{1}{4}$; (5) no solutions; (6) $-9 < x < -2.5$.

190. (1) (a) $m > -10$; (b) $-12 < m < -10$; (c) impossible;
(d) $m = -10$; (e) impossible; (2) (a) $m < -7.5$ and $m > 30$;
(b) $-7.5 < m < 0$; (c) impossible; (d) $m = -7.5$; (e) impossible.

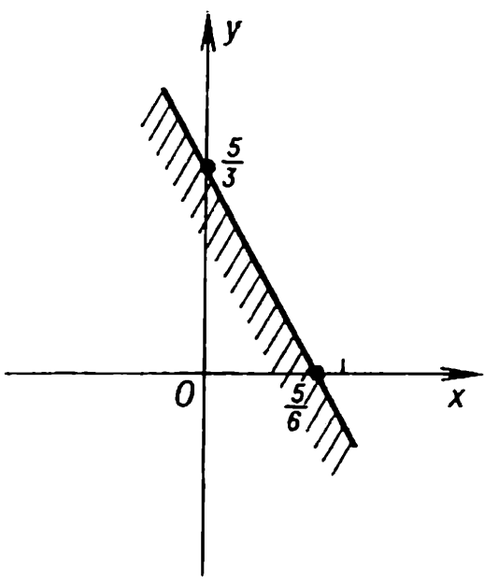
191. (2)



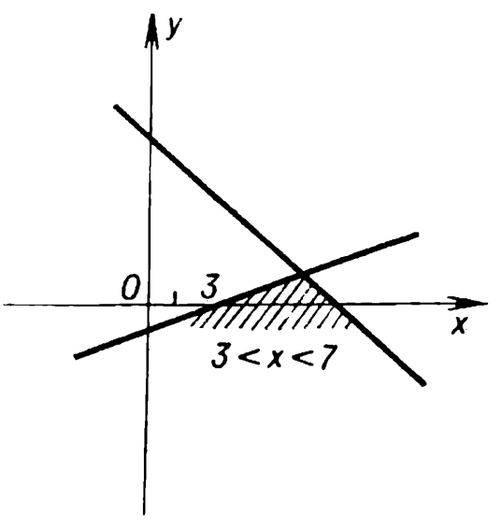
(4)



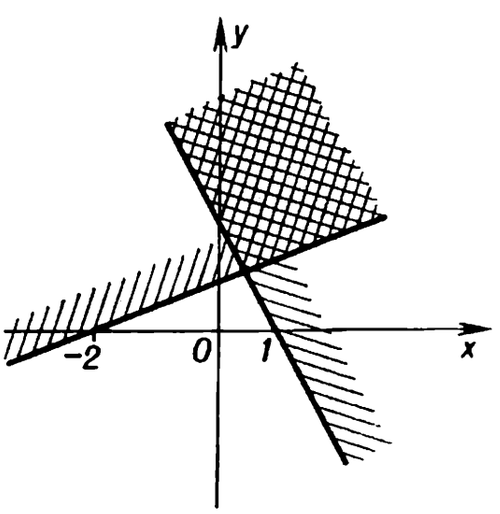
192. (2)



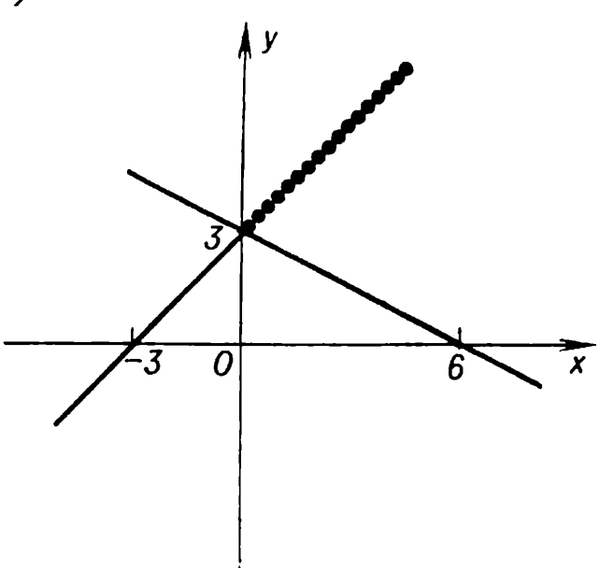
(5)



193. (2)



(3)



194. 74. **195.** $\frac{av}{b-c}$, $b > c$. **196.** 24. **197.** $92.5 < x < 102$.

198. 5 and 3; 6 and 4; 7 and 5. **199.** $x = 12, 11, \ldots, 1, 0$ m first-grade cloth; $y = 20-x$ m second-grade cloth. **212. 1.** (a) no, (b) yes. **2.** (a) no, (b) yes. **3.** (a) yes, (b) yes. **4.** (a) yes, (b) no, (c) yes. **220.** *(3)* $|x-4| < 2$; *(4)* $|x+1| \leqslant 3$; *(6)* $|2x-1| > 9$.

228. *(a)* $x \geqslant \frac{1}{2}$; *(b) null set; (c)* $x > \frac{1}{3}$. **244. 1.** *(a)* $10x+y$; *(b)* $10^2x+10y+z$; *(c)* $10^3x+10^2y+10z+t$. **2.** *(c)* $12x+y$, $12^2x+12y+z$, $12^3x+12^2y+12z+t$. **250.** *(1)* $x > 10$, $x < -30$, $x > 30$, $-5 < x < 5$; *(2)* $y \neq 0$, $x = 0$ and $x = y \neq 0$.

252. (3) $\frac{4ab}{c^2x^3y^5}$; (4) $\frac{5ab^{n-2}}{9c^3x^ny^2z^2}$. **253.** (1) $\frac{27ab}{1250y}$; (2) $\frac{n^2-m^2}{a^2-b^2}$; (3) ap. **254.** (3) $x^{2m}+x^my^n+y^{2n}$; (4) $a^{3n}+8$. **255.** (1) $\frac{44}{a^6}$.

256. 2. **260.** (5) $4a^{2n}-12a^{2n-1}+13a^{2n-2}-6a^{2n-3}+a^{2n-4}$; (6) $\frac{a^4-2a^3b+3a^2b^2-2ab^3+b^4}{a^2b^2}$. **261.** (2) $x^4+y^4+6x^2y^2-4x^3y-4xy^3$; (3) $16x^4+81a^4+216a^2x^2+216a^3x+96ax^3$; (4) $81m^4+216m^2n^2+16n^4-216m^3n-96mn^3$; (5) $\frac{1}{16}a^4+6a^2b^2+16b^4-a^3b-16ab^3$; (6) $a^8-8a^6b^3+24a^4b^6-32a^2b^9+16b^{12}$. **262.** (4) $8x^3-27y^3+1-36x^2y+54xy^2+12x^2+6x+27y^2-9y-36xy$; (5) $a^6-6a^5b+15a^4b^2-20a^3b^3+15a^2b^4-6ab^5+b^6$; (6) $64x^6+192x^5+240x^4+160x^3+60x^2+1$. **263.** (1) 5; (2) $\frac{1}{7}$; (3) $\frac{1}{9}$; (4) $\frac{1}{2}$. **273.** (1) $\frac{1}{4}$; (2) $1\frac{1}{3}$; (3) 1; (4) $1\frac{233}{360}$; (5) -1.5; (6) 4. **274.** (3) 14406; (4) $\frac{mp}{5n}$; (5) $-\frac{28a}{bx}$; (6) $x-a$.

275. (4) $32ab^7$; (5) $\frac{1}{y-x}$; (6) $\frac{a}{10b^nc}$; (7) $\frac{14y^nz^3}{5x}$.

276. (5) $\frac{(a+b)(3x-2y)}{a^2b^2xy}$; (6) $\frac{(b-a)xy}{ab}$. **277.** (4) $\frac{4x^2+3x+2}{x^2}$.

279. (1) $\frac{x+y}{x-y}$; (2) $\frac{m+n}{m-n}$; (3) $1-a^2$; (4) $\frac{1}{m-1}$. **280.** (1) $\frac{2}{17}$; (2) 1; (3) $\frac{(a+b+c)^2}{2bc}$; (4) 2; (5) $\frac{3a^2b-3ab^2+b^3}{(a-b)^4}$. **285.** (4) x any real number; (7) $0 \leqslant x \leqslant 0.4$; (8) $1 \leqslant x \leqslant 2$ and $x \geqslant 3$; (9) $x \geqslant 7$.

286. 1. $\sqrt[n]{-d}$ makes sense if n is even and $d \leqslant 0$ if n is odd and d is any real number. 2. $\sqrt[n]{m} > m$, if n is even and $0 < m < 1$, if n is odd and $0 < m < 1$ and $m < -1$. **289.** 1. $x \geqslant -2$, $x \leqslant 2$, $x \leqslant -3$ and $x \geqslant 3$, x any real number. **308.** 30 s. **309.** 13 km.

310. 14.7 m/s. **311.** 7.8 km/s. **313.** 28 m. **316.** (3) $\frac{3ab^2}{4}$; (4) $\frac{ab^2}{x^3}$; (5) $\left[\frac{a-b}{(a+b)^2}\right]^3$; (6) $\frac{(x+y)^2}{x^3(x-y)}$. **321.** (3) $2xz\sqrt[3]{2xz^2}$, $\frac{3}{4}x^2y^2\sqrt[4]{2x}$, a^3b^2; (4) $\frac{1}{3}\sqrt{a}$, $\frac{a^2}{x^3}\sqrt[3]{b}$, $\sqrt[n]{x}$, $\frac{a}{14}\sqrt{10}$, $\frac{b}{a}\sqrt[m]{a^{m-1}b^2}$.

322. (1) $\frac{a^3}{b^4}\sqrt{b}$, $\frac{2b}{9xy}\sqrt{6ay}$, $\frac{c}{3xy^2z^4}\sqrt[3]{ab^2x^2z^2}$, $\frac{1}{y}\sqrt{x-y}$;

(2) $\frac{1}{m^2}\sqrt{m(m-n)}$, $\frac{1}{xy}\sqrt{x^2+y^2}$, $\frac{1}{n}\sqrt[3]{m^3+n^3}$, $\frac{1}{x^3}\sqrt[5]{ax^4+b}$.

324. (1) $(\sqrt{5}-2)\sqrt{3}$; (2) $3(2-\sqrt{3})\sqrt{2}$; (3) $\frac{1}{2}(3-\sqrt{3})$;

(4) $\frac{2}{\sqrt{2}-1}\times\sqrt[4]{5}$; (5) $2(3-\sqrt{10})^2\sqrt{2}$; (6) $2(\sqrt{17}-4)^2\sqrt{6}$;

(7) $3(1-\sqrt{5})\sqrt[3]{2}$; (8) $(5-\sqrt{5})\sqrt[5]{(5-\sqrt{5})^2}$; (9) $(2-x)\sqrt{2-x}$;

(10) $x(x-3)^2\sqrt{x(x-3)}$; (11) $(4-x)(x-1)^2\sqrt{(x-4)(1-x)}$.

325. (1) $(x-2)\sqrt{x-2}$ for $x \geqslant 2$; (2) $5(5+a)^2\sqrt{2(5+a)}$ for $a \geqslant -5$; (3) $(5x-3)x\sqrt[3]{(5x-3)x^2}$; (4) $(3x+5)\sqrt[4]{3x+5}$ for $x \geqslant -\frac{5}{3}$;

(5) $\frac{1}{n}\sqrt{m^2-mn}$ for $\begin{cases} n > 0 \\ m \leqslant 0 \text{ and } m \geqslant n \end{cases}$ and $-\frac{1}{n}\sqrt{m^2-mn}$ for $\begin{cases} n < 0 \\ m = 0 \text{ and } m \leqslant n; \end{cases}$ (6) $\frac{1}{xy}\sqrt[3]{x^5+y^5}$; (7) $\frac{1}{|a-b|}\sqrt{b}$ for $b \geqslant 0$ and $a \neq b$; (8) $(2-a)\sqrt{2-a}$ for $a \leqslant 2$; (9) $x^2(\sqrt{2}-x)\sqrt[4]{\sqrt{2}-x}$ for $x \leqslant \sqrt{2}$; (10) $\frac{a^2}{b^2}\sqrt[3]{a(1+b+b^2)}$ for $b \neq 0$; (11) $|x-4|\sqrt[4]{2}$;

(12) $(x-4)^2\sqrt[4]{2}$; (13) $|x-4|^3\sqrt[4]{2}$; (14) $\frac{\sqrt{1-x}}{|x|}$ for $x < 0$ and $0 < x \leqslant 1$. **331.** (1) $\sqrt[3]{450}$; (2) $-\sqrt[4]{96}$; (3) $\sqrt{\frac{4}{3}}$; (4) $\sqrt[3]{\frac{3}{2}}$;

(5) $\sqrt{7}$; (6) $\sqrt[3]{0.2}$. **332.** (2) $\sqrt[3]{\frac{27mn}{32}}$, $\sqrt[3]{\frac{x^2}{y^2}}$, $\sqrt[3]{\frac{b}{a^2}}$; (3) $\sqrt[3]{\frac{m^5+1}{m^3}}$, $\sqrt{\frac{m+n}{m-n}}$, $\sqrt{\frac{m+n}{m-n}}$. **333.** (1) $\sqrt[5]{-\frac{16a^2}{243b}}$, $\sqrt[3]{1+a}$, $\sqrt{\frac{m-n}{m+n}}$, $\sqrt[3]{\frac{4v}{\pi}}$;

(2) $\sqrt[3]{8(1+x^4)}$, $\sqrt{mn(m+n)}$, $\sqrt{n^4-m^4}$. **334.** (1) $\sqrt[n]{mn}$, $\sqrt[n]{\frac{b^2}{a^2}}$, $\sqrt[n]{\left(\frac{x-y}{x+y}\right)^2}$; (2) $\sqrt[3]{\frac{40x^{2n}}{y^3}}$, $\sqrt[4]{a^{11}b^6}$, $-\sqrt[3]{\frac{b}{a}}$. **335.** (1) $\sqrt{m-3}$; (2) $-\sqrt{3-m}$; (3) $-\sqrt{\frac{y-x}{y+x}}$; (4) $-\sqrt{x-8}$. **337.** (2) $\sqrt{\sqrt{5}-1}$, $\sqrt{\sqrt{10}-3}$, $\sqrt[3]{5-3\sqrt{5}}$; (3) $\sqrt[3]{m-3}$, $\sqrt{x-2}$. **343.** (2) $\frac{1}{4b}\sqrt[3]{18a^2b}$, $\frac{1}{n^2p^3}\sqrt[3]{mp^2}$, $5\sqrt[4]{9a^3m^3n}$. **344.** (1) $\sqrt{2(a-b)}$, $\sqrt[3]{x^2-y^2}$, $\frac{1}{xy}\sqrt[3]{x^4y+xy^4+x^2y^2z}$; (2) $a\sqrt[n]{ab^{n-1}c}$, $\frac{1}{2m}\sqrt[2n]{2^{2n-1}m^nx^n}$, $\frac{1}{x(x-y)^2}\sqrt[m]{xy^{m-2}(x-y)^{m-2}}$. **345.** (1) $15\sqrt{3}$; (2) $2\sqrt{10}$; (3) $\sqrt[3]{6}$; (4) $-\sqrt[3]{30}$. **346.** (1) $3b\sqrt[3]{a^2b^2}$; (2) $2\sqrt[4]{4n^2m^2p^3}$; (3) $\frac{ab}{c^2}\sqrt[5]{20ab}$.

347. (1) $2xy\sqrt{y-8x}$; (3) $(x+y)\sqrt{x-y}$; (4) $2x^2y\sqrt[3]{1-3x^3y^3}$.

348. (1) $\sqrt{b-a}$; (2) $\frac{1}{b}\sqrt[4]{2b^2(1-12ab^2)}$; (3) $(x+1)\sqrt{3}$; (4) $\frac{1}{m}\sqrt[3]{m(n^2-m^2)}$. **349.** (1) $\frac{ab}{2}\sqrt[3]{a^2b^2(b^2-a^2)}$; (2) $\frac{1}{x^2-1}\sqrt{x-1}$; (3) $-\sqrt{a}$; (4) $x(1-x)$. **350.** (1) $\frac{c}{a}\sqrt[3]{5abc}$; (2) $\frac{1}{a(x-y)}\sqrt[m+n]{a^n}$; (3) $a\sqrt[4]{3a^2(a+b)}$. **359.** (1) ≈ 22.1; (2) ≈ -3.4.

360. (1) $\frac{1}{4}\sqrt{2}+3\sqrt{3}-7\sqrt[3]{2}$; (2) $-2.8\sqrt{5}$. **361.** (1) $-11.6\sqrt{2}$; (2) $11.5\sqrt[3]{9}$. **362.** (1) $6\sqrt[3]{a}(1-2a)$; (2) $2\sqrt[5]{b^2}(b-a)$; (3) 0; (4) $7\sqrt[4]{2m}$. **363.** (2) $-6b\sqrt[3]{a^2b^2}$; (3) $(0.25+3.3a)\sqrt{5x}$;

364. (1) $\sqrt{m^2-n^2}$; (2) $y\sqrt{x-2y}$; (3) 0. **365.** (1) $2y$; (2) $2x\sqrt{2}-6y$; (3) $\frac{1+a+a^2}{1+a}\sqrt[3]{a(1-a^2)}$. **367.** (1) 1733; (2) 9.24; (3) -1.8. **370** (1) $2\sqrt[3]{2}$; (2) 0.6; (3) 4; (4) $5a^4$; (5) $24\sqrt{30}$; (6) $-60\sqrt[3]{10}$. **371.** (1) 1.5; (2) $\frac{1}{120}$; (3) $6x^6y^6$; (4) $\frac{x}{y^2}\sqrt[n]{x^2y^{n-1}}$.

372. (1) -3; (2) 5; (3) 3; (4) $40\sqrt[3]{2}-4\frac{4}{9}$.

373. (1) $2ab^2+3a^2b+6a^3b^3$; (2) $\frac{1}{6}n^3-\frac{1}{4}m^2-mn$; (3) $\left(\frac{x-y}{xy}\right)^2\sqrt[3]{xy}$. **374.** (1) $14\sqrt{2}+2\sqrt{3}$. (2) $108x-45$; (3) 10; (4) $\sqrt[3]{ab}\left(\sqrt[3]{a^2}-\sqrt[3]{b^2}\right)$. **375.** (1) $\frac{7}{20}\sqrt{10}-\frac{7}{15}\sqrt{15}+\frac{7}{12}\sqrt{6}-4$; (2) 234;

(3) $15\sqrt[3]{4}+30\sqrt[3]{3}-17\sqrt[3]{2}-10\sqrt[3]{6}-10\sqrt[3]{12}-2$. **377.** (1) 2; (2) 3; (3) $5\sqrt{3}+3\sqrt{5}$; (4) $11\sqrt{11}-3\sqrt{3}$; (5) 5; (6) 3. **378.** (1) $a-b$; (2) x^2+y^2; (3) $x^3+a\sqrt{a}$; (4) $a\sqrt{a}+\sqrt[4]{b^3}$.

379. (3) $(\sqrt[4]{m}+\sqrt[8]{n})(\sqrt[4]{m}-\sqrt[8]{n})$; (5) $(a+b+\sqrt{2ab})(a+b-\sqrt{2ab})$; (6) $(a+b+\sqrt{ab})(a+b-\sqrt{ab})$ **380.** (1) $2\sqrt{x^2-1}$; (2) $\sqrt{m^2+a}$; (3) $-\frac{2}{\sqrt{m^2+1}}$. **382.** (1) $-xy^5\sqrt[3]{50}$; (2) $\sqrt[m]{2^n}-\sqrt[n]{3^m}$. **383.** (1) 4; (2) 2; (3) -2; (4) 123. **386.** (1) $\frac{1}{2}\sqrt{3}$; (2) 4; (3) $2\sqrt[3]{2}$; (4) $3\sqrt[4]{5}$; (5) 7.5; (6) $10\sqrt{2}$. **387.** (1) $\frac{3}{a}\sqrt{b}$; (2) $-\frac{a}{7b}\sqrt[3]{2}$; (3) $0.4ab$; (4) $\frac{x^2}{a^2}$; (5) $\frac{3a^2}{4b^2}$; (6) $\frac{n^2}{m^2}\sqrt[5]{m}$. **389.** (1) $60\sqrt{2}+40\sqrt{3}$; (2) 8; (3) -0.5; (4) 3. **390.** (1) $b-1$; (2) $\frac{36x^2y^2+4xy-1}{4xy}$; (3) $4x\sqrt[4]{y^3}+\frac{3}{2}y\sqrt[4]{8x^2y}-\sqrt{y}$. **391.** (1) $p^2-\sqrt{q}$; (2) $2y+3\sqrt{x}$; (3) $a+b$. **392.** (3) $x-\sqrt{xy}+y$; (4) $2x+\sqrt{6xy}+3y$.

393. (1) $-\sqrt[3]{4a^2}-3\sqrt[3]{ab}-3\sqrt[3]{2b^2}$; (2) $2\sqrt[3]{3a^2b}-\sqrt[3]{ab^2}$.

394. (5) $\sqrt[4]{m}(\sqrt[4]{n}-\sqrt[4]{m})$; (6) $\sqrt{a-b}(\sqrt{a-b}-1)$; (7) $\sqrt{a+b}(\sqrt{a-b}-1)$; (8) $\sqrt{b+c}(\sqrt{a}-\sqrt{b})$; (9) $\sqrt{m^2-mn+n^2}(\sqrt{m+n}-1)$; (10) $(a-2b)(\sqrt{a}+3\sqrt{b})$; (11) $(\sqrt{x}-1)(\sqrt{x}-3)$. **395.** (1) $(\sqrt{a}+\sqrt{b})(\sqrt{a}-\sqrt{b})$; (2) $(\sqrt{a}+2)(\sqrt{a}-2)$; (3) $(\sqrt{2}+a)(\sqrt{2}-a)$; (4) $(\sqrt{ab}+\sqrt{2})(\sqrt{ab}-\sqrt{2})$; (5) $(\sqrt{a}+b\sqrt{c})(\sqrt{a}-b\sqrt{c})$; (6) $(a\sqrt{ab}+\sqrt{5})(a\sqrt{ab}-\sqrt{5})$.

396. (2) $\frac{1}{n}\sqrt{2mn}$; (3) $\frac{1}{3}\sqrt[3]{xy^2}$. **402.** (2) $\frac{3}{32}$, 5; (3) $12, \sqrt[5]{625}$.

403. (1) 8.6481; (2) 2; (3) $-\frac{1}{12}$; (4) $\frac{35}{216}$. **404.** (1) 0.1; (2) -1; (3) -22.5; (4) 4. **407.** (1) 256; (2) $2.5ab^2$; (3) $\frac{\sqrt{a}}{a}$; (4) 2. **408.** (2) $\frac{9ab^2\sqrt[3]{a}-1}{b^2}$; (3) $\sqrt{x}+\sqrt[3]{x^2}+\sqrt[6]{x^5}-x\sqrt[6]{x}$; (4) $a^3+b^3+b\sqrt[3]{a}-2a\sqrt[6]{a^4b^3}+2ab\sqrt{ab}-2b^2\sqrt[6]{a}$; (5) $\sqrt{3}+\sqrt{a}$. (6) $m+\sqrt{mn}+n$. **409.** (1) $\frac{1}{4}\sqrt{x}$; (2) $-4\sqrt{a}$; (3) $\sqrt[3]{(m-n)^2}$;

(4) 2. **411.** (1) $a^2\sqrt[12]{a^5}$; (2) $a^3b\sqrt[12]{a^5b^7}$; (3) $\frac{\sqrt[6]{x(y-x)}}{x^2}$;

(4) $a^4b^2\sqrt[6]{243a^4b^2}$. **412.** (2) $2\sqrt{10}$; (4) $8\sqrt[6]{486}$; (6) $b\sqrt[4]{a^3b^2}$.

413. (1) $3\sqrt[4]{3}$; (2) $2\sqrt{2}+4\sqrt[3]{2}-4$; (3) $12-2\sqrt[6]{243}$;

(4) $9+\sqrt[4]{108}-2\sqrt[12]{243}$. **414.** (1) $\frac{a^2\sqrt[6]{a^5}-\sqrt[15]{a^{11}}-a\sqrt[3]{a^2}}{3a^2}$;

(2) $\frac{a\sqrt[4]{b^3}-\sqrt[6]{b^5}}{b}$; (3) $b\sqrt[8]{2a^4b}+ab\sqrt[12]{8a^5b^{10}}-ab^2\sqrt[4]{a^3b}$;

(4) $\frac{2x\sqrt[6]{mn^5}+2y\sqrt[3]{mn^2}}{nxy}$. **415.** (1) $m-\frac{4}{m^2}\sqrt{mn}$; (2) $a\sqrt[3]{a}-b\sqrt[3]{b}$;

(3) $2\sqrt[12]{a^7b^{10}}-b\sqrt[12]{a^5}+3b\sqrt[12]{a^4b}+48\sqrt[6]{a^5b}-12b\sqrt[4]{a^3b}-8ab$.

416. (2) $3x^{\frac{1}{3}}-3x^{-\frac{1}{3}}+2x^{-1}$; (3) $a\sqrt[3]{2b}+a\sqrt[6]{2b^4}+ab$.

417. (1) $m\sqrt{3mn}$; (2) $\sqrt[3]{xy^2}-\sqrt{xy}$; (3) $\sqrt[3]{x^2y}-\sqrt[4]{xy^2}$.

420. (1) m^5n, $\frac{b^2}{a^6}$, $a^3\sqrt[5]{ab^4}$; (2) $64a^2b^4$, $\frac{m+n}{m-n}\sqrt[3]{\frac{m+n}{m-n}}$;

(3) $\frac{1}{a^3(a+b)^5}$, $x\sqrt[n]{x^2}$, $\sqrt[n+1]{a^{3n}}$; (4) $-0.001a^2b\sqrt{2b}$, $\frac{a^6}{4b^{2n+2}}$;

(5) $\frac{16}{81a^4}$, $\frac{a-b}{a^3}$. **421.** (2) $4a+9b-12\sqrt{ab}$, $15+2\sqrt{15}+2\sqrt{21}+2\sqrt{35}$,

$10-2\sqrt{6}+2\sqrt{10}-2\sqrt{15}$; (3) $\sqrt[3]{a^2}-4\sqrt[3]{ab}+4\sqrt[3]{b^2}$,

$\frac{\sqrt[3]{x^2y^2}\left(3b\sqrt[3]{x^2y^2}+12\sqrt[3]{xy}+1\right)}{4y^2}$. **422.** (1) $2-4\sqrt[6]{3}+2\sqrt[3]{2}$, $\frac{m+2\sqrt{mn}+n}{mn}$,

$a^3+2ab\sqrt{ab}+b^3$; (2) $\sqrt[3]{36}-4\sqrt[3]{3}+2\sqrt[3]{2}$, $a+3\sqrt[3]{a^2b}+3\sqrt[3]{ab^2}+b$,

$a^2-3a\sqrt[3]{a^2}+3a\sqrt[3]{a}-a$; (3) $\sqrt[3]{25}+4\sqrt[6]{200}+8$, $55-63\sqrt{2}$,

$a^3+a^2-2a^2\sqrt{2}+\frac{a}{2}$. **424.** (1) 6; (2) 2; (5) $24+4\sqrt{30}$; (6) 2.

427. (1) 0.09; (2) 3. **429.** (1) $1+\sqrt{6}$; (3) $\frac{\sqrt{6}-\sqrt{2}}{2}$; (4) $\sqrt{2}+\sqrt{3}$;

(6) $|\sqrt{m}-\sqrt{n}|$; (7) $2\sqrt{x}+\sqrt{y}$. **430.** (4) $\sqrt[12]{x^7}$, $\sqrt[8]{\frac{x}{y^2}}$, $\sqrt[12]{\left(\frac{a-1}{a+1}\right)^5}$.

431. (1) $\sqrt[8]{\frac{x}{1+x}}$, a^2, $-\sqrt[12]{80x^{11}y^4}$; (2) $\sqrt[8]{x^7}$, $\sqrt[8]{8}$, $\sqrt[27]{x^{26}}$. (3) $\sqrt[8]{\frac{a^3}{b^8}}$,

$\sqrt[18]{\frac{32}{243}}$; (4) 1, $\sqrt[24]{a^7b^7}$. **432.** (1) $\sqrt[4]{18}+\sqrt[4]{2}$; (2) $\sqrt{2}+1$; (3) $\sqrt{3}-1$;

(4) $\sqrt{5}-2$; (5) $\sqrt{6}+\sqrt{2}$; (6) $2\sqrt{2}$; (7) -2. **433. 1.** (a) $\sqrt{2}-1$, (b) $2+\sqrt{3}$, (c) $\sqrt{7}-\sqrt{3}$, (d) $\sqrt{5}+\sqrt{2}$; (e) $1.5\sqrt{3}-2.5$. **2.** $x \leqslant -2$ and $x \geqslant 2$. **434. 2.** $\sqrt{a-1}$ if $a \geqslant 2$ and 2 if $1 \leqslant a \leqslant 2$.

435. (2) $\frac{\sqrt{6}}{4}, -\frac{\sqrt{12}}{3}, \frac{3\sqrt[3]{2}}{2}, (3+x)\sqrt[4]{3-x}, \frac{\sqrt[n]{x^2}}{x}$; (3) $\sqrt[n]{m^{n-1}}$, $2\sqrt[n+1]{t^2}, \frac{\sqrt{x^2-1}}{x+1}, \frac{\sqrt[10]{a^7}}{a}$; (4) $\frac{\sqrt[4]{m^3n}}{n}, \sqrt[5]{49}, \sqrt[n]{a+b}$; (5) $\frac{\sqrt{x^2-9}}{x-3}, \frac{\sqrt{a^2-b^2}}{2(a+b)}$, $\frac{\sqrt{a^{2n}-1}}{a^n+1}$. **436.** (10) $\frac{35-12\sqrt{6}}{19}$; (11) $\frac{m^3+2mn\sqrt{mn}+n^3}{m^3-n^3}$; (12) $\frac{156+5\sqrt{15}}{263}$. **437.** (1) $-\sqrt[3]{2\sqrt{6}+5}$; (2) $\frac{2\sqrt{5}-\sqrt{11}}{3}$; (3) $\sqrt{m-\sqrt{m^2-n^2}}$; (4) $\frac{\sqrt{x^2-9}-x}{3}$; (5) $-\frac{1+\sqrt{1-a^{2n}}}{a^n}$; (6) $m+\sqrt{m^2-n^2}$. **438.** (1) $\frac{10+7\sqrt{2}}{4}$; (2) $\frac{(\sqrt{a}+\sqrt{b})^2}{a-b}$; (3) $2(3+2\sqrt{2})$; (4) $153+108\sqrt{2}$; (5) $2(337-52\sqrt{42})$; (6) $362-209\sqrt{3}$; (7) $(2-\sqrt[4]{3})(4+\sqrt{3})$; (8) $-(\sqrt[4]{3}+\sqrt{2})(\sqrt{3}+2)$.

439. (1) $\frac{1}{2}(4+3\sqrt{3}+\sqrt{5}-2\sqrt{15})$; (2) $\frac{3}{2}(5\sqrt{2}-4\sqrt{3}+3\sqrt{6}-6)$; (3) $\frac{1}{30}(3\sqrt{10}-5\sqrt{6}+4\sqrt{15})$; (4) $\frac{\sqrt{m}(\sqrt{m}+\sqrt{n}-\sqrt{p})(m+n-p-2\sqrt{mn})}{m^2+n^2+p^2-2mn-2mp-2np}$; (5) $\frac{(\sqrt{a}-\sqrt{b})(\sqrt{a}+\sqrt{b}+\sqrt{c})(a-b-c+2\sqrt{bc})}{a^2+b^2+c^2-2ab-2ac-2bc}$; (6) $-2-\sqrt{2}-\sqrt{3}-\sqrt{6}$; (7) $\frac{1}{2}(\sqrt{10}+\sqrt{21}-\sqrt{14}-\sqrt{15})$; (8) $2(3\sqrt{3}-\sqrt{2}-\sqrt{6}+3)$.

440. (2) $\frac{1}{6}(\sqrt[8]{3}-\sqrt[8]{2})(\sqrt[4]{3}+\sqrt[4]{2})(\sqrt{3}+\sqrt{2})$; (3) $0.1(\sqrt[3]{9+4\sqrt{5}}+\sqrt[3]{9-4\sqrt{5}}-1)\sqrt{5}$; (4) $\frac{1}{37}(1+\sqrt[4]{108}-2\sqrt[4]{12})(8+\sqrt{27})$; (5) $\sqrt[3]{3}-\sqrt[3]{2}$; (6) $\sqrt{(\sqrt[3]{3}-\sqrt{2})(3\sqrt[3]{3}+2\sqrt[3]{9}+4)}$; (7) $-\frac{1}{4}(\sqrt[3]{2}+\sqrt{2})(2\sqrt[3]{2}+4\sqrt[3]{4}+4)$; (8) $\frac{1}{2}(2-\sqrt[4]{8})$; (9) $-\frac{1}{68}(4+\sqrt[3]{4})$. **441.** (1) $\frac{1}{\sqrt{5}+1}$; (2) $\frac{1}{3-\sqrt{7}}$; (3) $\frac{1}{2(\sqrt{5}+\sqrt{3})}$; (4) $\frac{1}{3(\sqrt{12}-\sqrt{10})}$; (5) $\frac{1}{(x+\sqrt{x^2-1})^2}$; (6) $\frac{(a-b)^2}{(\sqrt{a}+\sqrt{b})^5}$;

(7) $\dfrac{1}{\sqrt[3]{x^2}-\sqrt[3]{xy}+\sqrt[3]{y^2}}$. **443.** (1) 28; (2) $-2\sqrt{3}$; (3) $2\frac{1}{3}$;
(4) $-\frac{1}{2}(2+11\sqrt{2})$. **444.** (1) $9-\sqrt{11}+2\sqrt{7}$; (2) 1; (3) $14+6\sqrt{5}$;
(4) 3. **445.** (1) $\frac{1}{2}\sqrt[3]{11}$; (2) -2; (3) 5; (4) -2.

446. (1) $\sqrt{a^2+1}$; (2) $\frac{1}{x}(1+\sqrt{1-x^2})$; (3) $2+\sqrt[3]{a^2}$; (4) 4.

447. (1) $\frac{2}{3}n$; (2) $\dfrac{\sqrt{b}-1}{\sqrt{ab}}$; (3) $\sqrt{a-b}$; (4) a^2+a+1.

448. (1) $\frac{1}{2}(\sqrt{2+a}+\sqrt{2-a})$; (2) 3; (3) $-2\sqrt[4]{x}$; (4) 2.

449. (1) $\sqrt{1-a}$; (2) $\sqrt[3]{(x-y)^2}$. **450.** (1) $a+b$; (2) $-4\sqrt{x}$.

451. (1) $\dfrac{\sqrt{mn}}{n}$; (2) 2. **452.** (1) 10; (2) $\frac{1}{3}(56\sqrt{10}+12)$.

457. (1) -1 if $m > n$ and 1 if $m < n$; (2) 2 if $m > 2$ and -2 if $1 \leqslant m < 2$; (3) $\dfrac{(\sqrt{m}-1)\sqrt{1-m}}{\sqrt{m}}$ if $0 < m \leqslant 1$ and $\dfrac{(1-\sqrt{m})\sqrt{m-1}}{\sqrt{m}}$ if $m > 1$. **466.** $(-3, \pm 3\sqrt{3})$, $(3, \pm 3\sqrt{3})$. **471.** 24.5.

478. 2. (a) 15 km, (b) 10 km, 8h 30 min. **485.** 1. $y = x^2+1$.

488. 1. $s = \begin{cases} 4.5t \text{ for } 0 \leqslant t \leqslant 5 \\ 22.5 \text{ for } 5 \leqslant t \leqslant 5.5 \\ 0.5+4t \text{ for } 5.5 \leqslant t \leqslant 9. \end{cases}$

495. 1. (a) (2, 0), (−3, 0), (−4, 0), (0, −48), (b) (0, 3), (c) (0, 0), (3, 0), (−4, 0), (d) (0, 2). 2. (a) $x = -1$, $y < 0$ for $-\infty < x < -1$, $y > 0$ for $-1 < x < \infty$, (b) $x_1 = -1$, $x_2 = 2$, $y < 0$ for $-\infty < x < -1$ and for $2 < x < \infty$, $y > 0$ for $-1 < x < 2$, (c) $x = 0$, $y < 0$ for $-\infty < x < 0$, $y > 0$ for $0 < x < \infty$, (d) $x_1 = -2$, $x_2 = 2$, $y < 0$ for $-2 < x < 2$, $y > 0$ for $-\infty < x < -2$ and $2 < x < \infty$. **496.** (9) $x \neq 0$, $x \neq 2$; (10) $x \geqslant -1.5$;
(11) x any real number; (12) $x \leqslant 0.5$; (13) $x > 1\frac{1}{3}$;
(14) $x < -2$ and $x \geqslant 3$; (15) $1 < x < 2$ and $x > 2$.

497. (1) $-2 < x \leqslant 0$; (2) $-1 \leqslant x \leqslant 5$; (3) $-2 \leqslant x \leqslant 4$;
(4) no domain; (5) $-2 \leqslant x \leqslant 15$; (6) $x \leqslant -\sqrt{2}$ and $x \geqslant \sqrt{2}$;
(7) the complete set of real numbers. **498** (1) $(-\infty, \infty)$;

(2) $y \neq 0$; (3) $3 \leqslant y < \infty$; (4) $(-\infty, \infty)$; (5) $-8 \leqslant y < \infty$; (6) $-\infty < y \leqslant 5$; (7) $y \geqslant 0$; (8) $-\infty < y \leqslant 0$; (9) $2 \leqslant y < \infty$; (10) $0 \leqslant y \leqslant 1.5$; (11) $y < 0$ and $y \geqslant 1.5$; (12) $y = 1$ and $y = -1$.

499. (1) $-15 \leqslant y < \infty$; (2) $\frac{3}{4} \leqslant y < \infty$; (3) $-\infty < y \leqslant 4$; (4) $0 \leqslant y < \infty$; (5) $0 \leqslant y < \infty$; (6) $1 \leqslant y < 3$; (7) $0 < y < \infty$; (8) $0 \leqslant y < 1$ and $1 < y < \infty$; (9) $2 < y \leqslant 3$; (10) $-3 \leqslant y \leqslant 3$; (11) $-1 \leqslant y \leqslant 1$; (12) $-1 \leqslant y \leqslant 1$. **502.** (3) Decreases everywhere, except for $x = 0$; (4) $(-\infty, 0)$—increases; $(0, \infty)$—decreases; (5) $(-\infty, 1)$—decreases; $(1, \infty)$—increases; (6) $(-\infty, 2)$—decreases; $(2, \infty)$—increases; (7) $(-\infty, -2)$—decreases; $(-2, \infty)$—increases; (8) $(-\infty, 1.5)$—increases; $(1.5, \infty)$—decreases; (9) $(-\infty, -4)$ and $(-1.5, 1)$—decreases; $(-4, 1.5)$ and $(1, \infty)$—increases; (10) increases everywhere; (11) decreases everywhere; (12) decreases everywhere, except for $x = 0$. **503.** (2) Not obligatory. **504.** (1) 16; (2) 9; (3) 9; (4) 1; (5) 4; (6) 5; (7) none; (8) 4; (9) $\frac{2\sqrt{3}}{3}$; (10) 2; (11) 1; (12) $\frac{1}{4}\sqrt{34}$. **505.** (1) -9; (2) 5; (3) -9; (4) $-6\frac{1}{4}$; (5) $\sqrt{3}$; (6) -2; (7) 3; (8) 0; (9) 0; (10) none; (11) -24. **506.** 9 and 9. **507.** 25 and 50. **508.** 1, 5, 8, 9, 11, 12 even; 2, 4, 7 odd. **509.** (1), (3), (8) even; (2), (4), (9) odd.

511. (1) $y \to +0$ for $x \to +\infty$, $y \to -0$ for $x \to -\infty$; (5) $y \to -0$ for $x \to \pm\infty$, (7) $y \to +0$ for $x \to +\infty$; (8) $y \to \frac{1}{2}$ for $x \to \pm\infty$; (10) $y \to +\infty$ for $x \to \pm\infty$; (12) $y \to -\infty$ for $x \to \pm\infty$. **512.** (2) $y \to -\infty$ for $x \to -3-0$; $y \to +\infty$ for $x \to -3+0$; (3) $y \to -\infty$ for $x \to -2+0$ and for $x \to 2-0$, $y \to +\infty$ for $x \to -2-0$ and for $x \to 2+0$.

522. 1. $4x-3y=0$; **2.** $bx-ay=0$. **529. 1.** No. **535. 1.** 1.5. **2.** -4.5. **539. 2.** (a) $x > 2$, (b) $x < 2$, (c) $x > 5$, (d) $x \geqslant 6$, (e) $3 < x < 5$. **540. 1.** $k_1 = k_2$, $b_1 \neq b_2$. **543. 1.** $k = b = 1$.

547. 1. (a) (1, 1), (b) (1, 1), $(-1; -1)$, (c) (1, 1), $(-1, 1)$.

563. 2. (a) $(2, -9)$, (b) $\left(-1\frac{1}{2}, 2\frac{3}{4}\right)$, (c) $(-1, 4)$, (d) $\left(2\frac{1}{2}, 5\frac{1}{4}\right)$. **564. 1.** $y_{\text{least}} = -8$ for $x = 1$, $(-\infty, 1)$—decreases, $(1, \infty)$ — increases, $y > 0$ for $x < -1$ and $x > 3$, $y < 0$ for $-1 < x < 3$, $(1, -8)$ vertex. **566. 2.** (a) $\left(\frac{3}{4}, 2\frac{7}{8}\right)$, $y_{\min} = 2\frac{7}{8}$; (b) $\left(1\frac{1}{2}, 6\frac{3}{4}\right)$; (c) $(-2, 9)$, $y_{\max} = 9$;

(d) $(-1; 2)$, $y_{\min} = 2$. **568. 2.** 6 and 6; **570. 3.** (a) Yes; (b) no; (c) yes; (d) no. **571.** (3) ± 0.75; (4) ± 2.4; (7) ± 0.3.

572. 1. (a) ± 2, (b) 0; **3.** $c > -2$. **573.** (1) ± 2; (2) ± 4; (3) ± 2; (4) $-4\frac{2}{7}$; (5) 0, 1.5; (6) $0, \sqrt{3}$. **574.** (1) $\pm(2a-1)$; (2) ± 1; (3) $0, \frac{b^3-a}{b}$; (4) $a-b, b-a$; (5) $0, \frac{a^2+a+2b}{a-1}$; (6) $\pm 2a$.

579. (1) $-3, 2$; (2) $-3, 5$; (3) 4, 6. **580.** (1) 2, 5, 6; (2) $-4, 1\frac{5}{6}$; (3) $-\frac{34}{71}, 1$; (4) $-1, 8$. **581.** (1) $-\frac{7}{9}, 7$; (2) 2, 16; (3) $\frac{12}{13}, 6$; (4) $-3\frac{17}{12}, 5$. **582.** (1) 0.75, 1; (2) -1; (3) 2; (4) $4 \pm \sqrt{7}$; (5) $1\frac{3}{7}$; (6) 1. **583.** (1) $-2, 0$; (2) $-1, 4, 1$; (3) $-3, 4, 2$; (4) $-4, 5$. **584.** (1) -2.5; (2) 8; (3) $-2.5, 0$; (4) $\frac{1}{2}(5 \pm \sqrt{17}), \frac{1}{2}(5 \pm \sqrt{7}), 2.5$. **585.** (1) $-2, 0.5$; (2) $-1\frac{7}{8}, 1$; (3) $-3.5, -1$; (4) 2. **586.** (1) $-1.5, \sqrt{2}$; (2) $\frac{2}{3}(2\sqrt{6} \pm \sqrt{15})$; (3) $\frac{1}{4}(\sqrt{2} \pm 1)$; (4) $-\frac{1}{2}(\sqrt{6}+\sqrt{2})$; $\frac{1}{2}(\sqrt{6}-3\sqrt{2})$.

587. (1) $-0.5, 0.7$; (2) $-2.5, 0.1$; (3) $-1.3, -0.2$; (4) 0.4, 7.1.

588. (1) $8 \pm \sqrt{57}, 8 \pm \sqrt{73}$; (2) $-4, -2, -1.1$; (3) $-\frac{1}{6}, 1$; (4) $-1\frac{2}{3}, -\frac{2}{3}$; (5) $-2, -\frac{2}{3}, -\frac{1}{3}, 1$; (6) 1, 3.

589. (1) $-2, 1$; (2) -1; (3) 1; (4) 0, 1, 3, 4. **590.** (3) $1, \frac{1}{a}$; (4) $2p, 2q$. **592.** (1) $-0.5a, 2a$; (2) 0; $\frac{2ab}{a+b}$; (3) $-a, -b$; (4) $\frac{-(m+n+p) \pm \sqrt{m^2+n^2+p^2-mn-mp-np}}{3}$. **593.** (1) $a+b, a-b$; (2) $\frac{1}{3}m, \frac{1}{5}m$; (3) $\frac{1}{4}(a+3b), \frac{1}{6}(a+5b)$. **594.** (1) $m-2, n$; (2) $-a, b+3$; (3) $a, b-3$. **601. 1.** (a) $x^2+18x+56 = 0$; (b) $25x^2+45x+14 = 0$. **2.** (a) $108x^2-33x-1$; (c) $cx^2+bx+a = 0$.

602. 1. (a) $x^2-1 = 0$; (b) $x^2+8x+16 = 0$. (c) $x^2-px+q+0$; (d) $2x^2-9x+10 = 0$. **2.** (b) $x^2-10x+9 = 0$. **603.** (1) (a) p^2-2q, $\pm p\sqrt{p^2-4q}$; (b) $\frac{b^2-2ac}{a^2}$, $\pm\frac{b}{a^2}\sqrt{b^2-4ac}$; (2) (a) $-p(p^2-3q)$, $\pm(p^2-q)\sqrt{p^2-4q}$; (b) $-\frac{b}{a^3}(b^2-3ac)$, $\pm\frac{b^2-ac}{a^3}\sqrt{b^2-4ac}$;

605. 1. (a) ± 4, (b) ± 5, (c) ± 6. **2.** (a) 3, (b) 4, (c) 1.

606. (1) 3, 5; (2) -20; (3) 12. **607. 1.** $1\frac{14}{23}$, 3; **2.** (a) $-\frac{13}{16}$, (b) -10, (c) 2.25, (d) 1.25. **608. 2.** $3\frac{3}{8}$; $-15\frac{5}{8}$.

609. (1) $x^2+(p-2)x+1-p+q=0$;
(2) $x^2+(2q-p^2)x+q^2=0$; (3) $qx^2+2(p+q)x+(4+2p+q)=0$;
(4) $qx^2+p(q+1)x+(q+1)^2=0$; (5) $qx^2+(p^2+2q)x+q=0$;
(6) $p\sqrt{p^2-4q}\,x^2\pm 2(p^2-2q)x+p\sqrt{p^2-4q}=0$.

611. (3) $(x+5)(7-x)$; (5) $(2x+1)(4x+3)$;
(7) $n(2mx+1)(1-mnx)$; (8) $(x+a+b)(2x+a+2b)$.

612. (3) $(2ax-b)(ax+b)$; (4) $(2m-3ny)(m-ny)$;
(5) $(2x+a-b)(2x-a)$; (6) $(2a^2b-x)(ab^2+x)$. **613.** (2) $\frac{2(a-9)}{3(a+7)}$;
(3) irreducible; (4) $\frac{3a-8b}{2a+b}$; (5) $\frac{2m-n}{2n-m}$; (6) $\frac{x-1}{1-3x}$. **614. 3.** (a) ± 40,
(b) $\pm 8\sqrt{3}$. **616.** (1) 2.25; (2) $c > -\frac{1}{12}$; (3) $c < -2$; (4) $c \leqslant \frac{1}{32}$;
(5) $c > \frac{1}{3}$. **617. 1.** (a) ± 4, (b) $\pm 2\sqrt{3}$, (c) $b < -\sqrt{3}$, $b > \sqrt{3}$,
(d) $b < -\sqrt{40}$, $b > \sqrt{40}$, (e) b any number, (f) $-\sqrt{40} < b < \sqrt{40}$.
2. (a) $\frac{13}{14}$, (b) $\frac{5}{17}$, 1. **619. 1.** $a = n(n+1)$, where n is a natural number. **2.** $k \leqslant -2.55$. **621.** $b < 0$ and $c < 0$, $b > 0$ and $c < 0$.

622. 1. $a > 0$ and $c < 0$. **4.** $ac = -12$. **623. 2.** (a) $p \geqslant -2\frac{1}{8}$, (b) p any real number. **624.** 12 and 13. **625.** 53 and 35.

626. 7. **627.** $\frac{3}{7}$. **628.** 12. **629.** 51. **630.** 110 and 150.

631. 5 km/h. **632.** 17 m³. **633.** 160 m. **634.** 4 m and 5 m; 60 m². **635.** 5 cm; 0; 34 m². **636.** 0.5 m. **637.** 18 cm and 36 cm.
638. 22.5 cm/s; 30 cm/s; the bodies will meet at the vertex.

639. 24 kg and 30 kg. **640.** 410 and 350; 2 t and 3 t. **641.** 8 t and 17 t. **642.** 20. **644.** 16. **645.** 3 and 4. **646.** 50 km/h and 60 km/h. **647.** 6 km/h. **648.** 45 km/h and 48 km/h. **649.** At 6 P.M. **650.** 5 and 6. **652.** 2.4 km/h. **653.** 2.5 m and 3.5 m.

654. 5 and 7.5. **655.** 20 and 32. **656.** 15 h, 10 h, 6 h.
657. 8 min. **658.** 1 h 50 min and 2 h 12 min. **659.** 10 and 15.
660. 8 and 12. **661.** 6 h and 10 h. **662.** 11 and 14. **663.** 6 h and 8 h. **664.** 90 and 60 or 48 and 144. **665.** 6 and 4. **666.** 5 %.
667. 2 % and 3 %. **668.** 10 %. **669.** 20 l.

670. $\frac{1}{2p}(np+\sqrt{n^2p^2+4mnp})$. **671.** $\frac{1}{t}(d+\sqrt{d^2+t^2v^2})$ km/h.

672. $\frac{1}{2m}(mp+\sqrt{m^2p^2+4mnp})$; $\frac{1}{2p}(\sqrt{m^2p^2+4mnp}-mp)$ trees per row. **673.** $\frac{1}{2m}(\sqrt{m^2p^2+240mkp}-mp)$ and $\frac{1}{2m}(\sqrt{m^2p^2+240mkp}+mp)$. **674.** $\frac{a_2\sqrt{3}-a_1}{v_2\sqrt{3}-v_1}$. **675.** (1) -3, -2, 2.5, 3; (4) $-\frac{1}{3}$, 0, ± 2; (6) ± 1, ± 2. **676.** (3) 0.5, 1, 2; (4) a, $\frac{-a\pm\sqrt{a^2+4}}{2}$; (5) $\pm a$. **677.** (1) $-\frac{2}{3}$, $-\frac{1}{2}$, 3; (2) $\frac{1}{3}$, 3; (3) -3, 1; (4) -1, 5. **678.** (2) 1.5, $\frac{3}{4}(-1\pm i\sqrt{3})$; (3) ± 3, $\pm 3i$; (4) ± 0.5, $\pm 0.5i$; (5) -1, 1, $\frac{1}{2}(-1\pm i\sqrt{3})$, $\frac{1}{2}(1\pm i\sqrt{3})$; (6) ± 1, $\pm i$, $\frac{\sqrt{2}}{2}(-1\pm i)$, $\frac{\sqrt{2}}{2}(1\pm i)$; (7) $\frac{\sqrt{2}}{2}(-1\pm i)$, $\frac{\sqrt{2}}{2}(1\pm i)$; (8) $\pm i$, $-\frac{\sqrt{3}}{2}\pm\frac{1}{2}i$, $\frac{\sqrt{3}}{2}\pm\frac{1}{2}i$. **679.** (3) ± 2, $\pm 3i$; (5) ± 0.5; $\pm i\sqrt{2}$; (6) $\pm\frac{2}{3}$, $\pm\frac{1}{3}$. **680.** (1) ± 2, $\pm m$; (2) ± 0.5, $\pm a$ (3) ± 1, $\pm a^2$; (4) $\pm a$, $\pm b$. **681.** (1) $\pm\sqrt{a+b}$, $\pm\sqrt{a-b}$; (3) $\pm\frac{1}{a}\sqrt{a+a^2}$, $\pm\frac{1}{a}\sqrt{a-a^2}$; (4) $\pm(a+b)$, $\pm(a-b)$; **682.** (1) ± 2, $\pm i\sqrt{2}$; (2) ± 1; $\pm 1.5i\sqrt{2}$. **686.** (1) $x^4+(2q-p^2)x^2+q^2$; (2) $a^2x^4+(2ac-b^2)x^2+c^2$. **687.** (2) $(x+\sqrt{2})(x-\sqrt{2})(x+3)(x-3)$; (3) $(x+\sqrt{3})^2(x-\sqrt{3})^2$; (5) $(x^2+1)(4x^2+1)$; (6) $(x+3)(x-3)(x\sqrt{2}+1)(x\sqrt{2}-1)$; (7) $(x+\sqrt{a})(x-\sqrt{a})(x+\sqrt{b})(x-\sqrt{b})$; (8) $(x+1)(x-1)(x+\sqrt{ab})(x-\sqrt{ab})$. **688.** (2) $\frac{2a^2+3}{3(a^2-1)}$; (3) $\frac{5a^2-3b}{a^2+2}$. **689.** (2) $y>0$ for $2<x<3$, $y<0$ for $x<2$ and $x>3$, $y=0$ for $x=2$ and $x=3$; (3) $y>0$ throughout the x-axis; (4) $y>0$ everywhere, except for $x=0.5$, where $y=0$.

690. (1) $x<1$ and $x>4$; (2) $-4<x<0$; (3) $x\leqslant -1.5$ and $x\geqslant 5$; (4) $-2\frac{1}{3}<x<3\frac{3}{4}$; (5) $x<0$ and $x>4$;(6) $x=-3$.

691. (1) No solutions; (2) x any real number; (3) $-9 \leqslant x \leqslant -4$; (4) $x = \pm\sqrt{2}$; (5) no solutions; (6) x any real number.

692. (1) $x < -2$ and $x > 5$; (2) $x \neq 1.5$; (3) x any real number; (4) $x < -1$ and $x > 2$; (5) $x \leqslant 1$ and $x \geqslant 4$; (6) $x \leqslant \frac{-9-\sqrt{41}}{2}$ and $x \geqslant \frac{-9+\sqrt{41}}{2}$; (7) $3 < x < 6$; (8) x any real number; (9) $7-\sqrt{38} < x < 7+\sqrt{38}$. **693.** (2) $x < -2$, $\frac{1}{4} < x \leqslant 1$ and $x \geqslant 4$; (3) $x < 1$, $1.5 < x < 2$ and $x > 3$; (4) $x > 4$; (5) $-1 < x < 3$; (6) $1 < x < 1.5$ and $x > 2$. **694.** (1) $x < -1$ and $x > 2.5$; (2) $-4 < x < 1$; (3) $-2 < x < -1$ and $x > 0$; (4) $-4-\sqrt{21} < x < -1$ and $x > \sqrt{21}-4$; (5) x any real number; (6) $x < -1$, $0 < x < 0.5$ and $x > 1$. **695.** (1) $-3 < x < -2$ and $-1 < x < 1$; (2) $-2 < x < 1.25$, $-1 < x < 1$ and $x > 5$; (3) $x < -5$, $-2.5 < x < -1$ and $x > 0$. **696.** (1) $x < 1-\sqrt{6}$, $x > 1+\sqrt{6}$, $1-\sqrt{2} < x < 1+\sqrt{2}$; (2) $1 < x < 5$; (3) x any real number, except for $x = 4$; (4) $-6 < x < -1$ and $2 < x < 3$.

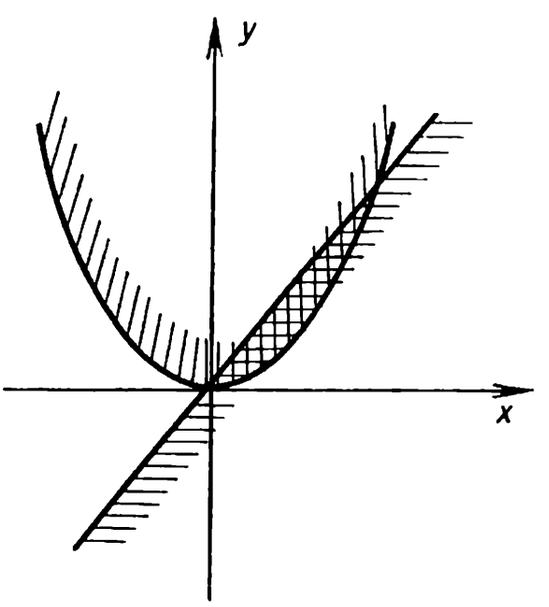
697. (1) $-3 < x < -2$ and $2 < x < 3$; (2) $x < -5$, $-2 < x < 2$ and $x > 5$; (3) $x < -0.4$, and $x > 0.4$; (4) $-0.5(9+\sqrt{101}) < x < -1$, $0 < x < \frac{1}{2}(-9+\sqrt{101})$.

698. (1) $x < -1$, $3 < x \leqslant 4$ and $x \geqslant 7$; (2) $1 < x < 1.5$; (3) no solution; (4) $1 < x < 7$; (5) $-1 < x < 1$ and $2 < x < 4$; (6) $0.5(-3-\sqrt{13}) < x < 0.5(-1-\sqrt{5})$ and $0.5(-3+\sqrt{13}) < x < 0.5(-1+1\sqrt{5})$. **699.** (1) $6 < x < 9$; (2) $-\frac{2}{3} < x < -\frac{1}{3}$ and $2 < x < 2.5$; (3) $-3 < x < -1$ and $5 < x < 6$; (4) $-4 < x < -2$ and $4 < x < 4.5$. **700. 1.** 0, 2, 4, 6, 8; **2.** $\pm 19, \pm 17, \pm 15, \pm 13, \pm 11, \pm 9, -7, -5, -3$. **701. 1.** 10, 9, 8, 7, $\pm 6, \pm 5, \pm 4, \pm 3, \pm 2$; **2.** (0, 0), (0, 1), (1, 1), (1, 2), (2, 0), (2, 1). **702.** (1) $2 < x < 4$; (2) $x > 2$.

703. (3) $-0.4 \leqslant x \leqslant 2$; (4) $x \neq 1$; (5) x any real number; (6) no domain; (7) $x \leqslant -1$ and $x > 2$; (8) $-1 < x \leqslant 1$ and $x > 2$; (9) $x \neq -1$ and $x \neq -3$. **704.** (1) (a) -1, -0.2, (b) $a < -1$ and $a > -0.2$, (c) $-1 < a < -0.2$; (2) (a) 0.5 and 2, (b) $0.5 < a < 2$, (c) $a < 0.5$ and $a > 2$; (3) (a) $\frac{3}{7}$ and 5, (b) $\frac{3}{7} < a < 5$, (c) $a < \frac{3}{7}$ and $a > 5$. **705.** (1) $m > 9$; (2) $0 < m < 28$; (3) $m \leqslant 0$ and $m \geqslant 4$; (4) $-\frac{1}{2} < m < \frac{1}{3}$; (5) $m < -7.2$; (6) $m < -3\frac{9}{16}$;

(7) $-2 < m < 2$; (8) $-1 \leqslant m \leqslant 2$. **706.** (1) If $a > 0$, then $x < \frac{1-\sqrt{1+4a}}{2a}$ and $x > \frac{1+\sqrt{1+4a}}{2a}$; if $a = 0$ then $x < -1$; if $-\frac{1}{4} < a < 0$, then $\frac{1+\sqrt{1+4a}}{2a} < x < \frac{1-\sqrt{1+4a}}{2a}$; for $a \leqslant -\frac{1}{4}$ no solution; (2) if $|a| > \sqrt{24}$, then $\frac{-a-\sqrt{a^2-24}}{4} < x < \frac{-a+\sqrt{a^2-24}}{4}$; if $|a| \leqslant \sqrt{24}$, then there is no solution; (3) if $a < 1$, then $x < 1 - \sqrt{1-a}$ and $x > 1+\sqrt{1-a}$; if $a = 1$, then x is any real number, except for 1; if $a > 1$, then x is any real number; (4) if $a > 0$, then $x_1 < x < x_2$; if $a < -20$, then x is less than the least root of the trinomial and x is greater than the greatest root of the trinomial; if $-20 < a \leqslant 0$, then x is any number; if $a = -20$, then x is any number, except for $x = -0.5$.

707. (1)



709. (3) $-1 \leqslant x \leqslant 4$; (4) $x \geqslant 3$; (5) x any real number; (6) $-\sqrt{10} \leqslant x \leqslant \sqrt{10}$; (7) $x \geqslant 2$; (8) x any real number. **711.** (1) No; (2) yes, within the set of real numbers; (3) no; (4) yes.

712. (1) 4; (2) 3; (3) ± 3; (4) -2, 5; (5) no solution; (6) 53.

713. (1) 0; (2) 2; (3) -0.4, 1; (4) 0, 2, 9; (5) ± 3. **714.** (1) 7; (2) -2; (3) $\frac{1}{3}$, 4; (4) ± 1. **715.** (1) 6; (2) $-\frac{1}{3}$, 3; (3) 3, 5; (4) ± 1; (5) ± 2; (6) $1 \pm \sqrt{6}$. **716.** (1) -1, $-\frac{34}{49}$; (2) 7; (3) 0; (4) 1; (5) 10; (6) 3; (7) -2.5; (8) 2. **717.** (1) 5; (2) 1.25; (3) 13; (4) 1.5; (5) 3, 0.8; (6) -2. **718.** (1) 4; (2) 9; (3) 4, $1\frac{80}{361}$; (4) 4.

719. (1) ± 5; (2) 6; (3) ± 5; (4) ± 8. **720.** (1) -3; (2) $\pm\frac{2}{3}$; (3) $\pm 2, 3$; (4) 4. **721.** (1) 4; (2) 2, 4; (3) -1; (4) 36.

722. (1) -0.5; (2) 2, 34; (3) 1; (4) 5, 7; (5) ± 10; (6) 6.

723. (1) ± 3; (2) 11; (3) 11; (4) 3. **724.** (1) $a+1$; (2) a^2-2a; (3) $m+4$; (4) m; (5) m, n. **725.** (1) $\sqrt{mn}$ for $n \geqslant 4m \geqslant 0$; no solution for $0 \leqslant n < 4m$; (3) $z = k$ for $k > 0$ and $z = -0.5k$ for $k < 0$. **726.** (1) -2; (2) 25; (3) -0.3; (4) 1; (5) no roots; (6) 4.

727. (1) 4; (2) 27; (3) $\pm 0.5\sqrt{34}$; (4) 9.5, 42; (5) -1, 4; (6) 0, -5. **728.** (1) $\frac{2}{3}$, 1; (2) 0.5, 1; (3) 4; (4) -3, 7.

729. (1) $\frac{1}{3}$; (2) -25, 16; (3) 4, $-\frac{11}{13}$; (4) 2.

730. (1) 0.5 $\left(-1-\sqrt{17}\right)$, 2; (2) 1, 3; (3) $x \leqslant 1$ and $x \geqslant 3$.

731. (1) -1, 5; (2) 2; (3) 8; (4) $\frac{2}{3}$, 2. **732.** (1) $-m$;

(2) $\dfrac{p\left(\sqrt[7]{p}+\sqrt[7]{q}\right)^7}{1-\left(\sqrt[7]{p}+\sqrt[7]{q}\right)^7}$; (3) $\pm\sqrt{\dfrac{8n^3+15n^2m+6nm^2-m^3}{27m}}$;

(4) $\dfrac{15a^2b-b^3+6ab^2+8a^3}{27b}$. **733.** (1) $x \geqslant 50$;

(2) $x \leqslant -7$ and $0 \leqslant x \leqslant 5\frac{121}{144}$; (3) $x > 2$; (4) $x > 34+\sqrt{1188}$;

(5) $-\frac{1}{2} \leqslant x < 28-\sqrt{756}$; (6) $x < \frac{1}{3}$ and $x > 2$. **739. 1.** (a) 5 at point (0, 0); (b) -2 at point (1, -1). **2.** (a) 16 at point (1, 2); (b) 4 at point (0.8, -0.6). **740.** (1) $(-2, 3), (3, 8)$; (2) $(5, -2), (2, -5)$; (3) $(4, -3), (5, 0)$; (4) $(0, 0), (-1, 3)$; (5) $(6, 4), (8, 3)$; (6) $(-5, 3), (-2, -3)$. **741.** (1) $(3, 0)$; (2) $(-4.5, -6,5)$. $(3,1)$; (3) $(-1, 1)$, $\left(\frac{211}{173}, -\frac{147}{173}\right)$; (4) $(-2, -1), (0.25, 0.5)$. **742.** (1) $(-2a, a), (a, -2a)$; (2) $(a-b, a+b), (-a-b, b-a)$; (3) $(-a, -2a), (2a, a)$; (4) $(-1.5a, 0.5a), (-a, 1.5a)$; (5) $(a+1, a-1), (a-1, a+1)$.

743. (1) $\left(-\frac{14}{5}a, \frac{11}{5}a\right), (2a, -a)$; (2) $(a+1, a^2-1), (-a-1, 1-a^2)$; (3) $(m, m+1)$, $\left(\frac{1-m}{2}, \frac{3-m}{2}\right)$; (4) $(a+b, a-b)$.

744. (1) $(-15, -5), (3, 4)$; (2) $(-1, 7), (-2, 8)$; (3) $(3, 1), \left(2\frac{2}{3}, \frac{2}{3}\right)$; (4) $(3, -1)$. **745.** (1) $\left(\frac{1}{6}, \frac{1}{4}\right)$; (2) x any real number and $y = -\frac{1}{2}x$; (3) $(-1, -1), \left(\frac{13}{118}, -\frac{46}{177}\right)$;

(4) (1, 5), (−1, −1). **746.** (1) (−2, 0), (−2, −1) (1, 0), (1, −1); (2) (−1, 1), (−2, 0); (3) (4, 2), (11, 16); (4) (2, 3), (2.5, 3).

747. (1) (0, 2), (5, 2), $\left(\frac{-3+\sqrt{37}}{2}, \frac{-3+\sqrt{37}}{2}\right)$, $\left(\frac{-3-\sqrt{37}}{2}, \frac{-3-\sqrt{37}}{2}\right)$; (2) (0, 1), (1.5, −0.5), (1.5, 2.5), (3, 1); (3) (2, 1), (1, 2); (4) (6, 7), (7, 6), $(-5, \pm\sqrt{6}, -5\mp\sqrt{6})$. **748.** (1) (2, 1), (−1, −0.5); (2) (3, −1), $\left(-6\frac{3}{8}, -4\frac{3}{4}\right)$; (3) (1, 2), (2, 2), $(2\pm\sqrt{5.5}, \pm\sqrt{5.5})$; (4) (3, −4), (−3, 4), (4, −3), (−4, 3). **749.** (1) (1, 1), $(2\pm\sqrt{7}, 2\mp\sqrt{7})$; (2) (5, 5), (−5, −5); (3) (2, 1), (3, 0); (4) (1, −1), (−3, −1), $(-1\pm\sqrt{6}, 0)$. **750.** (1) (1, 4), (4, 1); (2) (2, 1), $\left(1\frac{1}{2}, 1\frac{1}{3}\right)$; (3) (−4, 0.4), (2, −0.8); (4) (9, 2), (−2, −9); (5) (−3, −0.5), (1, 1.5); (6) (0, 4, −4), $\left(\frac{3}{5}, -\frac{2}{3}\right)$. **751.** (1) (0, 0); (2) (0, 0); (3) x any real number, $y = 2x$; (4) (0, 0).

752. (2) (−2, 4); (3) no solution; (4) (0, 0), (8, 6), (−6, 8).
753. (1) (1, 1), (−1, −1), (1.4, −0.4), (−1.4, 0.4);

(2) (2, 6), (−2, −6), $\left(\pm\sqrt{\frac{8}{17}}, \pm 8\sqrt{\frac{8}{17}}\right)$;

(3) (2, 2), (−2, −2), (1, −1), (−1, 1); (4) (4, 2), (−4, −2).
754. (1) $(\sqrt{2}, 0)$, $(-\sqrt{2}, 0)$, (1, −2), (−1, 2); (2) (2, 1), (−2, −1), $\left(\pm\frac{4}{\sqrt{7}}, \pm\frac{1}{\sqrt{7}}\right)$; (3) (5, 5), (−5, −5); (4) (1, −1), $\left(\frac{2}{13}\sqrt{26}, -\frac{3}{13}\sqrt{26}\right)$, $\left(-\frac{2}{13}\sqrt{26}, \frac{3}{13}\sqrt{26}\right)$.
755. (1) (0, 0), (−0.5, 0), (−0.25, ±0.5);

(2) (a, b), $\left(\frac{1}{5}(3a-4b+2), \frac{1}{5}(4-4a-3b)\right)$;

(3) (−1.5, −2.5), $\left(-\frac{3}{8}, \frac{7}{8}\right)$, (3, −1), $\left(4\frac{1}{8}, 2\frac{3}{8}\right)$;

(4) $\left(\frac{2n-a+1}{2}, \frac{6n-3a-1}{2}\right)$, $(3n-a, n-a)$, $(a+n, a-n)$, $\left(\frac{3a+1-2n}{2}, \frac{a+2n-1}{2}\right)$. **756.** (1) $\left(\pm\frac{1}{2}, \pm\frac{2}{3}\right)$, $\left(\pm\frac{4}{\sqrt{29}}, \pm\frac{3}{\sqrt{29}}\right)$;

(2) $(\pm 3, \pm 2)$; (3) (1, 1), (−3, 0.2), (−1, −1), (3, −0.2), $\left(\frac{-1\pm\sqrt{19}}{4}, \frac{2(-3\pm\sqrt{19})}{5}\right)$, $\left(\frac{1\pm\sqrt{19}}{4}, \frac{2(3\mp\sqrt{19})}{5}\right)$; (4) $(\mp\sqrt{7}, 3)$.

757. (1) (9, 4); (2) $(\pm 1, \pm 16)$; (3) (4, 4); (4) $\left(-1\frac{1}{4}, -\frac{1}{4}\right)$, $\left(-\frac{25}{28}, \frac{3}{28}\right)$. **758.** (1) (1, 1); (2) $\left(2a^2, \frac{1}{2}\sqrt{8a^2-1}\right)$, $\left(2a^2, -\frac{1}{2}\sqrt{8a^2-1}\right)$, $\left(-2a^2, \frac{1}{2}\sqrt{8a^2-1}\right)$, $\left(-2a^2, -\frac{1}{2}\sqrt{8a^2-1}\right)$; (3) (3, 2), $\left(\frac{17}{27}, -1\frac{5}{9}\right)$; (4) (5, 4), (−5, −4), (15, −12), (−15, 12).

759. (1) (0, 0), (3, 2), (−4, −12); (2) (1, 2), $\left(2\frac{1}{3}, -2\right)$; (3) $\left(5\sqrt{\frac{20}{41}}, 4\sqrt{\frac{20}{41}}\right)$, $\left(-5\sqrt{\frac{20}{41}}, -4\sqrt{\frac{20}{41}}\right)$; (4) $(0, 2n)$, $(2n, 0)$, $(m, 2n)$ $(2n, m)$.

760. (1) $\left(\frac{am}{\sqrt{a^2+b^2}}, \frac{bm}{\sqrt{a^2+b^2}}\right)$, $\left(-\frac{am}{\sqrt{a^2+b^2}}, -\frac{bm}{\sqrt{a^2+b^2}}\right)$; (2) $\left(\frac{n(a-b)}{m-n}, \frac{m(a-b)}{m-n}\right)$, $\left(\frac{n(b-a)}{m+n}, \frac{m(a-b)}{m+n}\right)$; (3) $((a+b^2), (a-b)^2)$; (4) $\left(\frac{1}{a}\sqrt{a^2+1}, -\sqrt{a^2+1}\right)$, $\left(-\frac{1}{a}\sqrt{a^2+1}, \sqrt{a^2+1}\right)$.

761. (1) (2, 3), (3, 2); (2) (−3, −5), (5, 3); (3) (−2, −4), (4, 2); (4) (3, 4), (4, 3). **762.** (1) (1, 2, 3); (−1, −2, −3); (2) (−3, 2, −1); (3, −2, 1); (3) (1, 3, 2), (−1, −3, −2), $\left(3, 1, \frac{2}{3}\right)$, $\left(-3, -1, -\frac{2}{3}\right)$; (4) $(-2-\sqrt{10}, 3+\sqrt{10}, 2-\sqrt{10})$, $(-2+\sqrt{10}, 3-\sqrt{10}, 2+\sqrt{10})$; (5) $\left(\frac{1}{2}(3+\sqrt{17}), \frac{1}{2}(-7+\sqrt{17}), \frac{1}{2}(3-\sqrt{17})\right)$; $\left(\frac{1}{2}(3-\sqrt{17}), \frac{1}{2}(-7-\sqrt{17}), \frac{1}{2}(3+\sqrt{17})\right)$. **763.** (1) (2, 5, 1), $\left(-3\frac{1}{3}, -\frac{1}{3}, -4\frac{1}{3}\right)$; (2) (2, −5, −4), (−2, −1, 0); (3) (2, 1, 3), (−2, −1, −3). **764.** (1) (−3, 2, 4), (2, −3, 4), (−4, 3, −2), (3, −4, −2); (2) (−2, 1, 1), (1, 1, −2), (−1, −1, 2), (2, −1, −1); (3) $(0, 0, a)$, $(0, a, 0)$, $(a, 0, 0)$; (4) $\left(\frac{1}{3}, \frac{1}{4}, \frac{1}{2}\right)$, $\left(\frac{5}{12}, \frac{1}{5}, \frac{5}{8}\right)$.

765. 2. $y = x+2$ and $y = x-2$. **766.** 1. $\left(-\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}\right)$ for $a = \frac{1}{2}$. 2. (1, 1, −1). **768.** 5 and 3, 5 and −3, −5 and 3, −5 and −3. **769.** 10 and 11. **770.** 7 and 5. **771.** 21. **772.** $\frac{7}{9}$.

773. 99. **774.** 25. **775.** 24 and 6. **776.** $0.5(3+\sqrt{5})$ and $0.5(1+\sqrt{5})$ or $0.5(3-\sqrt{5})$ and $0.5(1-\sqrt{5})$. **777.** 12 and 9.

778. 80×20 (m). **779.** 80 cm and 18 cm. **780.** 18 cm, 24 cm and 30 cm. **781.** 2×0.5 (m). **782.** 40 km/h and 50 km/h.

783. 16 km/h and 12 km/h. **784.** 5 h 24 min and 6 h 45 min.

785. 80 km, 6 h 40 min. **786.** 40 km/h, 50 km/h.

787. 60 km/h, at 10.45 A.M. **788.** $\frac{2ac}{b-c}$ km/h, $\frac{a(b+c)}{b-c}$ km/h, $\frac{2ab}{b-c}$ km/h, $\frac{b(b+c)}{b-c}$ h, $\frac{2bc}{b-c}$ h, $\frac{c(b+c)}{b-c}$ h. **789.** $2\frac{7}{16}$ km, 6 km/h.

790. 400 ha, 3 t. **791.** 6, 10, 15. **792.** 1 kg and $\frac{11}{12}$ kg.

793. 9 and 7. **794.** 6 h and 9 h. **795.** 5 N and 12 N. **796.** 14 N and 48 N. **797.** ≈ 10.8 N; ≈ 26.9 N. **798.** 0.15 m/s²; 60 s.

801. (1) $y_{\text{great}} = 2$ for $x = 0$; $y_{\text{least}} = 0$ for $x = \pm 2$; (2) $y_{\text{least}} = \sqrt[3]{4}$ for $x = 0$; (3) $y_{\text{least}} = 2$ for $x = 0$. **802.** (1) $2\sqrt{2}$; (2) 2.52. **804.** (1) 3.5; (2) $a-1$, $a-2$; (3) 4; (4) $2\sqrt[3]{9}$, $\sqrt[3]{11}$, $\sqrt[3]{12}$; (5) 1.5. **805.** (1) No roots; (2) $x = 0$; (3) no roots; (4) 0, ± 2.

806. (1) 0.5, 2; (2) $0.5(3 \pm \sqrt{21})$; (3) -2, 0. **810.** -2.

811. (1) $-\frac{155}{238}$; (2) (a) $\frac{61\sqrt{97}}{32}$, $\pm 1\frac{9}{64}$, (b) $\frac{45\sqrt{39}}{16}$, $\pm 81\frac{13}{16}$.

814. (1) (8, 4), (4, 8); (2) (5, 4), (4, 5), (−5, −4), (−4, −5); (3) (2, 3), $\left(16, -\frac{1}{2}\right)$. **815.** (1) $x < -3$ and $x > 1\frac{2}{3}$; (2) $x < -2$ and $x > 5$; (3) $-4 < x < 1$; (4) $x < -1$ and $\frac{1}{2} < x < 1\frac{2}{3}$; (6) $x < -1$ and $x > 3$. **816.** (1) $x > 3$; (2) $x > 3\frac{2}{3}$; (3) $-4 < x < -1$; (4) $x < 0$, $0 < x < 1$ and $x > 3$; (5) $-1 < x < 0$ and $4 < x < 5$; (6) $x < -4$, $-3 < x < -1$ and $x > 0$. **819.** (1) $y_{\text{great}} = 3$ for $x \leqslant -2$, $y_{\text{least}} = -1$ for $x \geqslant 2$; (2) $y_{\text{least}} = 0$ for $x = -1$; (4) $y_{\text{least}} = -2$ for $x = -1$ and $x = 5$.

820. (2) $m = -1$, $y = -2\frac{1}{3}$; (3) 1; (4) $-3.4 < x < 3.25$.

821. 1. $\frac{\sqrt{\sqrt{3}-1}}{\sqrt[4]{3}} \approx 0.6$. 2. 0, 5. 3. $-1 < x < \frac{1}{2}$ and $x > 5\frac{2}{3}$.

822. 1. $h = 120$ m, $v_0 = 10$ m/s, $h_{\max} = 125$ m. 2. $\frac{\sqrt{1-a^2}}{2}$, $-1 < a < 1$. 3. $\pm 0.8\sqrt{5}$. **823.** 1. $l = \sqrt{\frac{x^2}{4} + \frac{4s^2}{x^2}}$. 2. $\sqrt{5}$. 3. $\frac{4x^2}{15}$.

824. 1. 0.6. 2. $\sqrt[3]{2}$. 3. (5, 1), (1, 5), (2, 3), (3, 2).

825. 1. $\sqrt{m} - \sqrt{n}$. 2. (−1, −1). **826.** 1. 59 km/h. 2. $1 \leqslant x \leqslant 3$.

827. 1. 60 h and 40 h. **2.** 49. **4.** $x \leqslant -9$ and $x \geqslant 5$.

828. 2. $-\frac{2}{3} < x < 4$. **3.** $-2\frac{2}{3}a$. **829. 1.** 20 km.

2. $\frac{3-\sqrt[4]{m^2-n}}{\sqrt[4]{n}}$ if $\frac{1}{\sqrt[4]{n}} > \frac{\sqrt{m}}{2}$ and $\frac{1}{\sqrt[4]{n}}$ if $\frac{1}{\sqrt[4]{n}} < \frac{\sqrt{m}}{2}$. **830. 1.** -1.

3. 45 km. **831. 1.** 1.3 m^3, 5 m^3, 6 m^3. **3.** -2. **4.** $x < 1\frac{2}{3}$.

832. (1) 20 km/h, 12 km/h. **2.** $y_{\text{great}} = 1\frac{1}{3}$. **3.** 1. **833. 1.** 4 s and 6 s. **2.** 14 uv. **834. 1.** 25 min. **2.** $\sqrt[3]{a}$. **835. 1.** $a-b$.

2. $x < -2$, $-1 < x < 1$ and $x > 2$. **3.** 90 g, 180 g, $d = 2\frac{4}{7}$.

838. 2. (a) 2, 30°; (b) 4, 180°. **842. 1.** $\sqrt{34}$ and $\sqrt{34}$. **2.** 22. **3.** 20. **844.** $\overrightarrow{OA} = 5\boldsymbol{i}$, $\overrightarrow{OB} = 7\boldsymbol{j}$, $\overrightarrow{OC} = 5\boldsymbol{i}+7\boldsymbol{j}$, $\overrightarrow{AC} = 7\boldsymbol{j}$.

847. $\overrightarrow{CD} = 4\boldsymbol{m}-4\boldsymbol{n}$, $\overrightarrow{AM} = 4\boldsymbol{n}+2\boldsymbol{m}$, $\overrightarrow{AN} = 6\boldsymbol{m}+2\boldsymbol{n}$.

848. $2\boldsymbol{b}-\sqrt{3}\boldsymbol{a}$. **849.** $12\sqrt{3}$. **851. 2.** $\overrightarrow{OC} = \frac{1}{2}(\boldsymbol{a}+\boldsymbol{b})$, $\boldsymbol{a} = 2\overrightarrow{OC}-\boldsymbol{b}$. **852.** ≈ 8.2 N. **853.** 50 kgf and ≈ 86.6 kgf.

854. 1. $a_x = x_2-x_1$, $a_y = y_2-y_1$. **855. 1.** (a) $|\boldsymbol{a}| \cos\alpha$, (b) -3, (c) 0.71, (d) -5, (e) 0, (f) -0.65. **2.** (a) $\frac{x}{\cos\alpha}$, (b) 10, (c) 38.6, (d) 3. **856.** (1) (5, 1); (2) $(-2, 4)$; (3) $(-6, 11)$.

860. $-5\sqrt{3}$, $5\sqrt{3}$, 0. **861.** (a) $\{1, -1\}$, (b) $\{5, -3\}$, (d) $\left\{\frac{1}{2}, -\frac{1}{4}\right\}$, (e) $\{5, -4\}$. **863. 3.** $0.6\boldsymbol{i}+0.8\boldsymbol{j}$. **864. 1.** $\{-4, 4.5\}$, $\{-2, -5.5\}$, $\left\{1, 2\frac{1}{6}\right\}$, $\left\{-2\frac{1}{4}, -\frac{23}{40}\right\}$. **2.** $\{8, 2\}$, ≈ 8.2. **3.** $\sqrt{5}$, 63°26′. **866.** (2) $\{3, 0\}$, $\{3, 8\}$; (3) $\overrightarrow{ON} = 3\boldsymbol{i}+8\boldsymbol{j}$. **867.** (1) $-3\boldsymbol{i}+4\boldsymbol{j}$; (2) $-\boldsymbol{i}$; (3) $\boldsymbol{i}+4.5\boldsymbol{j}$; (4) 0. **868.** 4 and $2\sqrt{2}$. **869.** -1, -0.5.

870. 0. **874. 1.** 22 units of work. **2.** 10 units of work.

875. 2. (a) 102°32′, (b) 36°52′, (c) 169°54′, (d) 163°54′.

876. (1) $A = 18°26'$; (2) 10°18′. **877. 1.** ± 0.6. **2.** $a = b$.

886. 900°, 60°. **892. 1.** 1.0472, 1.3090, 2.3562, 1.5708.

2. $\smile AB = 1.0472$, $\smile BC = 2.0944$, $\smile CD = 1.7453$, $\smile AD = 1.3963$, $A = 1.9199$, $B = D = 1.5708$, $C = 1.2217$.

894. 1. 137°31′, 206°17′, 355°14′. **2.** (a) ≈ 31.25 cm, (b) 62.83 cm. **3.** 1429 m. **901.** (1) 370; (2) 48.8; (3) 1.56; (4) 10.9; (5) 0.35. **902. 1.** 140°. **2.** 0.5236. **903. 2.** ≈ 2.87 m.

904. 1. ≈ 62.83 cm^2. **2.** ≈ 9.4 units. **3.** 3.2. **905. 1.** 7.33. **2.** 1.25 $\frac{\text{rad}}{\text{s}}$. **906. 1.** 157. 1 $\frac{\text{rad}}{\text{s}}$, 15.7 m/s. **2.** 3000.

907. 1. 25.12 $\frac{\text{rad}}{\text{s}}$. **2.** 600. **908. 1.** 8 s. **2.** ≈ 28.28 cm/s, ≈ 508.94 m. **909.** 0.196 rad, 11°15′. **910. 1.** (a) 1 mil $\approx 3.6'$, $1' \approx 0.28$ mil, $1° \approx 16.7$ mils, (b) 10, 50, 200, 800. **2.** 1°12′.

911. 1. $\frac{R}{1000}$. **2.** $R = \frac{1000l}{n}$. **3.** 800 m.

920. (1) (a) $\pi k < x < \frac{\pi}{2}+\pi k$, (b) $\frac{\pi}{2}+\pi k < x < \pi+\pi k$, $k = 0, \pm 1, \pm 2, \ldots$; (5) (a) $-\frac{\pi}{3}+2\pi k < x < \frac{2\pi}{3}+2\pi k$, (b) $-\frac{4}{3}\pi+2\pi k < x < -\frac{\pi}{3}+2\pi k$; (6) $y > 0$ everywhere, except for $x = \frac{\pi}{2}+2\pi k$. **921.** (2) $\pi k-\frac{\pi}{4} < x < \frac{\pi}{4}+\pi k$; (3) $4\pi k-\pi < x < \pi+4\pi k$; (8) $-\frac{\pi}{2}+\pi k < x < \pi k$.

924. 2. (a) [0, 2], (b) [0, 2], (c) [1, 5], (d) [1, 7], (e) [0, 1], (f) [1, 2], (g) [1, ∞], (h) $\left[-\frac{5}{4}, \frac{1}{4}\right]$. **925. 2.** (a) $\left(\frac{3\pi}{2}, 2\pi\right)$, (b) $\left(\frac{\pi}{2}, \pi\right)$.

927. (1), (3), (4), (7), (8) no; (2), (5), (6), (9), (10) yes.

929. (1) $\frac{\pi}{3}+2\pi k < x < \frac{2\pi}{3}+2\pi k$; (2) $2\pi k-\frac{7\pi}{6} < x < 2\pi k+\frac{\pi}{6}$; (3) $2\pi k+\frac{5\pi}{6} < x < \frac{7\pi}{6}+2\pi k$; (4) $\pi k-\frac{\pi}{3} < x < \frac{\pi}{3}+\pi k, \ +\frac{\pi k}{2}$; (6) $\frac{\pi k}{2}+\frac{\pi}{6} \leqslant x \leqslant \frac{\pi}{3}+\frac{\pi k}{2}$; (7) $2\pi k+\frac{\pi}{6} < x < \frac{5\pi}{6}+2\pi k$ and $2\pi k-\pi < x < 2\pi k$; (8) $2\pi k+\frac{\pi}{6} < x < \frac{2\pi}{3}+2\pi k$, $2\pi k-\frac{2\pi}{3} < x < -\frac{\pi}{6}+2\pi k$. **930.** (1) $x \neq \frac{\pi}{2}+\pi k$; (3) $x \neq 2\pi k$; (4) $2\pi k \leqslant x \leqslant \frac{\pi}{2}+2\pi k$; (5) x real number; (6) $\pi k \leqslant x \leqslant \frac{\pi}{4}+\pi k$; (7) $\frac{\pi}{2}+2\pi k < x < \frac{3\pi}{2}+2\pi k$; (8) $2\pi k-\frac{\pi}{6} \leqslant x \leqslant \frac{\pi}{6}+2\pi k$; (9) $\pi k-\frac{\pi}{2} < x < \frac{\pi}{3}+\pi k$ and $\pi k+\frac{\pi}{3} < x < \frac{\pi}{2}+\pi k$; (10) $x \neq \frac{\pi}{2}+2\pi k$; (11) $x \neq -\frac{\pi}{4}+\pi k$. **932.** (4) $y_{\text{least}} = 0.5$ if $\sin x = -1$; (6) none; (7) 5 and 4; (8) 4 and 0. **934. 1.** 0.7446 and -0.7446 for $x = 1$ and $x = -1$, respectively; **2.** 1 and 0.6675

for $x = 0$ and $x = \pm 1$, respectively. **936.** (1) $\frac{7}{3}\sqrt{3}$; (2) $6\sqrt{3}-2$; (3) $-b$; (4) $-(n+p)$. **937.** (1) 5; (2) -1; (3) 0; (4) -0.5. **938.** (1) 1; (2) 0; (3) $\frac{1}{3}(3+\sqrt{3})$; (4) 0. **939.** (1) $(m-n)^2$; (2) $a+b$; (3) $a+b$; (4) a^2. **943.** (1) 2π; (2) π; (3) 2π; (5) $\frac{\pi}{3}$; (6) $2\pi a$; (7) π; (8) π; (9) π. **944.** (1) 2π; (2) π; (3) 6π; (4) π; (5) $\frac{\pi}{2}$; (6) π. **946.** (1) (b) $\sin\alpha = \frac{9}{41}$, (c) $\sin\alpha = \frac{3}{5}$, (d) $\sin\alpha = \frac{3}{4}$; (2) (a) $\cos\alpha = \frac{3\sqrt{3}}{14}$, (b) $\sin\alpha = -\frac{\sqrt{5}}{3}$, (c) $\sin\alpha = -\frac{9}{41}$. **947. 1.** (a) $\cos A = \pm 0.96$, (b) $\sin A = \frac{\sqrt{3}}{2}$, (c) $\sin A = \frac{\sqrt{7}}{4}$. **2.** $\cos A = 0.8$. **948.** (1) $\cos\alpha = -\frac{8}{17}$; (2) $\sin\alpha = -0.6$; (3) $\sin\alpha = -0.8$; (4) $\sin\alpha = -\frac{\sqrt{3}}{2}$; (5) $\cos\alpha = -0.4\sqrt{6}$; (6) $\sin\alpha = -0.3\sqrt{10}$. **949.** (1) $\cos x = 0.96$; (2) $\sin x = 0.5$; (3) $\sin x = -0.747$; (4) $\sin x = -0.555$. **951. 1.** $\sin\alpha = \frac{a}{b}$, $\cos\alpha = -\frac{1}{b}\sqrt{b^2-a^2}$, $\tan\alpha = \frac{-a}{\sqrt{b^2-a^2}}$. **2.** $\cos\alpha = -\frac{b^2}{\sqrt{a^4+b^4}}$, $\sin\alpha = -\frac{a^2}{\sqrt{a^4+b^4}}$. **952. 1.** $3\frac{4}{7}$. **2.** $-\frac{1}{3}$. **3.** $\frac{8}{17}$. **953.** (1) $\frac{12}{25}$; (2) $-\sqrt{3}$. **954.** (1) $\frac{\tan^2\alpha+1}{\tan^2\alpha-\tan\alpha+1}$, $\frac{1+\cot^2\alpha}{1-\cot\alpha+\cot^2\alpha}$; (2) $\frac{1+\tan^2\alpha}{\tan^2\alpha}$, $1+\cot^2\alpha$; (3) $\frac{4\tan\alpha-2}{5+3\tan\alpha}$, $\frac{4-2\cot\alpha}{5\cot\alpha+3}$. **955. 1.** (a) 23, (b) 110. **2.** (a) m^2+2, (b) m^3+3m. **3.** (a) $0.5(p^2-1)$, (b) $\pm\sqrt{2-p^2}$, (c) $0.5(1+2p^2-p^4)$. **956.** (1) $\pm 1\frac{1}{3}$; (2) $\pm\frac{8\sqrt{65}}{65}$; (3) $\pm\sqrt{15}$; (4) ± 0.6; (5) non-existent; (6) ± 1. **958.** (3) $1-\sin\alpha$; (4) 2; (5) 1; (6) $\frac{\cot\alpha}{1+\sin\alpha}$. **959.** (4) $\frac{2}{|\sin\alpha|}$; (5) $\sin\alpha\cos\alpha$. **960.** (1) $\cos^2\alpha$; (2) $-\sin^2\beta$; (3) $\sin^2 x$; (4) $\cos^2\alpha$. **961.** (3) $\sin^2\alpha$; (5) $\frac{1}{\sin^2\alpha}$; (6) $\tan^6\alpha$. **962.** (5) $\cos\alpha$; (6) 1. **963.** (1) $\sin\alpha\cos\alpha$; (2) 1; (3) $\cos\alpha$; (4) $0.5\sin\alpha$. **964.** (1) $\frac{2}{\sin\alpha}$; (2) $\frac{2}{\cos\beta}$; (3) $\frac{2}{\cos^2\alpha}$; (4) $\frac{2}{\sin^2\beta}$; (5) $\frac{1}{\sin\alpha}$; (6) $\frac{1}{\sin\alpha}$.

965. (3) $\frac{2}{|\cos\alpha|}$; (4) $\frac{2}{|\sin\alpha|}$; (5) 4 if $\pi k < \alpha < \frac{\pi}{2}+\pi k$ and -4 if $\pi k+\frac{\pi}{2} < \alpha < \pi+\pi k$, where $k = 0, \pm 1, \pm 2, \ldots$.

966. (1) $2k\pi+\frac{\pi}{2} \leqslant x \leqslant \frac{3\pi}{2}+2\pi k$; (2) $2\pi k+\frac{\pi}{2} < x < \frac{3\pi}{2}+2\pi k$; (3) $2\pi k-\pi < x < 2\pi k$; (4) $2\pi k < x < \pi+2\pi k$. **967.** (1) $2\tan\alpha$; (2) $\sin\alpha$; (3) $|\cos\alpha|+1$. **968. 1.** No. **2.** $\alpha \neq \frac{\pi}{2}k$, $\beta \neq \frac{\pi}{2}k$ (simultaneously). **978.** (1) $k\pi$; (2) $k\pi$; (3) $\frac{\pi}{4}+\frac{\pi}{2}k$; (4) $\frac{\pi}{4}+\pi k$; (5) $24°28'+360°k$ and $155°32'+360°k$; (6) $-63°26'+180°k$ and $26°34'+180°k$.

979. (2) $25x^2+9y^2-225 = 0$; (3) $x^2-2y-1 = 0$; (4) $x^2-y-2 = 0$. **981.** (1) $2\pi k-\pi < x < 2\pi k$ and $2\pi k+\frac{\pi}{6} < x < \frac{5\pi}{6}+2\pi k$; (2) $2\pi k+\frac{\pi}{3} < x < \frac{\pi}{2}+2\pi k$ and $2\pi k-\frac{\pi}{2} \leqslant x \leqslant 2\pi k-\frac{\pi}{3}$; (3) $2\pi k+\frac{\pi}{2} < x < 2\pi k+\frac{3\pi}{2}$; (4) $2\pi k-\frac{\pi}{2} < x < 2\pi k$ and $2\pi k-\pi < x < 2\pi k-\frac{5\pi}{6}$; (5) $2\pi k < x < \frac{\pi}{2}+2\pi k$ and $2\pi k+\frac{\pi}{2} < x < \pi+2\pi k$; (6) $2\pi k-\frac{5\pi}{6} < x < 2\pi k-\frac{\pi}{6}$; (7) $\pi k+\frac{\pi}{6} \leqslant x \leqslant \frac{5\pi}{6}+\pi k$; (8) $2\pi k+\frac{\pi}{6} < x < \frac{5\pi}{6}+2\pi k$. **983.** (1) 1; (2) 0.625. **984.** (1) 1; (2) $0.5(a^2-1)$. **985.** (1) 7; (2) 1. **987.** (2) -3.789, 0.4316, -0.3307; (3) 0.3166, 2.420, 0.1349. **988.** (2) $\sin 34°$, $-\sin 42°$, $-\tan 6°$, $-\cot 14°$; (3) $-\cos 5°$, $-\cos 28°$, $-\tan 10°$, $-\cot 4°$; (4) $\sin 7°$, $-\sin 4°$, $\cot 42°$, $\tan 5°$; (5) $\cos 14°$, $\sin 32°$, $-\tan 20°$, $\tan 30°$. **989.** (2) $-\sin 39°$, $\cos 56°$, $\tan 86°$, $-\cot 82°$; (3) $-\tan 0.2\pi$, $\sin 0.1\pi$, $\cos 0.4\pi$, $-\cot 0.3\pi$; (4) $-\sin(2\pi-5)$, $-\cos(4-\pi)$, $\tan(2\pi-5)$, $-\cot(\pi-2)$. **990.** (1) $\cos 64°$, $-\sin 62°$, $-\cot 17°$, $\tan 76°$; (2) $\cos 72°$, $-\sin 60°$, $\cot 28°$, $-\tan 36°$.

991. (3) $-\cos 33°40'$, $\sin 3°6'$, $\tan 14°48'$, $-\cot 3°48'$.

993. (1) $169°58'$; (2) $302°54'$; (3) $246°56'$; (4) $100°21'$; (5) $208°23'$; (6) $248°14'$; (7) $345°4'$; (8) $192°26'$. **994.** (1) $\frac{\sqrt{5}}{3}$, $\frac{2\sqrt{5}}{5}$; (2) 0.8, 0.75; (3) $\frac{\sqrt{5}}{5}$, $\frac{1}{2}$. **995.** (2) $-\frac{1}{\sqrt{17}}$; (3) $-1\frac{1}{3}$.

996. (1) -0.8; (2) $\frac{1}{\sqrt{26}}$; (3) $-\frac{1}{8}$. **999.** (1) 1.3347; (2) -1.2478; (3) 1. **1000.** (1) $\frac{3}{4}$; (2) $\frac{\sqrt{3}-2}{2}$; (3) 1; (4) 1. **1001.** (1) 0; (2) -1; (3) 0; (4) 0; (5) 0. **1002.** (1) $\frac{2-\sqrt{3}}{2}$; (2) $\sqrt{3}-5$; (3) 0; (4) $\frac{15}{16}$. **1003.** (1) 0; (2) 1; (3) 1; (4) 1. **1004.** (1) $\sin\alpha$; (2) 0; (3) 1; (4) 1. **1005.** (3) $2\sin\alpha$; (4) $\sin\alpha$.

1006. (1) $-\frac{2}{3}(2\sqrt{3}+3)$; (2) -1; (3) $-\frac{\sqrt{3}}{2}$; (4) $\frac{2-\sqrt{3}}{2}$.

1007. (1) 0; (2) $\cos 20^\circ(\sin 20^\circ-\cos 20^\circ)$; (3) $-\cot^3 40^\circ$; (4) $\tan 35^\circ+1$. **1008.** (1) $-2\sin 18^\circ$; (2) $\tan 18^\circ$; (3) 1.

1011. (1) 0.5; (2) 0; (3) $\pm\frac{2\sqrt{5}}{5}$; (4) $\pm\frac{\sqrt{3}}{3}$. **1012.** (1) 0.2831; (2) 2.9372; (3) -2.3028; (4) -0.293. **1014.** Even: (2), (4), (6), (8). Odd: (1), (3), (9). **1015.** Even: (2), (3), (4), (6), (8). Odd: (9).

1016. (1) $\frac{\pi}{6}$ and $\frac{5\pi}{6}$; (2) $\frac{3\pi}{4}$ and $\frac{7\pi}{4}$; (3) $0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi$; (4) $0, \pi, 2\pi$; (5) $\frac{3\pi}{4}, \frac{7\pi}{4}$; (6) $0 \leqslant x \leqslant 2\pi$. **1022.** (1) $y=\frac{1}{3}x-\frac{2}{3}$; (2) $y=4-x$; (3) $y=\frac{1}{x}$; (4) $y=\frac{2x}{3}$; (5) $y=\frac{x}{x-1}$; (6) $y=x^3$; (7) $y=1+x^3$; (8) $y=\frac{2x+3}{3x-2}$. **1023.** Have: (1), (2), (4), (6), (8). Have not: (3), (5), (7). **1032.** (4) $\left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$; (5) $[0, \pi]$; (6) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$; (7) $\left(-\pi, -\frac{\pi}{2}\right)$; (8) $(\pi, 2\pi)$. **1034.** (4) $y=\sin\frac{x-5}{3}$; (5) $y=\frac{1}{5}\tan\frac{x+3}{4}$; (6) $y=2\cos(4x-8)$. **1035.** (1) $-2 \leqslant x \leqslant 2$; (2) x any real number; (3) $-0.5 \leqslant x \leqslant 0.5$; (4) $x \geqslant 0$; (5) $0 \leqslant x \leqslant 1$; (6) $x \neq 2$; (7) $x \geqslant 4$; (8) $x \leqslant -3$ and $x \geqslant -1$; (9) $0 \leqslant x \leqslant 1$. **1036.** (1) x any real number; (2) $x \neq 0$; (3) $x \leqslant -0.5$ and $x \geqslant 0.5$; (4) $0 \leqslant x \leqslant 1$; (5) $0 \leqslant x \leqslant 0.5$; (6) $x \leqslant -\frac{3+\sqrt{5}}{2}; \frac{\sqrt{5}-3}{2} \leqslant x \leqslant \frac{3-\sqrt{5}}{2}$ and $x \geqslant \frac{3+\sqrt{5}}{2}$. **1038.** (1) $\frac{\pi}{3}$; (2) 0; (3) $-\pi$; (4) $\frac{\pi}{2}$; (5) $\frac{\pi}{2}$; (6) 2π. **1039.** (1) 95°; (2) $\frac{\pi}{2}$; (3) π; (4) 0; (5) 0; (6) π. **1040.** (7) 1; (8) -1. **1041.** (3) 0.6; (4) $\frac{\sqrt{5}}{5}$; (5) 0.8; (6) $\frac{8}{15}$; (7) $-3\frac{3}{7}$; (8) 2.223; (9) -0.2748.

1042. (2) $-30°$; (3) α if $x = 2k\pi+\alpha$ or $x = 2k\pi+\pi-\alpha$; $-\alpha$ if $x = 2k\pi-\alpha$ or $x = 2k\pi+\pi+\alpha$ (α a positive acute angle); (4) $\frac{\pi}{4}$; (5) 170° (6) 40°; (7) α if $x = 2k\pi\pm\alpha$, $\pi-\alpha$ if $x = 2k\pi+\pi\pm\alpha$ (α a positive acute angle); (8) 0; (9) $\frac{2\pi}{5}$. **1044.** (1) $y = \frac{\sqrt{1-x^2}}{|x|}$, $|x| \leqslant 1$, $x \neq 0$; (2) $y = \frac{|x|}{\sqrt{1-x^2}}$, $-1 < x < 1$; (3) $y = \frac{\sqrt{x^2-1}}{|x|}$, $|x| > 1$; (4) $\frac{1}{x}$, $x \neq 0$, $|x| < 1$.

1045. (a) $\arcsin x = \arccos\sqrt{1-x^2} = \arctan\frac{x}{\sqrt{1-x^2}} = \operatorname{arccot}\frac{\sqrt{1-x^2}}{x}$ for $x > 0$, $\arcsin x = -\arccos\sqrt{1-x^2} = \arctan\frac{x}{\sqrt{1-x^2}} = -\operatorname{arccot}\frac{\sqrt{1-x^2}}{|x|}$ for $x < 0$;

(b) $\arccos x = \pi-\arcsin\sqrt{1-x^2} = \pi-\arctan = \frac{\sqrt{1-x^2}}{|x|} = \operatorname{arccot}\frac{x}{\sqrt{1-x^2}}$ for $x < 0$; (d) $\operatorname{arccot} x = \arctan\frac{1}{x} = \arcsin\frac{1}{\sqrt{1+x^2}} = \arccos\frac{x}{\sqrt{1+x^2}}$ for $x > 0$,

$\operatorname{arccot} x = \pi-\arctan\frac{1}{|x|} = \pi-\arcsin\frac{1}{\sqrt{1+x^2}} = \arccos\frac{x}{\sqrt{1+x^2}}$ for $x < 0$. **1047.** (2) 223°2′ and 316°58′; (3) 131°58′; 228°2′; (5) 40°17′ and 319°43′; (6) 11°19′ and 191°19′. **1048.** (3) 17°26′; (4) 137°4′. **1050. 1.** (a) $\frac{\pi}{2}+2\pi k$, (b) $-\frac{\pi}{2}+2\pi k$, (c) $2\pi k$, (d) $\pi+2\pi k$. **2.** (c) $\frac{\pi}{2}+2\pi k$, (d) $\pi+2\pi k$. **1051.** (a) $k\pi$, $\pm\frac{\pi}{6}+2k\pi$, $\pm\frac{2}{3}\pi+2k\pi$, $-\frac{\pi}{4}+k\pi$; (b) $\frac{\pi}{6}+k\pi$, $\frac{5}{6}\pi+k\pi$, $(-1)^k\,17°46'+180°k$, $\pm 113°35'+360°k$.

1052. (1) $(-1)^k\,64°10'+180°k$; (2) $\pm\frac{\pi}{3}+2\pi k$; (3) $74°3'+180°k$; (4) $\frac{2\pi}{3}+\pi k$; (5) $-\frac{\pi}{2}+2\pi k$; (6) $\pi+2\pi k$.

1053. (5) $-196°16'+360°k < x < -16°16'+360°k$; (6) $-65°43'+360°k < x < 65°43'+360°k$. **1054.** (4) $30°+180°k$, 30° and 210°; (5) $(-1)^k\,10°+60°k$, 10°, 50°, 130°, 170°, 250°, 290°;

(6) $-\frac{\pi}{6}\pm\frac{\pi}{4}+2\pi k, \frac{\pi}{12}$ and $1\frac{7}{12}\pi$; (7) $\frac{3}{2}\pi+2\pi k$, $\frac{3}{2}\pi$;
(8) $-75°+300°k$, 225°; (9) $\pm 30°+180°k$, 30°, 150°, 210°, 330°.

1055. (5) $\frac{\pi}{10}+\frac{\pi}{5}k$; (6) $\frac{\pi}{2}k$. **1056.** (3) $\frac{\pi}{4}+2.5+\frac{\pi}{2}k$;
(4) $\pm 15°+90°k$; (5) $(-1)^k 16°22'+180°k$; (6) $\pm 149°2'+360°k$.

1057. (1) $\frac{\pi}{3}+(-1)^k\frac{\pi}{6}+\pi k$; (2) $\frac{\pi}{8}+\frac{\pi}{2}k$; (3) πk;
(4) $-3°47'+60°k$; (5) $45°+180°k$ and $-15°+180°k$;
(6) $-10°40'+60°k$. **1058.** (1) $\pm\frac{\pi}{3}+2\pi k$; (2) $\frac{\pi}{2}+2\pi k$;
(4) $\pm 18°53'+90°k$; (5) $\pm 30°+180°k$; (6) $\pm\frac{\pi}{4}+\pi k$.

1059. (1) $(-1)^k\frac{\pi}{6}+\pi k$; (2) $31°43'+90°k$ and $39°21'+90°k$;
(3) πk and $-\frac{\pi}{6}+\pi k$; (4) $\frac{\pi}{8}+\frac{\pi}{2}k$; (5) $\pi+2\pi k$ and $\pm\frac{\pi}{3}+2\pi k$;
(6) $\arctan\frac{1}{2}+\pi k$ and $\frac{\pi}{4}+\pi k$. **1060.** (1) $(-1)^k 60°32'+180°k$;
(2) $\pm 74°17'+360°k$; (3) $\pm 70°34'+360°k$; (4) $(-1)^k 48°51'+180°k$.
1061. (1) πk and $\frac{\pi}{4}+\frac{\pi}{2}k$; (2) $\frac{\pi}{2}k$ and $\frac{5\pi}{6}+\pi k$;
(3) $180°k$ and $(-1)^k 23°35'+180°k$; (4) πk; (5) $\frac{\pi}{4}k$; (6) $\frac{\pi}{2}k$.

1062. (1) $-45°+180°k$, $\pm 54°44'+180°k$; (2) $2\pi k$ and $\frac{\pi}{4}+\pi k$;
(3) $(-1)^k\frac{\pi}{3}+2\pi k$; (4) $-\frac{\pi}{4}+\pi k$; (5) $2\pi k$ and $\frac{\pi}{4}+\pi k$;
(6) $\frac{\pi}{2}+2\pi k$, $x=\arctan\frac{1}{2}+\pi k$. **1063.** (1) $\frac{\pi}{4}(2k+1)$;
(2) $\frac{\pi}{3}(3k\pm 1)$ and $\frac{\pi}{2}(2k+1)$; (3) πk and $\frac{\pi}{4}(2k+1)$;
(4) $\frac{\pi}{5}k$ and $\frac{\pi}{4}k$; (5) $\frac{5}{3}(3k\pm 1)$, $\frac{\pi}{8}(2k+1)$; (6) $\frac{\pi}{4}(2k+1)$.

1064. (2) $-\frac{\pi}{12}+\frac{\pi}{3}k$; (3) $\pm\frac{\pi}{6}+\pi k$; (4) $\pm\frac{\pi}{3}+\pi k$; (5) $180°k$ and $59°2'+180°k$; (6) $\pm 63°26'+180°k$. **1065.** (1) $71°34'+180°k$, $81°52'+180°k$; (2) $45°+180°k$, $81°52'+180°k$; (3) $\frac{\pi}{2}k$, $\frac{\pi}{2}(4k+1)$ and $\arctan 3.5+\pi k$. **1066.** (3) $45°+90°k$ and $\pm 63°26'+180°k$;
(5) $180°k$, $-61°56'+180°k$. **1067.** (1) $\pm\frac{2\pi}{3}+2\pi k$; (2) $2\pi k$;

(3) $\pm\frac{\pi}{3}+2\pi k$; (4) $(-1)^k 19°28'+180°k$. **1068.** (1) $\pm 70°32'+360°k$;
(2) $-45°+180°k$ and $-26°34'+180°k$; (3) $\pm\frac{\pi}{6}+\pi k$;
(4) $\frac{\pi}{4}(2k+1)$. **1069.** (1) $\frac{\pi}{4}+\pi k$; (2) $2\pi k$, $-(-1)^k\frac{\pi}{3}+2\pi k$;
(3) $-45°+180°k$, $59°2'+180°k$; (4) $(-1)^k\frac{\pi}{12}+\frac{\pi}{2}k$, $\frac{\pi}{4}+\pi k$.

1070. (1) $\frac{\sqrt{3}}{2}$; (2) no solution; (3) ± 2, ± 0.5; (4) -2, -1.

1071. (2) $-\frac{5}{6}$; (3) 0.5; (4) $\sqrt{3}$; (5) 0.8; (6) $-\frac{4}{3}$. **1072.** (1) $\frac{2}{5}$;
(2) $\frac{1}{17}$; (3) $\sqrt{\frac{\sqrt{5}-1}{2}}$; (4) 0. **1073.** (1) $\left(\frac{3}{5}, 1\frac{1}{3}\right)$; (2) $\left(\frac{\sqrt{1-m^2}}{2}, \frac{1}{m}\right)$.

1074. (1) in the first; (2) in the second; (3) in the third.

1077. (1) 0 and 0.96; (2) $-\frac{33}{65}; \frac{63}{65}$. **1078.** (1) $-\frac{84}{85}$ and $-\frac{36}{85}$;
(2) $\frac{84}{85}$ and $\frac{36}{85}$. **1079.** (1) $-1\frac{7}{19}$ and $\frac{14}{29}$; (2) 0 and $1\frac{1}{119}$.

1080. (1) $\frac{3\sqrt{3}-4}{10}$; (2) $\frac{\sqrt{21}-3}{8}$; (3) $-\frac{1}{7}$.

1081. (1) $\frac{1}{34}(15\sqrt{3}-8)$ and $\frac{1}{34}(15-8\sqrt{3})$; (2) $-\frac{120}{169}$ and $-\frac{24}{25}$;
(3) $-\frac{416}{425}$ and $\frac{304}{425}$. **1082.** (1) $-\frac{4}{5}$; (2) $\frac{2}{\sqrt{125}}$.

1088. 2. $\alpha+\beta = k\pi$, $\alpha \neq \frac{\pi}{2}(2n+1)$, $\beta \neq \frac{\pi}{2}(2m+1)$, $\alpha = k\pi$,
$\beta \neq \frac{\pi}{2}(2n+1)$ and $\beta = k\pi$, $\alpha \neq \frac{\pi}{2}(2n+1)$. **1089.** 1. $\frac{143}{145}$.
2. $\frac{255}{1313}$ or $\frac{735}{1313}$. **1093.** (3) $\frac{\sqrt{3}}{3}$; (4) $-\sqrt{3}$. **1094.** (1) 0.5; (2) 1;
(3) -0.5; (4) 1; (5) $\sqrt{3}$. **1095.** (1) 0; (2) 1; (3) 0.5; (4) $0.5\sqrt{2}$.

1096. (1) 1; (2) 2; (3) 2; (4) -2. **1097.** (1) $0.5\sqrt{3}$; (2) 0.5;
(3) 0; (4) 2. **1098.** (1) 2; (2) -1; (3) $\frac{1}{2}$; (4) 1. **1099.** (1) $\sin\alpha$;
(2) $\cos\alpha$; (3) $\sin 15°$; (4) $\cos(\alpha-\beta)$. **1100.** (1) $\tan\alpha\cot\beta$; (2) 1;
(3) $\tan\beta$; (4) $\sqrt{3}\tan\alpha$. **1101.** (1) $\cos 2\beta$; (2) 1; (3) 0;
(4) $-\sin(\alpha+\beta)$. **1102.** (1) 0.5; (2) 0; (3) $\frac{\sqrt{3}}{2}$. **1103.** (1) 0.5;
(2) 1.5; (3) 1.5; (4) 1.5. **1104.** (1) 1; (2) 0; (3) $\frac{\cos(\alpha+\beta)}{\cos(\alpha-\beta)}$;
(4) 1. **1105.** (3) 0; (4) $-\tan 75°$; (5) 1; (6) 1. **1106.** (2) 1;

(3) $\dfrac{\cos\frac{3}{4}\alpha}{\sin\frac{1}{4}\alpha}$; (4) $\tan\alpha$; (5) $\cos 2\alpha$; (6) $2[1+\cos(\alpha-\beta)]$.

1115. (1) $\frac{\pi}{2}(2k+1)$; (2) πk; (3) πk; (4) no solution; (5) $\frac{2\pi}{3}k$; (6) $-25°+180°k$. **1116.** (1) $\frac{\pi}{2}(2k+1)$ and $\frac{\pi}{4}(2k+1)$; (2) $\frac{\pi}{3}k$ and $\frac{\pi}{4}(2k+1)$; (3) $\frac{\pi}{12}(24k+1)$; (4) $\frac{\pi}{4}(2k+1)$. **1117.** (1) $\pm\frac{3}{4}\pi+2\pi k$; (2) $\pm 54°33'+360°k$; (3) $-(-1)^k 30°+180°k$; (4) $-\frac{\pi}{4}+\pi k$.

1118. (1) Non-existent; (2) $-\frac{44}{117}$; (3) 1; (4) $-\frac{77}{85}$.

1119. (1) $-\frac{345}{533}$; (2) $-\frac{31}{49}$; (3) $\frac{8}{31}$; (4) -2.

1120. 1. (a) $-\arcsin\frac{429}{629}$; (a) $\arcsin\frac{21}{221}$. **2.** (a) $-\arccos\frac{63}{65}$; (b) $\arccos\frac{64}{1025}$. **3.** (a) $\arctan\frac{1}{2}$; (b) $-\arctan\frac{1}{2}$. **1126.** (1) 0.5; (2) $\frac{\sqrt{3}}{2}$; (3) $\sqrt{3}-1$; (4) $\frac{\sqrt{21}}{14}$. **1127.** (1) 0.96, -0.28, $-3\frac{3}{7}$, $-\frac{7}{24}$; (2) $-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}, -1$. **1129. 1.** $-\frac{240}{289}, \frac{161}{289}, -\frac{240}{161}$. **2.** $-\frac{120}{169}, \frac{119}{169}$. **1130. 1.** 0.8, -0.6, $-1\frac{1}{3}$. **2.** $\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}, 1$. **1131.** (1) $\frac{12}{5}, -\frac{120}{119}$; (2) $-\frac{1}{7}$; (3) $\frac{204}{325}, \frac{116}{845}$. **1132.** $3\frac{60}{109}, \frac{91}{109}, \frac{60}{91}, \frac{91}{60}$.

1133. 1. $\frac{4}{21}$. **2.** $56\frac{1}{4}$. **1134.** $\sin\alpha = 0.96$, $\cos\alpha = -0.28$.

1136. 1. $\sin 2\alpha = \sin\alpha$ for $\alpha = \pi k$ and $\alpha = \pm\frac{\pi}{3}+2\pi k$, $\sin 2\alpha = 2\sin\alpha$ for $\alpha = k\pi$. **2.** Can. **1137.** $y_{\max} = 2$ for $x = 45°$ and $225°$, $y_{\min} = -2$ for $x = 135°$ and $x = 315°$; $y = 0$ for $x = 90°, 180°, 270°$. **1138. 1.** $\pm\frac{4(a-b)\sqrt{ab}}{(a+b)^2}, \frac{a^2-6ab+b^2}{(a+b)^2}, \pm\frac{4(a-b)\sqrt{ab}}{a^2-6ab+b^2}$. **2.** $\pm\frac{4a(1-a^2)}{(1+a^2)^2}, \frac{6a^2-1-a^4}{(1+a^2)^2}, \pm\frac{4a(a^2-1)}{a^4-6a^2+1}$. **3.** $\frac{9}{16}, \pm\frac{5}{16}\sqrt{7}, \pm\frac{9}{175}\sqrt{175}$. **1139. 1.** $-1\frac{1}{3}, \frac{13}{84}$. **2.** $\frac{8\sqrt{10}-1}{27}, \frac{14-7\sqrt{5}}{27}, \frac{\sqrt{8}-28\sqrt{5}}{81}$. **1140.** (1) 60° or 120°; (2) 15° or 75°. **1141. 1.** -0.96.

2. -0.28. **1142.** (1) 0.342; (2) 0.383; (3) 0.766; (4) 0.6428; (5) -0.5592; (6) 0.9345; (7) 5.671; (8) 2; (9) -0.3526; (10) 0.5403, -0.9388, -0.6956. **1144.** (5) $\cos 4\alpha$; (6) $-\frac{1}{\cos\alpha}$. **1145.** (1) -1; (2) 1; (3) -1; (4) $\cos 4\alpha$. **1146.** (1) $\sin 80°$; (2) $\cos\alpha$; (3) $-\cos 40°$; (4) 1. **1147.** (1) 1; (2) 1; (3) $-2\cot\alpha$; (4) $0.5\sin 2\alpha$; (5) $\cos 2\alpha$; (6) $\tan 2\alpha$; **1148.** (1) Yes; (2) no; (3) no. **1152. 1.** $0.4\sqrt{5}$, $-0.2\sqrt{5}$, -2. **2.** $\pm 0.3\sqrt{10}$, $\pm 0.1\sqrt{10}$, ± 3 or $\pm 0.1\sqrt{10}$, $\pm 0.3\sqrt{10}$, $\pm\frac{1}{3}$. **1153.** $\frac{\sqrt{6}-\sqrt{2}}{4}$, $\frac{\sqrt{6}+\sqrt{2}}{4}$, $2-\sqrt{3}$ and $\frac{\sqrt{2-\sqrt{2}}}{2}$, $\frac{\sqrt{2+\sqrt{2}}}{2}$, $\sqrt{2}-1$. **1154. 1.** 0.6, -0.8. **2.** -3. **1155.** 0.8; 0.6 and 0.6; 0.8. **1156.** $\frac{1}{4}\sqrt{5}$, $\frac{1}{4}\sqrt{11}$, $\frac{1}{11}\sqrt{55}$. **1157.** $\frac{1}{5}\sqrt{24}$; $\frac{1}{5}$ and $\frac{1}{5}$; $\frac{1}{5}\sqrt{24}$. **1159.** (1) $|\sin 2\alpha|$; (2) $\left|\cos\frac{\alpha}{8}\right|$; (3) $\left|\cot\frac{\alpha}{12}\right|$; (4) $\left|\tan\frac{3\alpha}{2}\right|$; (5) $\left|\cot\frac{\alpha}{2}\right|$; (6) 1. **1160.** (1) $\frac{1}{4}\sin 2\alpha$; (2) $\sin 2\alpha$; (3) $2\tan 2\alpha$; (4) $8\cos^2 x$. **1161.** (1) $-2\cos\alpha$; (2) $\sin 2\alpha$; (3) $\left|\cos\left(\frac{\pi}{4}+\frac{\alpha}{2}\right)\right|$; (4) $2\left|\sin\left(\frac{\pi}{4}+\alpha\right)\right|$; (5) $-\sin 2\alpha$; (6) $\sin 2\alpha$. **1162.** (1) 1; (2) -1; (3) $\cos\frac{\alpha}{4}$; (4) $\sin\frac{\alpha}{4}$. **1163.** (1) No; (2) yes, $\alpha = \pm 229°8' + 720°k$, $k = 0, \pm 1, \pm 2, \ldots$. **1167.** (1) 2; (2) $\sqrt{3}$; (3) -1; (4) $-0.5\sqrt{2}$. **1168.** (1) π; (2) 2π; (3) 2π; (4) π; (5) 2π. **1169.** (1) $\frac{1}{2}$ and $-\frac{1}{2}$; (2) 1 and -1; (3) 1 and -1; (4) none; (5) 3 and -1; (6) 3 and 0, **1171.** (1) $(-1)^k\frac{\pi}{6}+\frac{\pi}{2}k$; (2) $\pm\frac{\pi}{3}+\pi k$; (3) πk and $\pm\frac{\pi}{3}+2\pi k$; (4) $\frac{2\pi}{3}k$; (5) $\pm\frac{\pi}{6}+\pi k$; (6) $\pm\frac{\pi}{6}+\pi k$, πk.

1172. (1) πk; (2) $2\pi k$ and $\pi(4k+1)$; (3) $4\pi k$ and $\pi(2k+1)$; (4) $2\pi k$; (5) $2\pi k$, $\frac{\pi}{2}(4k+1)$; (6) $\pi(2k+1)$ and $\frac{\pi}{2}(4k+1)$.

1173. (1) $\frac{\pi}{4}(2k+1)$; (2) πk, $\frac{\pi}{4}+\frac{\pi}{2}k$; (3) $\pm 26°44' + 360°k$; $\pm 124°3' + 360°k$; (4) $-90° + 360°k$, $(-1)^k 22°2' + 180°k$; (5) $(-1)^k\frac{\pi}{12}+\frac{\pi}{2}k$; (6) πk and $\frac{\pi}{4}(2k+1)$. **1174.** (1) $180°k$,

$\pm 26°44' + 360°k$, $\pm 124°3' + 360°k$; (2) πk, $\pm\frac{\pi}{3} + \pi k$; (3) $\frac{\pi}{2}(2k+1)$, $\pm\frac{\pi}{6} + \pi k$; (4) $-22°30' + 90°k$; (5) $\frac{\pi}{2}(2k+1)$ and $(-1)^k\frac{\pi}{6} + \pi k$; (6) no solution. **1175.** (1) $\frac{\pi}{2}k$; (2) $-\frac{\pi}{2} + 2\pi k$ and $(-1)^k\frac{\pi}{6} + \pi k$; (3) $4\pi k$ and $\pm\frac{2}{3}\pi + 4\pi k$; (4) $\pm\frac{\pi}{3} + \pi k$; (5) $\frac{\pi}{4}(4k+1)$ and $\pi k - \arctan 2$. **1176.** (1) $\frac{\pi}{6} + \frac{\pi}{3}k$, $\pm\frac{\pi}{12} + \frac{\pi}{2}k$; (2) $2\pi k$, $\frac{\pi}{4} + \pi k$; (3) $\pm\frac{\pi}{6} + \pi k$; (4) $-\frac{\pi}{2} + 2\pi k$, $\frac{\pi}{4} + \pi k$; (5) $\frac{\pi}{4}(2k-1)$ and πk; (6) $\frac{\pi}{4}(2k+1)$ and $\pm\frac{\pi}{12} + \frac{\pi}{2}k$.

1177. (1) $\frac{24}{25}$; (2) $\frac{3\sqrt{10}}{10}$; (3) $-\frac{527}{625}$; (4) $-\frac{3\sqrt{15}}{16}$; (5) $-\frac{8\sqrt{5}}{81}$; (6) $-3\frac{3}{7}$. **1178.** (1) $-\frac{336}{625}$; (2) $1\frac{5}{8}$; (3) 2; (4) $\frac{41+7\sqrt{33}}{8}$.

1179. (1) $\frac{12}{13}$; (2) $\frac{5}{13}$; (3) $1\frac{191}{289}$; (4) 0; (5) 0; (6) 0.8.

1184. (1) $\pm\frac{\pi}{6}$; (2) 0, $\frac{1}{2}$; (3) $\sqrt{3}$; (4) 0. **1185.** (1) 0, $\pm\frac{\sqrt{7}}{8}$; (2) no solution; (3) 0; (4) $\pm\frac{1}{3}$, 0. **1186.** (1) 0.1; (2) 0; (3) 3; (4) ± 0.5. **1187.** (1) $\cos 10°$; (2) $-\sqrt{3}\sin 6°$; (3) $\sin 10°$; (4) $\cos 10°$; (5) $\frac{1}{2}\sqrt{3}$; (6) $-\cos 20°$. **1188.** (1) 0; (2) $2\sin 25°$; (3) $-2\cos 40°\sin 5°$; (4) $\sqrt{3}\sin 70°$; (5) $-\sqrt{2}\cos 25°$; (6) $2\cos 67°30'\sin(\alpha - 22°30')$. **1189.** (1) $2\sin(\alpha - 20°)\cos 40°$; (2) $2\cos\alpha\cos 40°$; (6) $2\cos 2\alpha\cos\alpha$. **1190.** (1) $\sqrt{2}\cos(45° - \alpha)$; (2) $\sqrt{2}\sin(\alpha - 45°)$; (4) $\sin(\alpha+\beta)\sin(\alpha-\beta)$; (5) $\sin(\alpha+\beta)\sin(\beta-\alpha)$; (6) $\frac{\sqrt{3}}{3}$. **1191.** (1) $4\sin 14°\cos 10°\cos 4°$; (2) $4\sin 25°\cos 35°\cos 10°$; (3) $-4\sin 12°\cos^2 28°$.

1193. (3) $-2\sin^2 8°30'$; (6) $\sin 10°$. **1194.** (1) $\tan 15°$; (2) $\cot^2\left(\frac{\pi}{4} - \frac{\alpha}{2}\right)$; (3) $\tan 20°$; (4) $\tan^2\frac{\alpha-\beta}{2}$.

1195. (3) $4\cos\alpha\sin\left(30° + \frac{\alpha}{2}\right)\sin\left(30° - \frac{\alpha}{2}\right)$; (5) $-4\cos\alpha\sin^2\frac{\alpha}{2}$; (6) $2\sqrt{2}\sin\frac{\alpha}{2}\sin\left(\frac{\alpha}{2} - 45°\right)$.

1196. (1) $-\cot^2\frac{\alpha}{2}$; (2) $-\cot^2\left(45^\circ-\frac{\alpha}{2}\right)$;
(3) $-4\sin 11^\circ\cos^2 28^\circ 30'$; (4) $4\sin 50^\circ\cos 40^\circ\sin 10^\circ$.

1197. (1) $4\sin 2\alpha\cos^2\frac{\alpha}{2}$; (2) $4\cos 3\alpha\cos\left(30^\circ+\frac{\alpha}{2}\right)\cos\left(30^\circ-\frac{\alpha}{2}\right)$;
(3) $4\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha}{2}\cos\frac{\beta}{2}$; (4) $4\cos\alpha\sin\frac{3\alpha+\beta}{2}\cos\frac{\alpha+\beta}{2}$.

1198. (1) $4\cos\alpha\cos\frac{3\alpha}{2}\cos\frac{\alpha}{2}$;
(2) $4\cos 2\alpha\cos\left(45^\circ+\frac{\alpha}{2}\right)\cos\left(45^\circ-\frac{3\alpha}{2}\right)$; (3) $4\sin^2 20^\circ\sin 24^\circ$;
(4) $4\sin 40^\circ\sin 5^\circ\cos 35^\circ$. **1199.** (1) $2\cos\left(45^\circ-\frac{\alpha}{2}\right)$;
(2) $2\sin\left(\frac{\alpha}{2}-45^\circ\right)$; (3) $2\sqrt{2}\cos\alpha$; (4) $2\cos\alpha$.

1200. (9) $\frac{2\sin(60^\circ+\alpha)}{\cos\alpha}$. **1201.** (2) $-2\tan\frac{\pi}{12}$; (3) $\frac{2}{\sin 72^\circ}$;
(4) $\frac{2\sin(\alpha-60^\circ)}{\cos\alpha}$; (5) $\frac{2\sqrt{3}\sin(60^\circ+\alpha)}{\sin\alpha}$; (6) $\frac{\sqrt{2}\sin(\alpha-45^\circ)}{\sin\alpha}$.

1202. (1) $\frac{\cos 2\alpha}{\sin 3\alpha\cos\alpha}$; (2) $\frac{\sqrt{2}\sin(45^\circ+\alpha)}{\cos\alpha}$; (3) -2; (4) $-2\tan 2\alpha$;
(5) $\frac{\sin(\alpha+\beta)\sin(\alpha-\beta)}{\cos^2\alpha\cos^2\beta}$; (6) $\frac{\sin(\beta+\alpha)\sin(\beta-\alpha)}{\sin^2\alpha\sin^2\beta}$. **1203.** (1) $\frac{1}{2\sin 2\alpha}$;
(2) $\frac{1}{\sin 2\alpha}$; (3) $\frac{\cos 3\alpha}{\cos\alpha}$; (4) $-\cot\alpha$. **1204.** $2\tan\alpha\cos^2\frac{\alpha}{2}$;
(2) $2\cot\alpha\cos^2\left(45^\circ-\frac{\alpha}{2}\right)$; (3) $\frac{2\sqrt{2}\sin(45^\circ+\alpha)\cos^2\frac{\alpha}{2}}{\cos\alpha}$;
(4) $\frac{2\sqrt{2}\sin(45^\circ-\alpha)\cos^2\frac{\alpha}{2}}{\cos\alpha}$; (5) $\frac{\sin(\beta-\alpha)}{\cos\alpha\sin\beta}$; (6) $-\tan\alpha\tan 2\alpha\tan 3\alpha$.

1205. (1) $\frac{\sqrt{3}\cos 10^\circ}{8}$; (2) $\frac{\cos(\alpha+\beta)\cos(\alpha-\beta)}{\cos^2\alpha\cos^2\beta}$;
(3) $-\frac{\cos(\alpha+\beta)\cos(\alpha-\beta)}{\sin^2\alpha\sin^2\beta}$; (4) $2\sqrt{\tan\alpha}\sin\left(45^\circ+\frac{\alpha}{2}\right)$
if $4\pi k\leqslant\alpha<\frac{\pi}{2}+4\pi k$; $-2\sqrt{\tan\alpha}\sin\left(45^\circ-\frac{\alpha}{2}\right)$
if $4\pi k+\pi\leqslant\alpha<\frac{3\pi}{2}+4\pi k$; $-2\sqrt{\tan\alpha}\sin\left(45^\circ+\frac{\alpha}{2}\right)$
if $4\pi k+2\pi\leqslant\alpha<\frac{5\pi}{2}+4\pi k$; $2\sqrt{\tan\alpha}\sin\left(45^\circ-\frac{\alpha}{2}\right)$
if $4\pi k+3\pi\leqslant a<\frac{7\pi}{2}+4\pi k$.

1206. (1) $2\sin\left(15^\circ+\frac{\alpha}{2}\right)\cos\left(15^\circ-\frac{\alpha}{2}\right)$;
(2) $2\sin(15^\circ+\alpha)\sin(15^\circ-\alpha)$; (3) $-4\sin 5^\circ\cos 55^\circ$;
(4) $2\sin(60^\circ+\alpha)$; (5) $2\sin(\alpha-30^\circ)$;
(6) $8\sin\left(\frac{\alpha}{2}+24^\circ18'\right)\cos\left(\frac{\alpha}{2}-24^\circ18'\right)$. **1207.** (1) $\frac{3\sin(\alpha-53^\circ8')}{\cos 53^\circ8'}$;
(2) $6\sin 37^\circ38'\cos 2^\circ22'$; (3) $\frac{3\cos 3^\circ8'}{\cos 53^\circ8'}$; (4) $-2\sin 42^\circ$;
(5) $\frac{15\sin(73^\circ4'-\alpha)}{\cos 28^\circ4'}$. **1208.** (1) $4\sin(30^\circ+\alpha)\sin(30^\circ-\alpha)$;
(2) $\sin(\alpha+36^\circ52')\sin(\alpha-36^\circ52')$; (3) $-\frac{\sin(63^\circ26'+\alpha)}{\sin 63^\circ26'\sin\alpha}$;
(4) $\frac{3\sin(33^\circ41'+\alpha)}{\cos\alpha\cos 33^\circ41'}$; (5) $\frac{4\sin(60^\circ+\alpha)\sin(60^\circ-\alpha)}{\cos^2\alpha}$;
(6) $\cot\frac{\alpha+\varphi}{2}\tan\frac{\alpha-\varphi}{2}$, where $\varphi=\arcsin\frac{c}{b}$ if $|c|\leq|b|$ and $b\neq 0$;
$\tan^2\frac{\varphi}{2}$, where $\varphi=\arccos\left(\frac{b}{c}\sin\alpha\right)$ if $|c|>|b|$; -1 if $b=0$.

1211. (3) $\frac{1}{2}\cos 14^\circ-\frac{1}{4}\sqrt{3}$; (4) $\frac{1}{2}\sin 5\alpha+\frac{1}{2}\sin\alpha$;
(5) $\cos 4\alpha+\cos 2\alpha$; (6) $\frac{1}{2}\cos\alpha-\frac{1}{2}\cos 7\alpha$.

1213. (1) $0.5\sin 2\alpha+0.5\sin 2\beta$; (2) $0.5\cos 40^\circ+0.25$;
(3) $\cos 54^\circ+\cos 22^\circ+\cos 18^\circ+\cos 14^\circ$;
(4) $\cos 9\alpha+\cos 7\alpha+\cos 3\alpha+\cos\alpha$;
(5) $0.5\sin 2\alpha+0.5\sin 4\alpha-0.5\sin 6\alpha$;
(6) $2\cos(2\alpha-2\beta)+2\cos(2\beta-2\gamma)+2\cos(2\gamma-2\alpha)+2$.

1214. 1. (a) $\frac{1}{2}(\sqrt{3}-1)$, (b) $\frac{1}{4}(\sqrt{3}-2)$, (c) $\frac{1}{4}(\sqrt{3}-\sqrt{2})$.

2. (a) $\frac{1}{2}(1-\cos 4\alpha)$, (b) $1+\sin 2\alpha$, (c) $1-\sin 2\alpha$,
(d) $2\cos\alpha-\cos 2\alpha-1$, (e) $\frac{1}{4}+\frac{1}{2}\sin 4\alpha-\frac{1}{4}\cos 8\alpha$,
(f) $1.5+2\cos 2\alpha+0.5\cos 4\alpha$, (g) $2(\sin 3\alpha+\sin\alpha)$,
(h) $1.5\sin 2\alpha+0.5\cos 4\alpha-0.5\sin 6\alpha-0.25\cos 8\alpha-0.25$.

1216. (1) $\frac{\sqrt{3}}{2}$; (2) $\frac{1}{2}$; (3) $\sin 3\alpha$; (4) $-\cos\alpha$; (5) $\frac{1}{2}$; (6) $\frac{1}{2}$.

1217. (1) $-\frac{3}{4}$; (2) $\frac{1}{4}$; (3) -1; (4) 0; (5) 0. **1218.** (1) $\sin 2\alpha$;
(2) $\cos 2\alpha$; (3) $\cot 2\alpha$; (4) $4\cos^2\frac{\alpha}{2}$; (5) $\tan\alpha$. **1219.** (1) $\sqrt{a^2+b^2}$,
$-\sqrt{a^2+b^2}$, 2π; (3) $\frac{1}{4}(\sqrt{2}+2)$, $\frac{1}{4}(\sqrt{2}-2)$, π; (4) 1, 0.5, $\frac{\pi}{2}$; (5) 2,
-2, π; (6) 1, $\frac{1}{4}$, $\frac{\pi}{2}$. **1220.** (1) 2; (2) -0.5; (3) -0.5; (4) 0.125.

1225. (1) $k\pi$, $\frac{\pi}{6}(2k+1)$; (2) $\frac{\pi}{7}k$; (3) $\frac{\pi}{3}k$; (4) $\frac{\pi}{6}(6k+1)$; $\frac{\pi}{6}(6k+5)$; (5) $\frac{\pi}{4}(4k-1)$, $\frac{\pi}{8}(4k+1)$. **1226.** (1) $\frac{\pi}{2}k$; (2) $\frac{\pi}{3}(2k+1)$; (3) $\frac{\pi}{4}k$; (4) $-18°+72°k$ and $6°+24°k$; (5) $-\frac{\pi}{4}+\pi k$; (6) πk, $\frac{\pi}{2}+2\pi k$. **1227.** (1) $\frac{\pi}{4}+\pi k$, $\frac{\pi}{2}+\pi k$; (2) $\frac{\pi}{3}k$; (3) $\frac{\pi}{2}k$, $\frac{\pi}{8}(2k+1)$; (4) πk, $\pm\frac{\pi}{3}+\pi k$. **1228.** (3) $45°+(-1)^k\,30°+180°k$; (4) $\frac{\pi}{6}+(-1)^k\frac{\pi}{4}+\pi k$; (5) $-77°28'+(-1)^k 49°24'+180°k$; (6) no solution. **1229.** (1) $-17°\pm 138°+360°k$; (2) $41°+180°k$; (3) $\pm 60°+180°\ k$; (4) $\frac{\pi}{2}(2k+1)$; (5) πk.

1230. (1) $\frac{\pi}{6}+\frac{2\pi}{3}k<x<\frac{\pi}{2}+\frac{2\pi}{3}k$; (2) $-\frac{4}{3}\pi+2\pi k<x<2\pi k-\frac{\pi}{3}$; (3) $\pi k<x<\frac{\pi}{2}(2k+1)$; (4) $\frac{2\pi k}{3}-\frac{7\pi}{18}\leqslant x\leqslant\frac{\pi}{18}+\frac{2\pi k}{3}$.

1231. (1) $\alpha-\beta=2k\pi$ and $\alpha+\beta=\pi(2k+1)$; (2) $\alpha+\beta=2k\pi$ and $\alpha-\beta=2k\pi$; (3) $\alpha-\beta=k\pi$, $\alpha\neq\frac{\pi}{2}(2k+1)$, $\beta\neq\frac{\pi}{2}(2k+1)$; (4) $\alpha+\beta=\frac{\pi}{2}+2k\pi$ and $\alpha-\beta=\pi(2k+1)-\frac{\pi}{2}$; (5) $\alpha+\beta=2k\pi$, $\alpha=2k\pi$, β any angle; $\beta=2k\pi$, α any angle.

1232. $\frac{2b\cos^2\frac{\alpha}{2}}{\cos\alpha}$, 47.8 cm. **1247.** (4) $A=\sqrt{2}$, $\omega=\frac{\pi}{2}$, $T=4$, $\varphi_0=3$; (5) $A=\frac{1}{3}$, $\omega=2\pi$, $T=1$, $\varphi_0=-6\pi$; (6) $A=1$, $\omega=\pi$, $T=2$, $\varphi_0=\frac{\pi}{4}$. **1248.** **1.** $s=-6\cos\frac{\pi}{6}t$; **2.** 0, −6, 6.

1249. (1) $5\sin(x+0.93)$; (2) $13\sin(2x+\pi-1.18)$; (3) $\sin(3x+\pi+0.93)$; (4) $0.94\sin(4x+2\pi-1.01)$.

1251. (1) $\pm\frac{2\pi}{3}+2\pi k$, $\frac{\pi}{4}+\pi k$; (2) $2\pi k$, $(-1)^k\left(-\frac{\pi}{2}\right)+2\pi k$; (3) $(-1)^k\frac{\pi}{8}+\frac{\pi}{2}k$; (4) $\frac{\pi}{4}+\pi k$, $\pm\frac{\pi}{6}+\frac{\pi}{4}+2\pi k$. **1252.** (1) $\pi+2\pi k$, $(-1)^k\frac{\pi}{6}+\pi k$; (2) $-\frac{\pi}{4}+\pi k$, $\pm\frac{\pi}{6}+\pi k$; (3) $\frac{\pi}{2}k$, $\pm\frac{2\pi}{3}+2\pi k$; (4) $-\frac{\pi}{4}+\pi k$. **1253.** (1) $\frac{\pi}{4}(4k-1)$, $\frac{\pi}{12}(24k+1)$, $\frac{\pi}{12}(24k-7)$; (2) πk, $(-1)^k\frac{\pi}{18}+\frac{\pi}{3}k$; (3) $\frac{2\pi}{3}k$, $\frac{\pi}{4}(4k+1)$, $\frac{\pi}{2}(4k-1)$;

(4) $\frac{\pi}{2}(2k+1)$, $\frac{\pi}{3}(3k\pm1)$. **1254.** (1) $\pm\frac{\pi}{3}+\pi k$, $\frac{\pi}{4}(2k+1)$, $(-1)^k\frac{\pi}{12}+\frac{\pi}{2}k$; (2) $\pm\frac{\pi}{6}+\pi k$; (3) $\frac{\pi}{2}+\pi k$, $\frac{\pi}{4}+\pi k$; (4) $-\frac{\pi}{4}+\pi k$, $\pm\frac{\pi}{4}-\frac{\pi}{4}+2\pi k$; (5) $k\pi$, $\frac{\pi}{8}+\frac{\pi}{2}k$. **1255.** (1) πk, $\frac{\pi}{4}+\pi k$; (2) $\frac{\pi}{4}+\pi k$, $\pm\frac{\pi}{4}-\frac{\pi}{4}+2\pi k$; (3) $\frac{\pi}{2}+2\pi k$; (4) $\frac{\pi}{2}(2k+1)$, $\frac{\pi}{4}+\pi k$. **1256.** (1) $-\frac{\pi}{4}+\pi k$; (2) $\frac{\pi}{4}+\pi k$; (3) $\pm\frac{\pi}{6}+\pi k$; (4) no solution; (5) no solution; (6) $\pm\frac{2\pi}{3}+2\pi k$. **1257.** (1) $\frac{\pi}{8}k$, where $k\neq 8n+4$; (2) $\frac{\pi}{3}k$; (3) $\frac{\pi}{4}(2k+1)$; (4) $\frac{\pi}{14}(2k+1)$, where $k\neq 7n+3$; (5) $\frac{\pi}{3}(3k\pm1)$. **1258.** *Hint.* Use the derived proportions. (1) $-59°+180°k$; (2) $-72°+180°k$; (3) $-20°-(-1)^k 65°47'+180°k$; (4) $-75°+180°k$; (5) $29°6'+180°k$; (6) $20°+180°k$.

1259. (1) $5°-(-1)^k 7°24'+90°k$; (2) no solution; (3) $20°\pm16°47'+180°k$; (4) $-\frac{\pi}{4}\pm\frac{\pi}{3}+\pi k$; (5) $\pm\frac{\pi}{6}+\pi k$.

1260. (1) $\pm\frac{\pi}{12}+\frac{\pi}{2}k$; (2) $75°+180k$, $15°+180°k$; (3) πk; (4) $(-1)^k 21°35'+90°k$. **1261.** (1) $\frac{\pi}{8}(2k+1)$, $\frac{\pi k}{2}$; (2) $\pm70°32'+360°k$; (3) $\pm\frac{2\pi}{3}+2\pi k$; (4) $\frac{\pi}{3}k$. **1262.** (1) $\frac{\pi}{2}k$, $\pm\frac{\pi}{6}+\pi k$; (2) $\pm45°\pm69°18'+360°k$; (3) $\frac{\pi}{2}(2k+1)$, $\pm\frac{\pi}{6}+\pi k$; (4) $45°k$; $\pm9°+90°k$; $\pm27°+90°k$. **1263.** (1) $\pi(2k+1)$, $\frac{5\pi}{6}+4\pi k$, $\frac{\pi}{6}+4\pi k$; (2) $\frac{\pi}{2}(4k+1)$; (3) πk, $(-1)^k\frac{\pi}{6}+\pi k$; (4) $\pm\frac{\pi}{6}+\frac{\pi}{2}k$. **1264.** **2.** 0. **1266.** $(2+\sqrt{7}, 2-\sqrt{7})$, $(2-\sqrt{7}, 2+\sqrt{7})$. **1267.** (1) $(28°34', 15°51')$, $(74°9', 61°26')$; (2) $(81°5', -14°5')$, $(-14°5', 81°5')$; (3) $(63°35', 33°3')$, $(33°3', 63°35')$; (4) $(71°36', 6°6')$, $(6°6', 71°36')$. **1268.** $51°4'$, $128°56'$. **1269.** $109°28'$. **1270.** $49°48'$. **1271.** $20°42'$, $62°6'$, $97°12'$.

1272. $70°12'$, $19°48'$. **1273.** $\arcsin\frac{v_1}{v}$. **1278.** (3) $\frac{1}{6}$, $\frac{1}{720}$, $\frac{1}{3628800}$; (4) 11, 23, 39; (5) 12, 96, 1536; (6) 0, $\frac{1}{3}$, $\frac{1}{5}$.

1280. No. 11, No. 105, does not belong to the sequence, No. 89.

1281. *Hint.* Represent the general term of the sequence in the form of a difference. (1) 0.99; (2) $\frac{9}{19}$; (3) $\frac{10}{21}$. **1288.** (a) 1, (b) $2k$.

1289. (a) 13, (b) 6. **1290. 1.** $\sin^2\alpha$. **2.** $\frac{1}{\cos^2\alpha}$. **3.** 8. **4.** 5.

1291. 1. 5050, $\frac{n(n+1)}{2}$. **2.** 2500, n^2. **3.** 2550, $n(n+1)$.

1293. (1) $a_8 = 34$, $S_8 = 132$; (3) $a_1 = -6$, $S_{10} = 120$; (6) $n = 20$, $S_{20} = 330$; (8) $n = 16$, $d = -3$; (10) $d = -6$, $a_7 = -21$; (11) $a_1 = 6$, $d = -4$; (14) $n = 8$, $a_8 = 3$. **1294.** 55.

1295. $a_1 = 3$, $d = 4$. **1296.** 0. **1297.** $-5, -3, -1, \ldots$.

1298. $\frac{1}{2}, 2, 3\frac{1}{2}, \ldots$ and $6\frac{1}{2}, 5, 3\frac{1}{2}, 2, \ldots$. **1299.** 40, -24.

1300. (1) $-5, -3, -1, \ldots$; (2) $2, 0, -2, \ldots$; (3) $-4, -1, 2, \ldots$. **1301.** (1) $1, -2, -5, \ldots$; (2) $4, 5, 6, \ldots$ and $6, 5, 4, \ldots$.

1302. 1. 7. **2.** $2n-1$. **1303.** (a) $-n$ if n is even; n, if n is odd; (b) $-\frac{n}{2}$ if n is even; $\frac{n+1}{2}$ if n is odd. **1304. 1.** 100.

2. 36. **1305.** $\frac{1}{2}, 1, 1\frac{1}{2}, \ldots$. **1306.** 2. 9, 16 .... **1307.** $-1, 3, 7, \ldots$. **1308.** 25. **1310.** 14. **1311.** 1, 3, 5, ... **1312.** mn.

1313. 90. **1314.** $\div +7, 9, 11\ldots$. **1315.** 9 or 16. **1316.** 432.

1317. 39. **1318.** 36°52′, 53°8′. **1319.** (1) 55; (2) 1; (3) 6.

1321. Yes. **1324.** No. **1325.** (a) $\pi, 2\pi, 3\pi, \ldots$, (b) no, (c) 60°, 240°, 420°, ... **1326.** -1, 0 and 1. **1331.** In 20 s.

1332. 10 m/s². **1333.** 7 s. **1334.** 11 m. **1335.** In 3 hours.

1336. In 10 min and 15 min. **1337.** 4.6 m. **1338.** 22a.

1339. 14 m. **1340.** 15. **1341.** 1540. **1346.** (a) 2, (b) 2916, (c) -1. **1347.** (1) 242; (2) $17\frac{13}{16}$; (3) 21.5; (4) $-123\frac{37}{256}$.

1348. (1) -0.5 and 50.4; (2) $\pm\frac{1}{3}$, -30.25 and -15.25; (3) 4 and 127.5. **1349.** (1) 3 and 65535; (2) -32 and -5187; (3) 3584 and 2387; (4) $7\frac{19}{32}$ and $21\frac{43}{96}$. **1350.** (1) 3 and 768; (2) 2 and $\frac{1}{8}$; (3) $-162, -32$; (4) 1, -2048. **1351.** (1) 10; (2) 8; (3) 5; (4) 6.

1353. (1) $a_1 = 256$, $S_8 = 510$; (2) $q = 3$, $S_7 = 1093$ or $q = -3$, $S_7 = 547$; (3) $a_1 = -\frac{243}{160}$, $a_6 = \frac{1}{5}$; (4) $q = \frac{1}{2}$, $n = 7$; (5) $n = 5$, $S_5 = -\frac{211}{243}$; (6) $a_1 = -4.5$, $S_4 = 90$. **1354.** (1) 2, 6,

18, ... and -162, -54, -18, ...; (2) 3, -6, 12. **1355.** (1) 36, $\frac{1}{3}$, 8; (2) 1, 3, 4. **1357. 2.** 1048576. **1361. 1.** No.

1363. $\sqrt[n-m]{\frac{l^{p-m}}{k^{p-n}}}$ for $n > m$. **1364.** $\frac{2}{9}\left[\frac{10(10^n-1)}{9}-n\right]$.

1365. $\frac{\sqrt{5}-1}{2}$, $\frac{\sqrt{2(\sqrt{5}-1)}}{2}$. **1366.** 2, 6, 18 and 18, 6, 2; $2(\sqrt{55}+8)$, -6, $2(8-\sqrt{55})$ and $2(8-\sqrt{55})$, -6, $2(8+\sqrt{55})$.

1367. 1, 2, 4 and 4, 2, 1. **1368.** 5, 25, 125. **1369.** $2^{64}-1$.

1370. $q = 4$, a_1 any number. **1371.** 96, 48, 24 and 12, 6, 3.

1372. 3, 6, 12, 24 and 24, 12, 6, 3. **1373.** 2, 5, 8, ...; 3, 6, 12, ... and $12\frac{1}{2}$, $13\frac{1}{6}$, $14\frac{5}{6}$, ...; $\frac{2}{3}$, $\frac{25}{3}$, ... **1374.** 24, 27, 30, ...

1375. 125.

1376. $\begin{cases} \div\ 2, 6, 10, \ldots \\ \vdots\ 2, 6, 18, \ldots, \end{cases}$ $\begin{cases} \div\ 2, -2, -6, \ldots \\ \vdots\ 2, -2, 2, \ldots, \end{cases}$

$\begin{cases} \div\ 1, 1+2\sqrt{2}, 1+4\sqrt{2}, \ldots \\ \vdots\ 1, 1+2\sqrt{2}, 9+4\sqrt{2}, \ldots, \end{cases}$ $\begin{cases} \div\ 1, 1-2\sqrt{2}, 1-4\sqrt{2}, \ldots \\ \vdots\ 1, 1-2\sqrt{2}, 9-4\sqrt{2}, \ldots, \end{cases}$

and so on — a set of solutions. **1377.** 4, 12, 36 and $\frac{4}{9}$, $-\frac{20}{9}$, $\frac{100}{9}$.

1378. $q = \frac{3}{2}$, a_1 any number. **1379.** The arithmetic one.

1380. $\begin{cases} \div\ 2, 5, 8, 11, 14, \ldots \\ \vdots\ 2, 4, 8, 16, 32, \ldots, \end{cases}$ $\begin{cases} \div\ 2, 5, 8, 11, 14, \ldots \\ \vdots\ 2, -4, 8, -16, 32, \ldots. \end{cases}$

1381. 3, 15, 27 and 3, 9, 27. **1382.** 4, 11, 18 and 19, 11, 3.

1383. 18, 7, -4 and 3, 7, -4 and 3, 7, 11. **1384.** 5, 15, 45 and 45, 15, 5. **1385.** 4, 8, 12, 18 and 17.5, 12.5, 7.5, 4.5. **1386.** 3, 7, 11, 15. **1387.** 1, 3, 9, 27. **1388.** 243 l. **1389.** 5. **1390.** (1) 7; (2) $\frac{(a^{2n}-1)(a^{2n+2}+1)}{a^{2n}(a^2-1)}+2n$. **1393.** (1) x any real number, $0 < y < \infty$; (2) $-\infty < x < \infty$, $4 \leqslant y < \infty$; (3) $x \neq 0$, $0 < y < \infty$, except for $y = 1$; (4) $-\infty < x < \infty$, $2.5 \leqslant y \leqslant 10$; (5) $x \leqslant -2$ and $x \geqslant 2$, $0 < y \leqslant 1$; (6) $-1 \leqslant x \leqslant 1$, $2^{-\frac{\pi}{2}} \leqslant y \leqslant 2^{\frac{\pi}{2}}$; (7) $-\infty < x < \infty$, $-3 < y < \infty$; (8) $x \leqslant -2$, $0 \leqslant y < \infty$; (9) $x \geqslant 0$, $1 \leqslant y < \infty$. Even: 2 and 5; odd: none.

1394. (1) $y_{\min} = 1$; (2) $y_{\min} = 1$, $y_{\max} = 2$; (3) $y_{\max} = 1$ for $x = \pm 2$, $y_{\min} = \frac{1}{9}$ for $x = 0$; (4) $y_{\max} = 1$ for $x = 0$; (5) $y_{\max} = 4$

for $x=\frac{\pi}{2}+2\pi k$, $y_{\min}=\frac{1}{4}$ for $x=-\frac{\pi}{2}+2\pi k$; (6) $y_{\max}=4$ for $x=\frac{\pi}{2}+\pi k$, $y_{\min}=1$ for $x=\pi k$; (7) $y_{\max}=9$ for $x=\pi+2\pi k$, $y_{\min}=1$ for $x=2\pi k$; (8) $y_{\max}=6$ for $x=2\pi k$, $y_{\min}=3$ for $x=\pi+2\pi k$; (9) $y_{\min}=4$ for $x=2$. **1403.** (1) $x>-2$; (2) $x<-1.5$; (3) $x>-2$; (4) $-3<x<0$; (5) $x<-\frac{1}{8}$; (6) $x>1\frac{1}{6}$; (7) $x<0.4$. **1406.** (1) $m>n$; (2) $m>n$; (3) $m<n$. **1408.** (1) $0<x<1$; (2) $0<x<1$; (3) $-3<x<-2$; (4) $-1<x<0$. **1409.** (1) $a<1$; (2) $a<1$; (3) $a>1$; (4) $a>1$; (5) $a>1$; (6) $a>1$. **1410.** $a\left(\frac{2}{3}\right)^n$, where a is the initial intensity. **1418.** (c) 1.5. **1419.** (3) $-\frac{2}{3}$; (4) $-\frac{1}{6}$; (5) -0.3. **1420.** (3) 4; (4) 0.25; (5) $5\sqrt{5}$; (6) $2\sqrt{2}$. **1421.** (2) $\frac{1}{9}$; (3) $\frac{1}{1600}$; (6) -3; (7) $-2\frac{2}{3}$; (8) 0.5. **1422. 1.** a^a. **2.** (a) 0.25, (b) $\sqrt[N]{N}$. **1423.** (2) $\frac{1}{3}$; (4) 9; (7) 8; (8) 0.01; (9) 0.25; (10) 0.8; (11) 36; (12) 15; (13) 36; (14) 242; (15) $1\frac{1}{3}$. **1424.** (2) 2; (5) 0.25; (6) -2; (7) 0. **1425.** (2) 25; (3) 2, 4; (4) -1, 0.25; (5) 10, 0.1, $10^{\sqrt{2}}$, $10^{-\sqrt{2}}$. **1427. 2.** (a) 1.5, (b) 25, (c) $\sqrt[3]{100}$, (d) 2. **1428.** 16, 4, 1, $\frac{1}{4}$ and $\frac{1}{16}$, $\frac{1}{4}$, 1, 4. **1429.** (b) $y=\frac{1}{3}\log_2 x$, $x>0$, $-\infty<y<\infty$; (c) $y=-2\log_2 x$, $x>0$, $-\infty<y<\infty$; (d) $y=\log_3 x-1$, $x>0$, $-\infty<y<\infty$; (e) $|y|=\log_3 x$, $x\geqslant 1$, $-\infty<y<\infty$; (f) $y=\sqrt{\log_{\frac{1}{3}} x}$, $0<x\leqslant 1$, $0\leqslant y<\infty$; (g) $y=\arcsin\log_5 x$, $\frac{1}{5}\leqslant x<5$, $-\frac{\pi}{2}\leqslant y\leqslant\frac{\pi}{2}$; (h) $y=\sqrt{\log_5 x-1}$, $x\geqslant 5$, $0\leqslant y<\infty$. **1430.** (2) (b) 0, (c) ± 2, (d) $\frac{\pi}{2}+2\pi k$, (e) πk, (f) $\sin 1$. **1431.** (4) $x<-2$ and $x>2$; (5) $-\sqrt{3}<x<\sqrt{3}$; (6) $x\neq 0$; (9) $x<-2$ and $x>0$.

1432. (1) $x<-1$ and $x>2\frac{1}{3}$; (2) x any real number; (3) $x\neq 0.5$; (4) $-2<x<-1$ and $1<x<2$; (5) $x<-1$ and $x>2$; (6) x any real number; (7) $x<-1$ and $x>1$; (8) $x>1$ for $a>1$ and $0<x<1$ for $0<a<1$.

1433. (1) $2\pi k < x < \pi + 2\pi k$; (2) $\pi k < x < \frac{\pi}{2} + \pi k$;
(3) $\pi k - \frac{\pi}{4} < x < \frac{\pi}{4} + \pi k$; (4) $x \neq \pi k$; (5) $0 < x < 1$; (6) $x \geqslant 0$.

1439. (3) -1 and 0; (4) -3 and -2. **1440.** (6) $\log_a 5 > \log_5 a$ for $1 < a < 5$ and for $0 < a < \frac{1}{5}$; $\log_a 5 = \log_5 a$ for $a = 5$ and $a = \frac{1}{5}$; $\log_a 5 < \log_5 a$ for $a > 5$ and for $\frac{1}{5} < a < 1$.

1441. 2. (a) $\frac{1}{2} < m < 1$, (b) $0 < m < \frac{1}{3}$. **1445.** (1) $x > 1$; (2) $0 < x < 1$; (3) $x > 3$; (4) $0 < x < 0.75$; (5) $x \geqslant 1$ for $a > 1$ and $0 < x \leqslant 1$ for $0 < a < 1$; (6) $0 < x \leqslant 1$ and $-1 \leqslant x < 0$ for $a > 1$, $x \leqslant -1$ and $x \geqslant 1$ for $0 < a < 1$.

1450. (5) $\frac{1}{5}[3 \log_{10} a + 2 \log_{10} (a+b)]$;
(6) $2 \log_{10} a + 3 \log_{10} b - \left(\log_{10} c + \frac{1}{2} \log_{10} d\right)$;
(7) $\log 4 + 4 \log a + \frac{1}{5}(2 \log c - \log d)$;
(8) $\log 3 + \log m + 2 \log n - \left[\log 4 + \frac{1}{2}(\log 5 + \log m + \log n)\right]$;
(9) $2 \log p + 3 \log(m - n^2) - \frac{1}{2} \log q$. **1451.** (4) $3 \log n - 2 \log m$;
(5) $\log 3 + \frac{1}{2} \log \cos 2\alpha - 3 \log a$;
(6) $\frac{1}{3}(\log 4 + 2 \log \tan \alpha) - \frac{1}{2} \log a - \log 3$;
(7) $\log 2 + \frac{1}{3} \log a + \frac{3}{4} \log b$; (8) $\frac{4}{3} \log b - 2 \log a$;
(9) $\frac{m}{n}(\log m - \log n)$.

1452. (1) $\frac{1}{3}(\log 4 + 2 \log \sin \alpha) - (\log 5 - 3 \log b - 2 \log a)$;
(2) $-\frac{3}{4} \log a - \frac{2}{3} \log b$; (3) $\frac{1}{2}\left[\log a + \frac{1}{2} \log b\right] - \log 5$;
(4) $\frac{1}{2}\left[\log 2 + \frac{1}{2}\left(\log 6 + \frac{1}{2} \log 15\right)\right]$;
(5) $\log 3 + \frac{1}{3}\left(\log a + \frac{1}{2} \log b\right) - \log 5$;
(6) $\frac{1}{3}\left[\log 3 + \frac{1}{3}\left(\log 21 + \frac{1}{3} \log 6\right)\right]$;
(7) $\frac{1}{3}\left(2 \log a + \log b - \frac{3}{5} \log c\right)$; (8) $-\frac{3}{5}\left(2 \log a + 3 \log b + \frac{1}{4} \log c\right)$.

1453. (1) $\frac{1}{12}(7\log a-5\log b)$; (2) $\frac{1}{15}(5\log 5+8\log a+4\log b)$; (3) $\frac{2}{5}\log a+\frac{3}{4}\log c-3\log b$; (4) $\frac{1}{12}(11\log a-7\log b)$; (5) $\frac{1}{n}\left[\log a+\frac{1}{n}\left(\log b+\frac{1}{n}\log c\right)\right]$; (6) $\frac{1}{n}\left[\frac{1}{n}\left(\log b-\frac{1}{n}\log c\right)-\log a\right]$. **1454.** (4) $\log 2+\log(\log a)$; (5) $2\log(\log a)$; (6) $\log 3+\log(\log ab)-\log 2$.

1455. (2) $\log(\log 2)-\frac{1}{2}\log 2$; (3) $\log(\log 3)-\frac{2}{3}\log 3$; (4) $\log a+\log b+\log(\log 10)$; (5) $\log 10-\log 3$; (6) $\log[\log(\log 5)-\log 2]$. **1459.** (1) 13.5, $2.5\sqrt{2}$; (3) 48; (4) $\frac{7\sqrt{2}}{4}$. **1460.** (1) $\frac{\sqrt{a-b}}{\sqrt[5]{(a+b)^2}\sqrt[3]{b^2}}$; (2) $\sqrt[5]{(ab)^2}$; (3) $\sqrt[7]{\left(\frac{a}{b}\right)^4}$; (4) $\sqrt[4]{\left(\frac{a}{2}\right)^3}$. **1461.** (1) $\frac{a+b}{\sqrt[3]{a^4\sqrt{b^3}}}$; (2) $\frac{b}{(b+c)^n\sqrt[n]{b-c}}$; (3) $\frac{1}{a^2}\sqrt[3]{\frac{(a+b)\sqrt[4]{(a-b)^3}}{b\sqrt{c}}}$; (4) $\sqrt[4]{\frac{8(a-b)\sqrt[5]{(a-b)^4}}{c^9b^2(a+b)^3\sqrt[3]{b^2}}}$. **1462.** (1) $\frac{a^5\sqrt{a+b}}{\sqrt{mn^3}}$; (2) $\frac{1}{5}\sqrt[5]{\frac{3a^3\sqrt[3]{ab^2}}{c^2}}$; (3) $\sqrt[3]{\frac{m^2(m+n)\sqrt{m+n}}{(m-n)^4n\sqrt{n}}}$; (4) $\sqrt[4]{\frac{b\sqrt[3]{(a+b)^2}}{a^2\sqrt{a-b}}}$.

1463. (1) -2; (2) 1.5; (3) $-\frac{2a+1}{a}$; (4) $\frac{12-4a}{3+a}$.

1465. (2) $z^3\sqrt[5]{t^2}=2$; (3) $\sqrt[3]{x^2}=\sqrt{z}$; (4) $\sqrt[3]{x}=a^2y$. **1466.** (1) 3; (2) 1; (3) $1+\log_a b$; (4) 0. **1467.** Possible: (3) and (5).

1470. 1. (a) $\frac{a}{c}$, (b) $\frac{a}{9b}$, (c) $\frac{c}{2a}$. **2.** 0.75. **1471. 3.** 2, 10, 50 or 50, 10, 2. **1472.** -6. **1474.** 61. **1483.** (2) 2.9308, 2.5953, 2.8297; (3) $\bar{8}$.9308, 0.3381, 0.0546. **1484.** (2) $\bar{1}$.5897, $\bar{1}$.7691; (3) 0.7052, 0.5282, $\bar{2}$.4140; (4) 20.622. **1485.** (1) 1.0358; (2) 5.1195; (3) $\bar{3}$.8432; (4) 1.8007. **1488.** (9) 19.77; (10) -0.00079; (11) 2.956; (12) -0.4006. **1489.** (1) 0.5084; (2) 1.972; (3) 106.5; (4) -468.2.

1490. (1) 0.0567; (2) 94.26; (3) 0.1042; (4) 28.2. **1491.** (1) 1094; (2) 35.22; (3) 44.06; (4) 0.002. **1492.** (1) 0.3588; (2) -29.26; (3) 0.182; (4) -0.0016. **1493.** (1) 0.4410; (2) 0.3224; (3) 0.877; (4) 1.358. **1494.** (1) 7.416; (2) 5.029; (3) 3.713; (4) 1.881.

1495. (1) 0.0055; (2) 1.176; (3) 1.087; (4) -0.129.

1496. (1) 1.321; (2) 1.457; (3) -34.52; (4) 1.8076; (5) 59.7;

(6) 10.06. **1500.** (1) 0.1082; (2) 7.499; (3) 6.449; (4) 99.79; (5) 30.57; (6) 3.36; (7) 461; (8) 0.2576; (9) 3869.

1501. 1. 172.7 dm^2. **2.** 593.9 dm^2. **3.** 5.968 m.

1502. (1) 1.376 m^2; (2) 5738 cm^2. **1503. 1.** 21510 dm^3. **2.** 3.4 dm. **3.** 212.5 m. **1504.** 510 000 000 km^2; 1083×10^9 km^3. **1505.** 60.

1506. (3) $1\frac{1}{3}$; (4) 1.5; (5) $\frac{7}{9}$; (6) -8; (7) -0.5; (8) 1.8.

1507. (1) 1; (4) $-\frac{6}{7}$; (5) $-2\frac{1}{3}$; (6) -2.5; (7) 0.5; (8) $\pm\frac{1}{3}$; (9) $-\frac{5}{18}$. **1508.** (1) ±2; (2) 2; (3) 1.8; (4) 2; (5) -3; (8) -2, 1, 3; (9) 1.3; (10) -2. **1509.** (1) 20; (2) 0.5, 1; (3) 2.5, 5.2; (4) -0.5, 1.5; (5) 2; (6) 0.5, 3. **1510.** (1) 10; (2) 12; (3) 3; (5) -7, 1; (6) 5. **1511.** (1) 5; (2) 9; (3) $1\pm\sqrt{8}$; (4) 3.

1512. (1) 1; (2) 2; (3) 2; (4) 0 and 25. **1513.** (2) 3.585; (3) 2.096; (4) -5.128; (5) ±0.6907; (6) 2.383. **1515.** (1) $\pm\frac{\pi}{6}+\pi k$; (2) 0.5; (3) $-45°+180°k$ and $63°26'+180°k$; (4) $\frac{\pi}{4}(2k+1)$ and $(-1)^k\frac{\pi}{12}+\frac{\pi}{2}k$. **1516.** (2) 2; (3) 1; (4) 3; (5) 0; (6) 1. **1517.** (1) 2; (2) 1; (3) 1; (4) 3; (5) 3; (6) 10. **1518.** (1) 0; (2) -0.4369; (3) -1.701; (4) 2.896. **1519.** (1) 2; (2) 0, 0.8269; (3) 3; (4) 2.

1520. (1) -2; (2) 2; (3) 0, 0.25; (4) 6. **1521.** (1) 1, 0.2031; (3) 0.25, 0.5; (4) 2.25, 3. **1522.** (1) $\pm\frac{\pi}{6}+\pi k$, $\pm\frac{\pi}{3}+\pi k$; (2) -4, 4; (3) 8; (4) 0, -5.419. **1523.** (1) $\frac{\log_{10}(1+\sqrt{5})-\log_{10}2}{\log_{10}3-\log_{10}2}$; (2) 0.585; (3) -2; (4) 0.5. **1524.** (1) 2; (2) 3; (3) 0, 2 and 9; (4) 5. **1525.** (1) ±1, $\pm\sqrt{6}$; (2) 10^{10}; (3) 3; (4) 4. **1526.** (1) 31; (2) 9; (4) 2. **1527.** (1) -2.5; (2) 2; (4) 5. **1528.** (1) 6 and 14; (2) $3\frac{2}{3}$; (3) 5. **1529.** (1) 2; (2) $1\frac{1}{59}$; (3) $-\frac{1}{3}$, 7; (4) 1 and 5.

1530. (1) 3; (2) 2, 4; (3) 0.99; (4) 8. **1531.** (1) 1; (2) -2.322; (3) 4; (4) 4. **1532.** (1) 0.0001; (2) $100^{\sqrt{2}}$, $100^{-\sqrt{2}}$, 100, 0.01; (3) 0.1 and 0.001; (4) 0.1, 1, 10. **1533.** (1) 1.1, 11, 1.01, 101; (2) ±3; (3) 0, 1, 10; (4) 1, $10^{\sqrt[m-n]{m^n:n^m}}$. **1534.** (1) 1, -1; (2) $\frac{1}{32}$, 2; (3) 0.0001, 10; (4) -1; (5) 0.01, 10; (6) 0, 1, 10. **1535.** (1) 0.01, 100; (2) 4; (3) $\frac{1}{a}$, a^2; (4) 0.01, 10; (5) -3; (6) ≈0.3806, 1,

≈ 2.628. **1536.** (1) 1.53; (2) 2; (3) 3; (4) 25, $\sqrt{5}$; (5) ≈ 2.155; (6) 16; (7) b, $\frac{1}{b}$; (8) a. **1537.** (1) 3, $1+\frac{1}{\sqrt[4]{32}}$; (2) $\frac{1}{12}$; (3) 1, ac.

1538. (2) (4, 1); (3) (3, 3), (4, 2), (5, 1); (4) (3, 1); (5) (1, 2); (6) $\left(-\frac{1}{14}, -\frac{5}{14}\right)$. **1539.** (1) $\left(\frac{1}{4}, \frac{3}{4}\right)$, $\left(\frac{8}{11}, -\frac{4}{11}\right)$; (2) (100, 10); (3) (9, 7); (4) (10, 0.01); (5) (10, 4), (4, 10); (6) (4, 2).

1540. (1) (3, 9); (2) (1, 1), (4, 2); (3) (3, 3); (4) (3, 1), $\left(\frac{1}{3}, \frac{7}{3}\right)$; (5) (20, 16); (6) (2, 6). **1541.** (1) $x > \frac{1}{3}$; (2) $x < -1$ and $x > 5$; (3) $x > 8$; (4) $4 < x < 6$; (5) $-\frac{3}{5} < x < \frac{1}{3}$; (6) $x < -1$, $-1 < x < 1$ and $x > 4$; (7) $x > 2$; (8) $-1-\sqrt{2} < x < -2$ and $0 < x < \sqrt{2}-1$; (9) $x < -2.5$ and $x > 0$; (10) $0 < x < 1$.

1542. 1. (a) $0 < n \leqslant 5$; (b) $n > 5$. **2.** 7. **1543.** (1) $x > 1.55$; (2) $1 < x < 10$; (3) $x < 2$; (4) $7.5 < x < 8$; (5) $x > 3$.

1544. (1) $x < 2$ and $x > 3$; (2) $0.7 < x < 1$; (3) $x < -3$; (4) $-4 < x < -2$ and $0 < x < 2$. **1545.** (1) $2.5 < x < 4$ and $x > 4$; (2) $0 \leqslant x \leqslant 4$; (3) $\sqrt{6}-1 \leqslant x < 2$, $2 < x \leqslant 5$; (4) $x \geqslant 0.4$.

1546. (1) $2 \leqslant x < 2\frac{3}{4}$ and $x \geqslant 4$; (2) $1.17 < x < 2$; (3) $\frac{3-\sqrt{5}}{2} < x < \frac{6-\sqrt{6}}{4}$ and $\frac{6+\sqrt{6}}{4} < x < \frac{3+\sqrt{5}}{2}$; (4) $x < 2$.

1548. $\frac{2}{3}, \frac{\sqrt{5}}{3}, \frac{2}{\sqrt{5}}, \frac{\sqrt{5}}{2}, \frac{3}{\sqrt{5}}, \frac{3}{2}$. **1549.** (1) $\sin\alpha = \frac{4}{5}$, $\cos\alpha = \frac{3}{5}$; (2) $-\frac{2\sqrt{2}}{3}, \frac{1}{3}$; (3) 0.28, 0.96; (4) $\pm 0.6, -0.8, \mp 0.75$.

1550. (1) -0.5; (2) -1. **1551.** (1) -2; (2) $2\cos\alpha$; (3) $\sin 2\alpha$; (4) 1.5. **1552.** (1) $\tan^4\alpha$; (2) $\cot 3\alpha \cot\alpha$; (3) $-\cot^2\alpha \cot^2\beta$; (4) $2(\cos\alpha + a\sin\alpha)$. **1555.** (1) $\sqrt{2}\sin(2\alpha+45°)$; (2) $-4\cos\frac{\alpha}{2}\sin\frac{\beta}{2}\cos\frac{\alpha-\beta}{2}$; (3) $4\sqrt{2}\sin(2\alpha+45°)\cos\left(\frac{\alpha}{2}+30°\right)\cos\left(\frac{\alpha}{2}-30°\right)$. **1556.** (1) 1; (2) $-\frac{2\sqrt{13}}{13}$; (3) $-\frac{\sqrt{33}}{3}$; (4) $\frac{2+\sqrt{2}}{2}$. **1557.** (1) $\frac{\pi}{8}(2k+1), \frac{\pi}{2}k$; (2) $2\pi k, \frac{\pi}{2}\left(4k+\frac{\pi}{2}\right)$; (3) $\frac{\pi}{4}(2k+1), \pm\frac{2\pi}{3}+2\pi k$; (4) $\frac{\pi}{4}(2k+1)$, πk; (5) πk; (6) $\frac{\pi}{3}k, \frac{\pi}{2}(2k+1)$; (7) $\frac{\pi}{6}k, \pm\frac{\pi}{15}+\frac{2\pi}{5}k$.

1558. (1) $\frac{2\pi}{13}$; (2) $\frac{\pi}{4}$; (3) $-1, 0, 1$; (4) $-\frac{\sqrt{3}}{2}, 0, \frac{\sqrt{3}}{2}$.

1559. $4\sqrt{6}-4$, $4\sqrt{6}$ and $4\sqrt{6}+4$. **1560.** 2 cm, 4 cm, 6 cm. **1561. 1.** -4374; **2.** 3. **1562. 1.** -0.5; **2.** 7. **1563.** 421. **1564.** 381. **1565.** (1) 8, 4, 2, ...; 8, -4, 2, ...; -8, -4, -2, ...; -8, 4, -2, ...; (2) 3, 9, 27, ...; 3, -9, 27, ...

1566. 1. 2, 4, 8, ... **2.** 3905. **1567.** 5, 15, 45, ... **1568.** (1) -1.5, 0.5; (2) -1, 4; (3) 24; (4) $\pm 32°45' + 180°k$; (5) 6; (6) 0. **1569.** (1) 9; (2) $\frac{1}{3}$; (3) 6; (5) 2; (6) $\frac{5}{6}$.

1571. (1) 2; (2) 8; (3) 9; (4) 8. **1572.** (1) 0.3014; (2) 0.0481; (3) 1.184; (4) 2.583. **1573. 2.** 60° and 60°. **3.** $\frac{\pi}{2}k$.

4. $\frac{m^2-1}{m^2+1}$ if $m \geqslant 0$ and -1 if $m < 0$. **5.** $a^4-a^3+a^2-a+1$.

6. (a) $2\pi k < x < \pi+2\pi k$, (b) $2\pi k-\frac{\pi}{2} < x < \frac{\pi}{2}+2\pi k$, (c), (d) $\pi k < x < \frac{\pi}{2}+\pi k$. **1574. 2.** (a) No, (b) $2\pi k$, (c) $\frac{\pi}{2}+2\pi k$.

3. $-\frac{\pi}{3}+\pi k$. **4.** $n^4-mn^3+m^2n^2-m^3n+m^4$. **5.** $\frac{2n}{1-n^2}$ for $0 \leqslant n < 1$ and $n > 1$ and 0 for $n < -1$ and $-1 < n \leqslant 0$. **6.** 2.

1575. 1. $y_{\text{great}} = \begin{cases} m+n \text{ if } m > 0 \\ -m+n \text{ if } m < 0; \end{cases}$ $y_{\text{least}} = \begin{cases} -m+n \text{ if } m \geqslant 0 \\ m+n \text{ if } m \leqslant 0. \end{cases}$

2. Isosceles. **3.** $-\frac{1}{3} \leqslant x \leqslant 1$. **4.** $-\frac{\pi}{4}+\pi k$, $(-1)^k\frac{\pi}{12}-\frac{\pi}{4}+\frac{\pi}{2}k$. **5.** $5\frac{1}{3}$. **6.** 0 and $\frac{1}{9}$. **1576. 1.** $\frac{\pi}{2}$, 1 and $\frac{1}{2}$. **2.** $\frac{1}{2}$. **3.** $x = 2\pi k$, $y = 0$. **4.** (a) $0.1 < x < 10$, (b) $1 < x < 10\,000$. **5.** (a) πk and $-\frac{\pi}{8} \pm \arccos -\left(\frac{1}{2\sqrt{2}}\right)+2\pi k$, (b) $\frac{1}{2}$ and $-\frac{1}{3}$. **6.** $\frac{2}{\pi}, \frac{2}{5\pi}, \frac{2}{9\pi}$, ..., an infinite bounded decreasing sequence. **1577. 1.** $\frac{2}{\pi+2\pi k}, \frac{1}{2\pi k}, \frac{1}{\pi+2\pi k}$; even. **4.** $\tan\alpha$;

5. (a) 347479, (b) $-29512\frac{3}{266}$.

1578. 1. $y_{\text{great}} = \begin{cases} m+n \text{ if } m \geqslant 0, \\ n \quad \text{if } m < 0; \end{cases}$ $y_{\text{least}} = \begin{cases} n \quad \text{if } m \geqslant 0, \\ m+n \text{ if } m < 0. \end{cases}$

2. 1. **4.** $\pm\frac{\pi}{3}+\pi k$. **5.** $\cot\frac{\alpha}{2}$. **6.** $-\tan 2\alpha$. **1579. 1.** 2 least value. **2.** 30°, 60°. **3.** $-0.5 \leqslant x \leqslant 0.5$. **4.** $6°+360°k$ and $-90°+360°k$. **5.** $\dfrac{2\sqrt{2}\cos(45°-\alpha)\cos^2\frac{\alpha}{2}}{\cos\alpha} \approx 2.334$.

1580. 1. (a) Yes, (b) yes, (c) no. **2.** 13 and -13.

3. $2\cos(60°+2\alpha)$. **6.** πk, $\pm\frac{1}{2}\arccos\frac{1-\sqrt{17}}{4}+\pi k$. **7.** 9.

1581. 1. (a) $2\pi k+\frac{2\pi}{3} < \alpha < \pi+2\pi k$, $2\pi k-\frac{2\pi}{3} < \alpha < 2\pi k$; (b) πk, $\pm\frac{2\pi}{3}+2\pi k$. **2.** $-\frac{1}{16}$. **4.** 1953125. **6.** $\frac{\pi}{4}+2\pi k$. **7.** 1.

1582. 1. $x \neq \frac{\pi}{2}(2k+1)$. **2.** 0.8. **3.** $-\frac{\pi}{6}$. **4.** $\frac{\pi}{4}(2k+1)$, $\pm\frac{\pi}{12}+\frac{\pi}{2}k$. **5.** $\frac{1}{\sqrt{2}} < |y| < \sqrt{8}$. **6.** (a) $\frac{\pi}{4}+\pi k < \alpha < \frac{\pi}{2}+\pi k$; (b) $\frac{\pi}{4}+\pi k$, (c) $\pi k-\frac{\pi}{2} < \alpha < \frac{\pi}{4}+\pi k$.

1583. 1. $-0.8415 \leqslant x \leqslant 0.8415$. **2.** $-\frac{\pi}{4}+\pi k$, $\pm\frac{\pi}{3}+\pi k$.

3. $x > 0$. **4.** 2. **6.** $\frac{a}{b}$. **7.** 2. **1584. 1.** $-\frac{529}{576}$. **2.** $(-1)^k\frac{\pi}{6}+\pi k$.

3. 4. **4.** 10. **6.** $\therefore -\frac{3}{128}, \frac{3}{64}, \ldots$. **1585. 1.** $\frac{\pi}{4}$. **2.** $18°(2k+1)$.

3. $\pm\frac{\pi}{6}$. **4.** 2, 8. **5.** 0. **1586. 1.** 2. **2.** $1 < x \leqslant 2$. **3.** 0.

5. $(0.5n, 4.5n)$, $(4.5n, 0.5n)$. **1587. 1.** $x = 0.5+k$.

2. $18°26'+180°k$. **3.** 0. **5.** 5. **6.** 1, 5. **7.** yes. **1593. 2.** (b) $\sqrt{2}$, $\frac{3\pi}{4}$, $\sqrt{13}$, $-56°19'$, 5, $143°8'$, 5, $223°8'$. **1598. 1.** On a circle with the radius of r, and the centre: (a) at point (2, 0); (b) at point $(-1, 1)$. **2.** (a) On a circle of radius $r = 2$ with centre (0, 0); (b) in a ring between the circles with centre (0, 0) and $r_1 = 1$, $r_2 = 2$; (c) on a circle and outside it with centre (0, 0) and $r = 3$. **3.** (1) On a circle and inside it: centre $(0, -1)$ and $r = 1$; (3) in a ring between the circles with centre $(-3, 2)$ and $r_1 = 1$, $r_2 = 2$; (4) a locus of points equidistant from the points $(0, -1)$, (2, 0); (5) a point equidistant from three points: (1, 0), (3, 0), (0, 1). **1599. 1.** (a) On a circle of radius $r = 3$ with centre at the point (2, 0); (b) on a circle of

radius $r = 9$ with centre at the point $(-1, 0)$. **2.** (a) Inside a circle with centre at the point $(0, 1)$ and $r = 1$; (b) outside a circle with centre at the point $(-0.5, 0)$ and $r = 1$. **1600.** (4) 5; (5) 3; (6) $\sqrt{2}$; (7) 5; (8) 7. **1601.** $3 = 3(\cos 0 + i \sin 0)$; $-5 = 5(\cos 180° + i \sin 180°)$. **1603.** (1) Yes; (2), (3), (4), (5), (6) no; (4) $2.5(\cos 300° + i \sin 30°) = \frac{5\sqrt{2}}{2}\left(\cos\frac{\pi}{4} + i \sin\frac{\pi}{4}\right)$; (5) $-5(\cos 40° - i \sin 40°) = 5(\cos 140° + i \sin 140°)$; (6) $-3(\cos 15° + i \sin 15°) = 3(\cos 195° + i \sin 195°)$.

1610. 1. (a) $b_2 = -b_1$; (b) $b_1 \neq -b_2$. **1611.** (1) $(2, -1)$; (2) $(0, 7.5)$, $(1, 6)$; (3) x any number, $y = 1.5x$.

1619. (6) $(a\sqrt{5} + i\sqrt{b})(a\sqrt{5} - i\sqrt{b})$; (8) $(\sqrt{a} + i\sqrt{b})(\sqrt{a} - i\sqrt{b})$; (10) $(\sqrt{5} + i\sqrt[4]{3})(\sqrt{5} - i\sqrt[4]{3})$; (11) $\left(\frac{a}{\sqrt{2}} + i\sqrt{\frac{b}{3}}\right)\left(\frac{a}{\sqrt{2}} - i\sqrt{\frac{b}{3}}\right)$; (12) $(1 + i \tan\alpha)(1 - i \tan\alpha)$. **1620.** (7) $\frac{6}{5} + \frac{12}{5}i$; (8) $\frac{4}{5} - \frac{8}{5}i$; (9) $1 + i$; (10) $-\frac{12}{13} + \frac{8}{13}i$; (11) i; (12) $-i$. **1621.** (1) $-\frac{1}{2} + \frac{\sqrt{3}}{2}i$; (2) $\frac{2}{3} - \frac{5\sqrt{2}}{3}i$; (3) $1.8 + 1.6i$; (4) $-0.5 - 1.5i$; (5) $\frac{7}{9} + \frac{\sqrt{5}}{9}i$; (6) $\frac{3\sqrt{2} + \sqrt{3}}{4} + \frac{3 - \sqrt{6}}{4}i$; (7) i; (8) $-\sqrt{a}\,i$; (9) $\frac{a^2 - b}{a^2 + b} + \frac{2a\sqrt{b}}{a^2 + b}i$; (10) i.

1622. 1. (a) $\sqrt{40}$, (b) $\sqrt{68}$, (c) $\sqrt{533}$, (d) $\frac{\sqrt{533}}{41}$. **2.** 1. **1625.** (1) -1; (2) i; (3) i; (4) 1; (5) $-2i$; (6) $12 - 16i$; (7) $16 + 30i$; (8) $-5 - 12i$; (9) $24i$. **1626.** (1) $4 - a^2 + 4ai$; (4) 16; (6) $-\frac{1}{2} - \frac{\sqrt{3}}{2}i$.

1627. (1) $-2 + 2i$; (2) $10 - 9i\sqrt{3}$; (3) $-24i\sqrt{3}$; (4) -4; (5) $4i$; (6) $2b(3a^2 - b^2)i$. **1628.** (1) $x^2 - 2i$; (2) 64; (3) -1; (4) -1; (5) -1. **1631. 1.** (a) $ab = 0$, (b) $ab \neq 0$. **2.** (a) $b = 0$, $b = \pm a\sqrt{3}$, (b) $b \neq 0$, $b \neq a\sqrt{3}$. **1632.** (1) $0.5(1 - i)$; (2) $0.5i$; (4) $0.5(1 + 3i\sqrt{7})$; (5) 4; (6) $\frac{1}{8}$; (7) $-\cos 2\alpha + i \sin 2\alpha$.

1633. (1) $\pm(3 + 2i)$; (2) $\pm(3 - 2i)$; (3) $\pm(2 - 3i)$; (4) $\pm(1 + 2i)$; (5) $\pm(4 - 3i)$; (6) $\pm\frac{\sqrt{2}}{2}(\sqrt{3} - i)$; (7) $\pm\left(\sqrt{\frac{\sqrt{2} + 1}{2}} - i\sqrt{\frac{\sqrt{2} - 1}{2}}\right)$; (8) $\pm\sqrt{6}$. **1634.** (1) $\pm\frac{\sqrt{2}}{2}(1 + i)$; (2) $\pm\frac{\sqrt{2}}{2}(1 - i)$; (3) ± 2;

(4) $\pm\sqrt{5}$; (5) $\pm\sqrt[4]{8}\left(\sqrt{\frac{\sqrt{2}+1}{2}}+i\sqrt{\frac{\sqrt{2}-1}{2}}\right)$ and
$\pm\sqrt[4]{8}\left(\sqrt{\frac{\sqrt{2}-1}{2}}-i\sqrt{\frac{\sqrt{2}+1}{2}}\right)$; (6) $\pm\frac{3}{\sqrt[4]{2}}\left(\sqrt{\frac{\sqrt{2}+1}{2}}-i\sqrt{\frac{\sqrt{2}-1}{2}}\right)$ and
$\pm\frac{3}{\sqrt[4]{2}}\left(\sqrt{\frac{\sqrt{2}-1}{2}}+i\sqrt{\frac{\sqrt{2}+1}{2}}\right)$; (7) $\pm\sqrt{2}$, $\pm i\sqrt{2}$; (8) $\pm(i+1)$,
$\pm(1-i)$. **1635.** (3) $-1-i$, $1.5(-1+i)$; (4) $-3-i$, $-1+7i$.
1636. (3) $x^2+(1-i)x-4+7i=0$; (4) $x^2-x+1=0$.
1637. 1. $a>1.25$; **2.** $8+6i$. **1638.** (1) -1, $\frac{1}{2}(1\pm i\sqrt{3})$;
(2) $-1\pm i\sqrt{3}$; (3) $-\frac{1}{3}$, $\frac{1}{6}(1\pm i\sqrt{3})$; (4) $1\frac{2}{3}$, $\frac{5}{6}(-1\pm i\sqrt{3})$;
(5) ± 2, $\pm 2i$; (6) $\frac{3\sqrt{2}}{2}(-1\pm i)$, $\frac{3\sqrt{2}}{2}(1\pm i)$; (7) -1, $\frac{1}{2}(1\pm i\sqrt{3})$;
(8) 1, $\frac{1}{2}(-1\pm i\sqrt{3})$, 2, $-1\pm i\sqrt{3}$. **1640.** (1) $1.7-0.9i$; (2) $2i$;
(3) -2; (4) $-1.5i$. **1641. 2.** (1) 6; (3) $9\sqrt{2}$; (4) 2; (5) $8i$; (6) $12i$.
1642. (1) $(-4, 6)$; (2) $(5, 2)$, $(5, -2)$; (3) $(-2, 2)$; (4) $(2, -\sqrt{3})$,
$(4\sqrt{3}-8, 5-3\sqrt{3})$. **1644.** (1) $-729i$; (2) $8(1+i\sqrt{3})$;
(3) $0.5(-1+i\sqrt{3})$; (4) 1; (5) 64; (6) 1. **1645.** (1) $0.5(1+i\sqrt{3})$;
(2) $0.5(-\sqrt{3}+i)$; (3) 32; (4) $-72\sqrt{12}(\sqrt{3}+i)$; (5) 256; (6) -1;
(7) 1; (8) $-32i$. **1648.** (2) $2(\cos 270^\circ+i\sin 270^\circ)=-2i$;
$2(\cos 180^\circ+i\sin 180^\circ)=-2$. **1649.** (1) $\cos 40^\circ+i\sin 40^\circ$,
$\cos 160^\circ+i\sin 160^\circ$, $\cos 280^\circ+i\sin 280^\circ$; (2) $2(\cos 60^\circ+i\sin 60^\circ)$,
$2(\cos 150^\circ+i\sin 150^\circ)$, $2(\cos 240^\circ+i\sin 240^\circ)$,
$2(\cos 330^\circ+i\sin 330^\circ)$; (3) $\cos 50^\circ+i\sin 50^\circ$, $\cos 122^\circ+i\sin 122^\circ$,
$\cos 194^\circ+i\sin 194^\circ$, $\cos 266^\circ+i\sin 266^\circ$, $\cos 338^\circ+i\sin 338^\circ$;
(4) $\cos 10^\circ+i\sin 10^\circ$, $\cos 70^\circ+i\sin 70^\circ$, $\cos 130^\circ+i\sin 130^\circ$,
$\cos 190^\circ+i\sin 190^\circ$, $\cos 250^\circ+i\sin 250^\circ$, $\cos 310^\circ+i\sin 310^\circ$.
1650. (1) $\cos 60^\circ+i\sin 60^\circ$, $\cos 240^\circ+i\sin 240^\circ$;
(2) $4(\cos 40^\circ+i\sin 40^\circ)$, $4(\cos 160^\circ+i\sin 160^\circ)$,
$4(\cos 280^\circ+i\sin 280^\circ)$; (3) $\frac{1}{2}(\cos 30^\circ+i\sin 30^\circ)$,
$\frac{1}{2}(\cos 210^\circ+i\sin 210^\circ)$; (4) $\frac{1}{3}(\cos 210^\circ+i\sin 210^\circ)$.
1651. (1) 1, $\frac{1}{2}(-1\pm i\sqrt{3})$; (2) -1, $\frac{1}{2}(1\pm i\sqrt{3})$; (3) 2,
$-1\pm i\sqrt{3}$; (4) -3, $\frac{3}{2}(1\pm i\sqrt{3})$; (5) ± 1, $\pm i$; (6) $\frac{\sqrt{2}}{2}(1\pm i)$,

$\frac{\sqrt{2}}{2}(-1\pm i)$; (7) $\frac{5\sqrt{2}}{2}(1\pm i), \frac{5\sqrt{2}}{2}(-1\pm i)$;
(8) $\cos 22°30'+i\sin 22°30'$, $\cos 112°30'+i\sin 112°30'$, $\cos 202°30'+i\sin 202°30'$, $\cos 292°30'+i\sin 292°30'$; (9) ± 1, $\frac{1}{2}(1\pm i\sqrt{3}), \frac{1}{2}(-1\pm i\sqrt{3})$; (10) $\frac{1}{2}(\sqrt{3}\pm i), \pm i, \frac{1}{2}(-\sqrt{3}\pm i)$; (11) $\sqrt{2}(\cos 105°+i\sin 105°)$, -1 $-i$, $\sqrt{2}(\cos 345°+i\sin 345°)$; (12) $4\sqrt[4]{3}(\cos 52°30'+i\sin 52°30')$, $4\sqrt[4]{3}(\cos 142°30'+i\sin 142°30')$, $4\sqrt[4]{3}(\cos 232°30'+i\sin 232°30')$, $4\sqrt[4]{3}(\cos 322°30'+i\sin 322°30')$.

1653. 220.8 roubles. **1654.** 671.4 roubles. **1655.** 2235 roubles. **1656.** 1486 roubles. **1657.** In 23.5 years. **1658.** In 6 years 2 months. **1659.** The first one. **1660.** 26920 m^3. **1661.** During 5 years. **1662.** 268,500. **1663.** In 10 years. **1664.** 212,800. **1665.** In 54 years. **1666.** 8678 roubles. **1667.** 10%. **1668.** 13%. **1669.** 6%. **1670.** 290.7 roubles. **1671.** 6.7%. **1672.** 500 roubles; 2%. **1673.** 5514 roubles. **1674.** 256 roubles. **1675.** In 10.5 years. **1676.** 1963.8 roubles. **1677.** 1080.8 roubles. **1678.** 859 roubles. **1679.** 218.6 roubles. **1680.** 424.6 roubles, 823 roubles. **1682.** (1) 20; (2) 210; (3) 1344; (4) 6; (5) 1.

1683. (1) 0.2; (2) 5; (4) $n(n+1)$; (5) $(n-1)n(n+1)(n+2)$; (7) $(2n+2)(2n+3)\ldots(3n-1)$; (8) $(n+k-1)(n+k)$.

1684. (2) 696; (3) 6.2; (4) $6\frac{5}{6}$; (5) $\frac{1}{6}$; (6) 7; (7) 9; (8) $\frac{1}{13}$.

1685. (5) 4; (6) 182; (7) $8\frac{13}{14}$. **1686.** 15. **1687.** 10. **1688.** 56.

1689. 5, 20, 54. **1690.** $n+C_n^2$. **1691.** (a) 60, (b) 120. **1692.** 120, 2400. **1693.** (a) 600, (b) 600. **1694.** 48. **1695.** 42. **1696.** 495. **1697.** 24. **1698.** 380. **1699.** 66. **1700.** 504. **1701.** C_{30}^{12}. **1702.** 360. **1703.** 5040. **1704.** 300. **1705.** 8008. **1706.** 10. **1707.** 360; (a) 60, (c) 6. **1708.** 1,974,024. **1709.** 300. **1710.** 2739. **1711.** (3) $\frac{(m+n)!}{(2n)!}$; (4) $\frac{(m+n+2)!}{2}$.

1713. (1) $\frac{(m+1)!}{(m-1)!\cdot 2!}$; (2) $\frac{(m+n)!}{(n-1)!(m+1)!}$; (3) $\frac{(m+2)!}{(m-3)!\,5!}$; (4) $\frac{(m+n)!}{(m-n)!\,(2n)!}$. **1714.** (1) $\frac{n^2-7n+13}{n-3}$; (2) 0; (3) 1; (4) 0; (5) $m+3$; (6) $\frac{m}{m+2}$. **1715.** **1.** 220 **2.** 28 **3.** $A_{10}^n P_{10-n}$ is 10 times P_9.

1717. (1) 4; (2) 4; (3) 5; (4) 5; (5) 4; (6) 4; (7) 3 and 14; (8) 12; (9) 4; (10) 5; (11) 5; (12) 12. **1718.** (1) (17, 2), (2) a set of solu-

tions: $x = 4, 9, 14, 19, \ldots$; $y = 3, 6, 9, 12, \ldots$. **1719.** (1) 5; (2) 9; (3) 6; (4) 7. **1729.** For all natural n.

1731. (1) $x^4+10x^3+35x^2+50x+24$;
(3) $y^4-2y^3-13y^2+14y+24$;
(4) $b^5-15b^4+85b^3-225b^2+274b-120$;
(5) $a^4+2a^3b-13a^2b^2-14ab^3+24b^4$;
(6) $a^5+15a^4+85a^3+225a^2+274a+120$;
(7) $m^5+3m^4-23m^3-51m^2+94m+120$;
(8) $a^4+10a^3i-35a^2-50ai+24$. **1732. 1.** 26. **2.** $-10i$.

1733. (3) $x^8-16x^7+112x^6-448x^5+1120x^4-1792x^3-1792x^2-$
$-1024x+256$.

1734. (1) $32a^5+240a^4b+720a^3b^2+1080a^2b^3+810ab^4+243b^5$;
(2) $1-18x+135x^2-540x^3+1215x^4-1458x^5+729x^6$;
(3) $64m^6-576m^5n+2160m^4n^2-4320m^3n^3+4860m^2n^4-2916mn^5+$
$+729n^6$;
(4) $x^{14}-14x^{12}y+84x^{10}y^2-280x^8y^3+560x^6y^4-672x^4y^5+448x^2y^6-$
$-128y^7$;
(5) $81x^8+108x^6y^2+54x^4y^4+12x^2y^6+y^8$;
(6) $\frac{1}{32}a^5+\frac{5}{8}a^4b^2+5a^3b^4+2a^2b^6+40ab^8+32b^{10}$.

1735. (1) $m^2\sqrt[3]{m}-7m^2\sqrt[3]{n}+21m\sqrt[3]{m^2n^2}-35mn\sqrt[3]{m}+35mn\sqrt[3]{n}-$
$-21n\sqrt[3]{m^2n^2}+7n^2\sqrt[3]{m}-n^2\sqrt[3]{n}$;
(3) $x\sqrt[3]{x}-4x\sqrt{x}+6x\sqrt[3]{x^2}-4x\sqrt[6]{x^5}+x^2$;
(5) $x^3+6x^2\sqrt{xy}+15x^2y+20xy\sqrt{xy}+15xy^2+6y^2\sqrt{x}+y^3$.

1736. (1) $\dfrac{b\sqrt[3]{b^2}-5b\sqrt[3]{ab}+10b\sqrt[3]{a^2}-10a\sqrt[3]{b^2}+5a\sqrt[3]{ab}-a\sqrt[3]{a^2}}{ab\sqrt[3]{a^2b^2}}$;

(2) $\dfrac{m^9+6m^7n\sqrt{mn}+15m^6n^3+20m^4n\sqrt[4]{mn}+15m^3n^6+6mn\sqrt[7]{mn}+n^9}{m^3n^3}$;

(3) $8a^2-4b^2-8a\sqrt{a^2-b^2}$;
(4) $a^6+15a^5+30a^4+31a^3+18a^2+3a+1-$
$-2a(3a^4+10a^3+13a^2+6a+3)\sqrt{1+a}$;
(5) $x^4+4x^2-4+4x^2\sqrt{x^2-1}$; (6) $x^4+4x^2-4-4x^2\sqrt{x^2-1}$.

1737. (1) $-8i$; (2) $8i$; (3) $-72+72i\sqrt{3}$; (4) $2x^4-12x^2y^2+2y^4$;
(5) $-352+944i$. **1738.** (1) $101376x^5$; (2) $168\sqrt{2}x^6$; (3) $C_{16}^8x^{-4}$;
(4) (a) $-6435x^4y\sqrt[4]{y^3}$ and $6435x^3y^2\sqrt{x}$, (b) $256C_{17}^8m^3n^2$ and
$-512C_{17}^8m^{8/3}n^{27/2}$. **1739.** (1) $924x^6$; (2) $12012\sqrt{2}x^5$; (3) $153x^8y$;
(4) $495m^{10}\sqrt[3]{m^2}$; (5) $C_{18}^6x^{-1}y^6$. **1740.** (1) 84; (2) 5005; (3) 120;

(4) 5005; (5) C_{1000}^{200}. **1741.** (1) 12; (2) 12; (3) 15; (4) 9. **1742.** 14 and 7. **1743. 1.** $-4+4i$. **2.** $-8+8i$. **1744. 1.** 3003. **2.** $\frac{11}{3888}$.

1745. $84z^{-2.5}$. **1746.** $35a^5$. **1747.** (1) 60; (2) $36C_{24}^{10}$; (3) 3125, 56250, 105000, 42000, 3600, 32. **1748.** (2, 1.5, 5). **1749. 1.** $n+1$. **2.** 6. **1751.** (1) 0.976; (2) 1039.37; (3) 0.9996; (4) 0.999; (5) 1.0015; (6) 1.0003. **1752.** (6) $-3 < x < -\sqrt{2}$ and $\sqrt{2} < x < 3$; (8) $y \leqslant 1$ and $y \geqslant 4$. **1753.** (3) $-\frac{\pi}{12}, \frac{\pi}{12}, \frac{\pi}{3}$. **1754.** 16, 4, -36, -32. **1758.** (a) $a+b$; (b) $2a$. **1759.** Even: (1), (5), (6), (7), (8), (11), (12). Odd: (2), (4), (13). **1760.** (2) $0 \leqslant x \leqslant 3$; (3) $x \neq \pm 2$; (4) $x \leqslant -\sqrt{3}$ and $x \geqslant \sqrt{3}$; (5) $-1 \leqslant x < \sqrt{2}$ and $x > \sqrt{2}$; (6) $x \neq -2, x \neq -3$; (7) $-1 \leqslant x \leqslant 2$. **1761.** (1) $x \neq 0$; (2) $x < -4$ and $x > 4$; (3) $-1 \leqslant x \leqslant 4$; (4) $\pi k - \frac{\pi}{4} < x < \frac{\pi}{4} + \pi k$; (5) $1 < x \leqslant 3$; (6) $x \neq 0$. **1763.** (5) 0; (6) none; (7) 2; (8) -3.

1767. (2) $x \to \pm 2$; (3) none; (5) $x \to 1$ and $x \to 2$; (6) $x \to -1$ and $x \to 2.5$. **1768.** (1) $x \to \pm 1$; (2) $x \to \pm \infty$; (3) $x \to \pm \infty$; (4) $x \to -\infty$; (5) $x \to -1-0$ (from the left); (6) $x \to 0$.

1769. (1) $x \to 0$ and $x \to \frac{1}{\pi k}$ where $k = \pm 1, \pm 2, \ldots$; (2) $x \to \frac{\pi}{2}(2k+1)$; (3) $x \to -1$ and $x \to \frac{\pi}{2}(2k+1)$; (4) $x \to 2\pi k$; (5) $x \to \frac{\pi}{2}k$; (6) $x \to \frac{\pi}{4} + \frac{\pi}{2}(2k+1)$. **1770.** (1) $x \to \frac{\pi}{3}k$, $x \to \frac{\pi}{2}(2k+1)$; (2) $x \to \frac{\pi}{2}k$ and $x \to \frac{\pi}{8}(2k+1)$; (3) $x \to \pi(2k+1)$; (4) $x \to \frac{\pi}{3}(2k+1)$, $x \to \pi(2k+1)$; (5) $x \to \frac{\pi}{2}k$; (6) $x \to \frac{\pi}{3}k$. **1771.** (1) $x \to 2$; (2) $x \to -0.5$ and $x \to 1$; (3) $x \to \frac{\pi}{2}(2k+1)$; (4) $x \to \frac{\pi}{2}(2k+1)$; (5) $x \to 0.5(\sqrt{5}-1)$ and $x \to -0.5(\sqrt{5}+1)$. **1772.** (5) $x \to \pm 1$; (6) $x \to 0$; (7) $x \to \pm\sqrt{2}$; (8) $x \to -1$ and $x \to 2.5$. **1773.** (1) $x \to +0$ and $x \to -2-0$; (3) $x \to -\infty$; (4) $x \to \pi k$; (5) $x \to \pm \infty$; (6) $x \to 0$; (7) $x \to \frac{\pi}{2}k$.

1774. (1) $x \to \infty$ and $x \to -2+0$; (2) $x \to -0.5-0$; $x \to +0$; $x \to \pm \infty$; (3) $x \to \pm 1$; (4) $x \to \frac{\pi}{2} + 2\pi k$; (5) $x \to 2\pi k + 0$ and $x \to \pi(2k+1) - 0$; (6) $x \to \pi k + 0$ and $x \to \frac{\pi}{2}(2k+1) - 0$.

1779. (1) $\frac{\pi}{6}$; (2) $\frac{5\pi}{6}$; (3) $\frac{\pi}{2}$; (4) $\frac{3\pi}{4}$. **1780.** (1) 2; (2) none; (3) 2; (4) 0; (5) none; (6) none; (7) 0. **1782.** (1) -0.5; (2) $\frac{\pi}{3}$; (3) 1; (4) -1; (5) none; (6) 0; (7) none; (8) none.

1783. (1) None; (2) -2; (3) -4; (4) none; (5) 2; (6) 1.

1784. (1) $\frac{3}{8}$; (2) -1.8; (3) -0.6; (4) $\frac{2}{3}$; (5) 0; (6) $\frac{1}{3}$.

1785. (1) None; (2) $1\frac{5}{8}$; (3) $-\frac{11}{12}$; (4) $-\frac{\sqrt{2}}{2}$; (5) $\frac{1}{6}$; (6) -2.

1786. (1) 2; (2) 1; (3) 0.25; (4) 4; (5) -0.75; (6) -2.

1787. (1) 6; (2) $\frac{1}{8}$. (3) $\frac{2\sqrt{a}}{a}$; (4) $\frac{1}{3}$; (5) -3; (6) 2.

1788. (1) -1; (2) 0; (3) 2; (4) 0.5; (5) none; (6) 0; (7) 2.

1789. (3) 1; (4) 2; (5) 0.5; (6) 2; (7) $1\frac{1}{3}$; (8) 0; (9) 0.4.

1790. (1) 4; (2) none; (3) 4; (4) 7; (5) $-\frac{2}{\pi}$; (6) $\frac{1}{4}$; (7) $\frac{2}{\pi}$.

1791. (1) -6; (2) 8; (3) 0.5; (4) 0.5; (5) 0.5; (6) -0.5.

1792. (1) 2; (2) 1; (3) -6; (4) $-\frac{4}{\pi}$. **1793.** (1) 0.2; (2) 1; (3) 0.5; (4) none. **1794.** (4) e^2; (5) e^3; (6) e^{-4}. **1796.** (1) 0.5; (2) 0.1; (3) -0.191; (4) -0.4. **1797. 1.** 0. **2.** 0. **1798. 1.** 2.76 cm^2. **2.** 38.84 cm^2. **3.** 10.09 dm^3. **1799.** (4) $\frac{x^2\Delta x+x\Delta x^2-2x\Delta x-\Delta x^2-\Delta x}{(x+\Delta x-1)(x-1)}$;

(5) $\log_{10}\left(1+\frac{\Delta x}{x}\right)$; (6) $-2\sin\left(x+\frac{\Delta x}{2}\right)\sin\frac{\Delta x}{2}$;

(7) $2x\sin\frac{\Delta x}{2}\cos\left(x+\frac{\Delta x}{2}\right)+\Delta x\sin(x+\Delta x)$. **1800.** (1) (a) 2; (b) 4.2; (c) 0; (d) 2. **1801.** (4) $\frac{2\sin\frac{\Delta x}{2}\cos\left(x+\frac{\Delta x}{2}\right)}{\Delta x}$;

(5) $\log_{10}\left(1+\frac{\Delta x}{x}\right)^{\frac{1}{\Delta x}}$; (6) $3x^2+3+3x\Delta x+\Delta x^2$. **1802. 1.** 4.

2. 0.75. **1806.** (1) 1; (3) ± 2; (4) $\frac{\pi}{2}(2k+1)$; (5) none; (6) 0 (7) none; (8) $\frac{\pi}{2}(2k+1)$. **1807.** (2) $42\frac{2}{3}$; (3) $4\frac{1}{6}$; (4) 1.

1808. (1) $\frac{27\sqrt{2}}{2}$; (2) $-\frac{32}{63}$; (3) $2(\sqrt{2}+1)$; (4) $\frac{8\sqrt{3}}{21}$.

1809. (1) $2\sqrt{2}$; (2) $\frac{12+7\sqrt{3}}{6}$; (4) $\frac{10+7\sqrt{2}}{4}$; (5) 6; (6) 2.25.

1810. 1. $-\frac{2}{3}$. 2. $\frac{2}{3}$. **1811.** (1) 24; (2) 81; (3) 4.2.

1812. 1. 5.4. 2. $\therefore$ 48, 24, 12, ... **1813.** $\therefore 7, \frac{21}{4}, \frac{63}{16}, \ldots$.

1814. $\pm\frac{\sqrt{2}}{2}$. **1815.** 1. $\therefore a, \frac{4}{5}a, \ldots$. 2. $a, \frac{1}{4}a, \ldots$.

1818. (1) 1.25; (2) $1\frac{1}{3}$. **1819.** (1) $(-1)^k\frac{\pi}{4}+\pi k$; (2) 0.6.

1820. $1\frac{1}{3}$. **1821.** $3\frac{1}{3}$. **1822.** $4a(2+\sqrt{2})$, $2a^2$.

1823. 60 cm, $\frac{100\sqrt{3}}{3}$ cm^2. **1824.** $a^2\sqrt{3}$, $6a(2+\sqrt{3})$. **1825.** 64 dm^2.

1826. $0.5\pi a^2$, $2a^2$. **1827.** $\frac{1}{9}\pi a^2$, $\frac{2}{3}\pi a\sqrt{3}$. **1828.** $a(\sqrt{2}+1)$.

1829. $a(2+\sqrt{3})$. **1830.** 1. (a) 3; (b) 21. 2. (a) -2; (b) 8.

1831. 2. (a) 1; (b) -0.5; (c) k. **1832.** (1) -4; (2) 1; (3) 4; (4) -9; (5) $\frac{3}{2\pi}$. **1833.** 6.5. **1834.** 1. (a) 80; (b) 8, 64, 216.

2. (a) 4; (b) 2, 4, 6. **1835.** (1) 3; (2) 0; (3) 3; (4) 8; (5) -8; (6) 0.5; (7) -0.5; (8) $\frac{1}{6}$. **1836.** 1. 2, 5. **1837.** 5 A. **1840.** 1. 0, 3. 3. -1. **1841.** 1. 72 m/s. 2. $\left(6t-\frac{2}{t^2}\right)$ m/s, $\left(\frac{4}{t^3}+6\right)$ m/s^2.

3. (a) 10, 11; (b) 4, 2; (c) -2, 0; (d) $\sqrt{3}$, -1. **1843.** (2) $2x+1$; (3) $-\frac{8}{x^5}$. **1844.** (4) $3t^2+6t+2$; (5) $\frac{4x-6}{x^3}$; (6) $-\frac{n}{x^{n+1}}$.

1845. (4) $\frac{2x^5-x^2+4x-3}{x^4}$; (5) $\frac{3x\sqrt[6]{x}-2\sqrt[3]{x}+2}{\sqrt[3]{x^2}}$.

1846. (1) $8\sqrt[8]{x}+6\sqrt[5]{x}+3\sqrt{x}$; (2) $\frac{2-\sqrt[6]{x}}{x\sqrt[3]{x^2}}$. **1847.** (1) $\frac{\sqrt[6]{x}-1}{\sqrt[3]{x^2}}$; (2) $\frac{2\sqrt[6]{x}+3}{3\sqrt{x}}$; (3) $1.5\sqrt{x}+8x\sqrt[3]{x^2}$; (4) $\frac{5}{4x\sqrt{x}}-\frac{4}{3x\sqrt[3]{x^2}}$.

1848. (1) $\frac{5ax^4-cx^2+3b}{2x^2\sqrt{x}}$; (2) $\frac{11\sqrt[6]{x^5}}{6}$; (3) $\frac{5\sqrt[6]{t^5}}{t}$; (4) $\frac{3\sqrt{t}+2}{2}$; (5) $\frac{\sqrt[6]{x^5}}{6x^2}$. **1850.** 1. -4, 2. 2. 2, -3. **1851.** 2. x^2+3x+4.

1852. 1. (0, -2) and (1, 0). 2. 45° and 18°26′. **1853.** 1. 0.4. 2. (a) 0; 0.3 and 15.7; (b) 0, 0.2, 11.8; (c) $v<0$ means that the point moves along the axis in the negative direction; the path s decreases. **1854.** -2 m/s, 2 m/s^2. Positive acceleration means that

velocity increases; if the velocity can become equal to zero, the body will stop. If the velocity then can become positive, the body will start moving in the positive direction (s increases). **1855.** -3, 5, 9, $a = 0$ at time $t = 0.75$. **1856.** 262 m/s, 36. 7s.

1857. (a) 10.4 m/s; (b) 3 s; (c) 96 m. **1858.** 902.5.

1859. (a) 10; (b) 13. **1860.** 6. **1861.** (2) $12x^2+20x-14$; (3) $16x^3-36x^2-30x-4$; (4) $2x^2(3x^3-5x^2+8x-12)$.

1862. (1) $2x(3x^4-4x^2-a^4)$; (2) $\cos 2x$; (3) $4x(2x^6-15x^4+35x^2-25)$; (4) $\frac{e^x}{x}(x \ln x+1)$.

1863. (1) $x(2 \ln x+1)$; (2) $2^x x^3(4+x \ln 2)$; (3) $3^x[(x+1) \ln 3+1]$. **1864.** (4) $a^x e^x(\ln a+1)$; (5) $(2x-4) \tan x+\frac{1}{\cos^2 x}(x^2-4x+1)$. **1865.** (1) $\frac{2x^2+4x-4}{(x+1)^2}$; (2) $\frac{6x^2+6x+4}{(2x+1)^2}$; (3) $\frac{-x^4+2x^3-6x^2-2x+1}{(x^3-1)^2}$; (4) $\frac{3x^2+2x+1}{x^2(1+x)^2}$.

1866. (1) $\frac{x^4+4x^2-1}{x^2(x^2-1)^2}$; (2) $-\frac{1}{\sqrt{x}(1+\sqrt{x})^2}$; (3) $-\frac{\ln x+1}{x^2}$; (4) $\frac{4}{x(1+\ln x)^2}$. **1867.** (1) $\frac{1-x \ln x}{xe^x}$; (2) $\frac{2^x[x \ln 2(1-\ln x)+1]}{x(1-\ln x)^2}$; (3) $-\frac{2e^x}{(e^x-2)^2}$. **1868.** (1) $\frac{-2 \cos x}{(1+\sin x)^2}$; (2) $\frac{2^x e^x \ln 2+x 2^x \ln 2+xe^x-2^x-e^x 2^x-e^x}{(x-2^x)^2}$; (3) $\frac{\cos x+x \sin x}{\cos^2 x}$; (4) $\frac{(3e)^x[x(\ln 3+1)(2+\ln x)-1]}{x(2+\ln x)^2}$. **1869. 1.** $-\frac{4}{9a}$, $-\frac{1}{a}$.

2. $-\frac{2e}{(e-1)^2}$, $-\frac{2e}{(e-1)^2}$; $f'(0)$ is non-existent. **1870.** (1) $4(x+1)^3$; (2) $9(3x-2)^2$; (3) $-10x(1-x^2)^4$; (4) $8(x+1)(x^2+2x)^3$; (5) $4(2t-1)(t^2-t+1)^3$. **1871.** (1) $-\frac{6x}{(x^2+1)^4}$; (2) $-\frac{50}{(2x-3)^6}$; (3) $\frac{28x(3x-1)}{(x^2-2x^3)^8}$; (4) $\frac{x}{\sqrt{x^2+1}}$; (5) $\frac{2}{3\sqrt[3]{(2x-1)^2}}$; (6) $-\frac{1+x}{\sqrt{1-2x-x^2}}$.

1872. (1) $\frac{t^2}{\sqrt[3]{(t^3-1)^2}}$; (2) $-\frac{4x+1}{2\sqrt{(2+x)(3-2x)}}$; (3) $\frac{x^2-2x}{\sqrt[3]{(x^3-3x^2)^2}}$; (4) $-\frac{1}{2x}\sqrt{\frac{3}{x}}$. **1873.** (1) $(2x-1)(x+1)^2(10x+1)$; (2) $\frac{5t^2-6t}{\sqrt{2t-3}}$; (3) $\frac{2x^2-1}{\sqrt{x^2-1}}$; (4) $\frac{2x^2-x+1}{\sqrt{x^2+1}}$. **1874.** (1) $\frac{20x^2-4x-7}{2\sqrt{x-1}}$; (2) $\frac{3x^4+2x}{\sqrt[3]{(x^3+1)^2}}$;

(3) $-\frac{1}{2x\sqrt{2x}}$; (4) $\frac{3x+1}{2x\sqrt{3x}}$. **1875.** (1) $\frac{3x}{2\sqrt[4]{1+x^2}}$; (2) $\frac{2\sqrt{x}+1}{4\sqrt{x^2+x\sqrt{x}}}$;
(3) $\frac{2\left(1+\sqrt[3]{x^2}\right)^2}{\sqrt[3]{x}}$; (4) $\frac{1}{6\sqrt{x}\sqrt[3]{(1+\sqrt{x})^2}}$. **1876.** (1) $-\frac{24x^2}{(3x-2)^4}$;
(2) $\frac{6x}{(1-2x^2)\sqrt{3(2x^2-1)}}$; (3) $\frac{x+2}{(x+1)\sqrt{x+1}}$; (4) $\frac{2x+4}{(x^2+1)\sqrt{x^2+1}}$.

1877. (1) $\frac{a^2}{(a^2+x^2)\sqrt{a^2+x^2}}$; (2) $-\frac{a^2}{x^2\sqrt{a^2+x^2}}$; (3) $\frac{1}{(1-x)\sqrt{x^2-1}}$;
(4) $\frac{2x}{(1-x^2)\sqrt{x^4-1}}$. **1878.** (1) $\frac{x^3-3a^2x}{(x^2-a^2)\sqrt{x^2-a^2}}$;
(2) $-\frac{2x^2+a^2+2x\sqrt{a^2+x^2}}{a^2\sqrt{a^2+x^2}}$; (3) $\frac{a^2-a\sqrt{a^2-x^2}}{x^2\sqrt{a^2-x^2}}$; (4) $\frac{x+\sqrt{a^2+x^2}}{a^2\sqrt{a^2+x^2}}$.

1879. (1) $\frac{1}{x+1}$; (2) $\frac{2}{2x+1}$; (3) $\frac{2}{2x-3}$; (4) $\frac{2}{x-1}$; (5) $\frac{2}{3+2x}$.

1880. (1) $\frac{0.4343}{x}$; (2) $\frac{3.4744x}{4x^2+1}$; (3) $-\frac{1}{x}$; (4) $\frac{2\ln x}{x}$; (5) $\frac{2}{x}$;
(6) $\frac{1}{x(1+\ln x)}$. **1881.** (1) $\frac{2}{x^2-1}$; (2) $\frac{4x}{1-x^4}$; (3) $1+\ln x$;
(4) $\frac{2\sqrt{x}-1}{2x(\sqrt{x}-1)}$; (5) $\frac{1}{x\ln x}$; (6) $\frac{2}{x(x^2+1)}$. **1882.** (1) $\cot x$;
(2) $-\tan x$; (3) $\frac{2}{\sin 2x}$; (4) $-\frac{2}{\sin 2x}$; (5) $2\cot 2x$; (6) $-5\tan 5x$.

1883. (1) $\frac{8\ln x}{x}$; (2) $\frac{2x+3}{2(x^2+3x+1)}$; (3) $\frac{1}{\sqrt{x^2-1}}$; (4) $\frac{2x}{\sqrt{x^4-a^4}}$.

1884. (1) $\frac{1-2\ln x}{x^3}$; (2) $\frac{x-\sqrt{1+x^2}}{x\sqrt{1+x^2}}$; (3) $\frac{1}{\sqrt{2x+x^2}}$; (4) $\frac{1}{\sqrt{x}(x-1)}$.

1885. (1) $\frac{1}{2(1-x^2)}$; (2) $\frac{2}{\sqrt{x^2+1}}$; (3) $\frac{x(x^2+a^2-4)}{(x^2+a^2)^2}$; (4) $-\frac{1}{x(x+1)}$.

1886. (1) $1+\cos x$; (2) $3\cos 3x$; (3) $3\sin^2 x\cos x$;
(4) $\sin x+x\cos x$; (5) $\frac{1}{x^2}\sin\frac{1}{x}$; (6) $\frac{2}{\cos^2(2x+1)}$.

1887. (1) $-\cot^2 x$; (2) $\cos 2x+\sin x$; (3) $\frac{1}{2\sqrt{x+1}\cos^2\sqrt{x+1}}$;
(4) $-\frac{x\cos\sqrt{1-x^2}}{\sqrt{1-x^2}}$; (5) $-\frac{1}{3\sin^2\frac{x}{3}}$. **1888.** (1) $-6\cos^2 2x\sin 2x$;
(2) $-\frac{6\cos 3x}{\sin^3 3x}$; (4) $-\frac{1}{a}\sin\frac{t}{a}$; (5) $2.5\cos 5x-0.5\cos x$;
(6) $\cos 2x-2x\sin 2x$. **1889.** (1) $0.5\sin 2x\sin 4x(1+5\cos 4x)$;

(2) $\frac{4}{\sqrt{x}\cos^2 2\sqrt{x}}$; (3) $-\frac{\sin 2x}{\sqrt{\cos 2x}}$; (4) $-\frac{\cot x}{2\sqrt{\sin x}}$; (5) $-\frac{\sin^2 \frac{1}{x}}{x^2 \cos^4 \frac{1}{x}}$.

1890. (1) $\frac{3\sin 3x+3\sin 5x+\sin 2x\cos 3x}{\cos^2 3x}$; (2) $\frac{2-\cos 2x-\sin 2x}{\cos^4 x}$; (3) $\frac{4\sin 2x\cos 3x+3\sin 3x\sin^2 2x}{\cos^2 3x}$; (4) $\frac{1}{\cos x}\sqrt{\frac{1+\sin x}{1-\sin x}}$.

1891. (1) $\sin^3 x$, (2) $\tan^4 x$; (3) $-\cot^6 x$. **1892. 1.** 0, 8. **2.** 4, $8(2+\sqrt{3})^2$. **1893.** (2) $t = 2k$, where $k = 0, 1, 2, 3, \ldots$. (3) $\frac{\pi^2 a}{4}$;

1895. (1) $(-\infty, -2)$ and $(1, \infty)$—intervals of increase; $(-2, 1)$—interval of decrease. (2) $(-\infty, -0.25)$ and $\left(\frac{1}{6}, \infty\right)$—intervals of increase. $\left(-\frac{1}{4}, \frac{1}{6}\right)$—interval of decrease; (3) $(-\infty, -4)$ and $(-3, \infty)$—intervals of increase; $(-4, -3)$—interval of decrease. (4) $(0, 10)$—interval of increase; $(-\infty, 0)$, $(10, \infty)$—intervals of decrease. (5) $(-3, 0)$, $(1, \infty)$—intervals of increase; $(-\infty, -3)$, $(0, 1)$—intervals of decrease. (6) $(1, \infty)$—interval of increase; $(-\infty, 1)$—interval of decrease. **1896.** (1) $\left(\frac{3+\sqrt{5}}{2}, \infty\right)$—interval of increase; $\left(-\infty, \frac{3-\sqrt{5}}{2}\right)$—interval of decrease.

(2) $\left(\frac{-5-\sqrt{45}}{2}, -4\right)$, $(-1, 0)$, $\left(\frac{-5+\sqrt{45}}{2}, \infty\right)$—intervals of increase; $\left(-\infty, \frac{-5-\sqrt{45}}{2}\right)$, $(-4, -1)$, $\left(0, \frac{-5+\sqrt{45}}{2}\right)$—intervals of decrease. (3) $2\pi k-\frac{\pi}{3} < x < \frac{\pi}{3}+2\pi k$—intervals of increase; $2\pi k+\frac{\pi}{3} < x < \frac{5\pi}{3}+2\pi k$—intervals of decrease ($k = 0, \pm 1, \pm 2, \ldots$). (4) $2\pi k+\frac{3\pi}{4} < x < \frac{9\pi}{4}+2\pi k$—intervals of increase; $2\pi k+\frac{\pi}{4} < x < \frac{3\pi}{4}+2\pi k$—intervals of decrease.

(5) $\left(\frac{-3+\sqrt{17}}{2}, \infty\right)$—interval of increase; $\left(-\infty, \frac{-3-\sqrt{17}}{2}\right)$—interval of decrease. (6) $\left(-2\frac{1}{3}, -\frac{1}{3}\right)$—interval of increase; $\left(-\frac{2}{3}; 1\right)$—interval of decrease. **1897.** (1) $(-\infty, -1)$, $(-1, 0)$—intervals of increase; $(0, 1)$, $(1, \infty)$—intervals of decrease; (2) the function increases everywhere except for the point $x = 0$;

(3) $(-\infty, -1)$, $(1, \infty)$—intervals of increase; $(-1, 1)$—interval of decrease. (4) $(0.5, 1)$—interval of increase; $(-\infty, 0)$, $(0, 0.5)$, $(1, \infty)$—intervals of decrease. (5) $(-\infty, 0)$—interval of increase; $(0, \infty)$—interval of decrease; (6) the function decreases everywhere except the points $\frac{\pi}{2}(2k+1)$, at which it is not defined and the points πk at which the derivative of the function is equal to zero.

1899. 1. (a) increases; (b) increases; (c) decreases; **2.** (a) decreases; (b) increases; (c) increases. **1900.** (1) $y_{\min} = -3.25$; (2) $y_{\min} = -1\frac{1}{2}$; (3) $y_{\min} = -4.5$, (4) $y_{\max} = 0.25$; (5) no extrema; (6) $y_{\max} = 21$. **1901.** (1) $y_{\max} = 0$, $y_{\min} = -1$; (2) $y_{\max} = 12$, $y_{\min} = -52$; (3) no extrema; (4) $y_{\max} = 25$, $y_{\min} = -2$; (5) $y_{\min} = -9$; (6) $y_{\max} = \frac{1}{16}$, $y_{\min} = 0$.

1902. (1) $y_{\max} = \frac{4\sqrt{2}+16}{7}$, $y_{\min} = \frac{16-4\sqrt{2}}{7}$; (2) no extrema; (3) $y_{\max} = 2$, $y_{\min} = 10$; (4) $y_{\max} = -1$, $y_{\min} = 11$; (5) $y_{\max} = 1$, $y_{\min} = 9$; (6) $y_{\max} = 1$, $y_{\min} = 16$. **1903.** (1) $y_{\min} = 0$; (2) $y_{\max} = \sqrt{2}$, $y_{\min} = -\sqrt{2}$; (3) $y_{\min} = -0.1$; (4) $y_{\min} = -e^{-1}$; (6) $y_{\max} = 1$. **1904.** (1) $42x^5+6x$; (2) $12x^2+18x+6$; (3) $\frac{15x^2+6x-1}{4x\sqrt{x}}$; (4) $(2-x^2)\sin x+4x\cos x$; (5) $\frac{4}{(x-1)^3}$; (6) $-\frac{6x}{(1+x^3)^3}$; (7) $2\cos 2x$; (8) $\frac{1}{x}$; (9) $\frac{e^{\sqrt{x}}(\sqrt{x}-1)}{4x\sqrt{x}}$; (10) $\frac{1-x^2}{(1+x^2)^2}$.

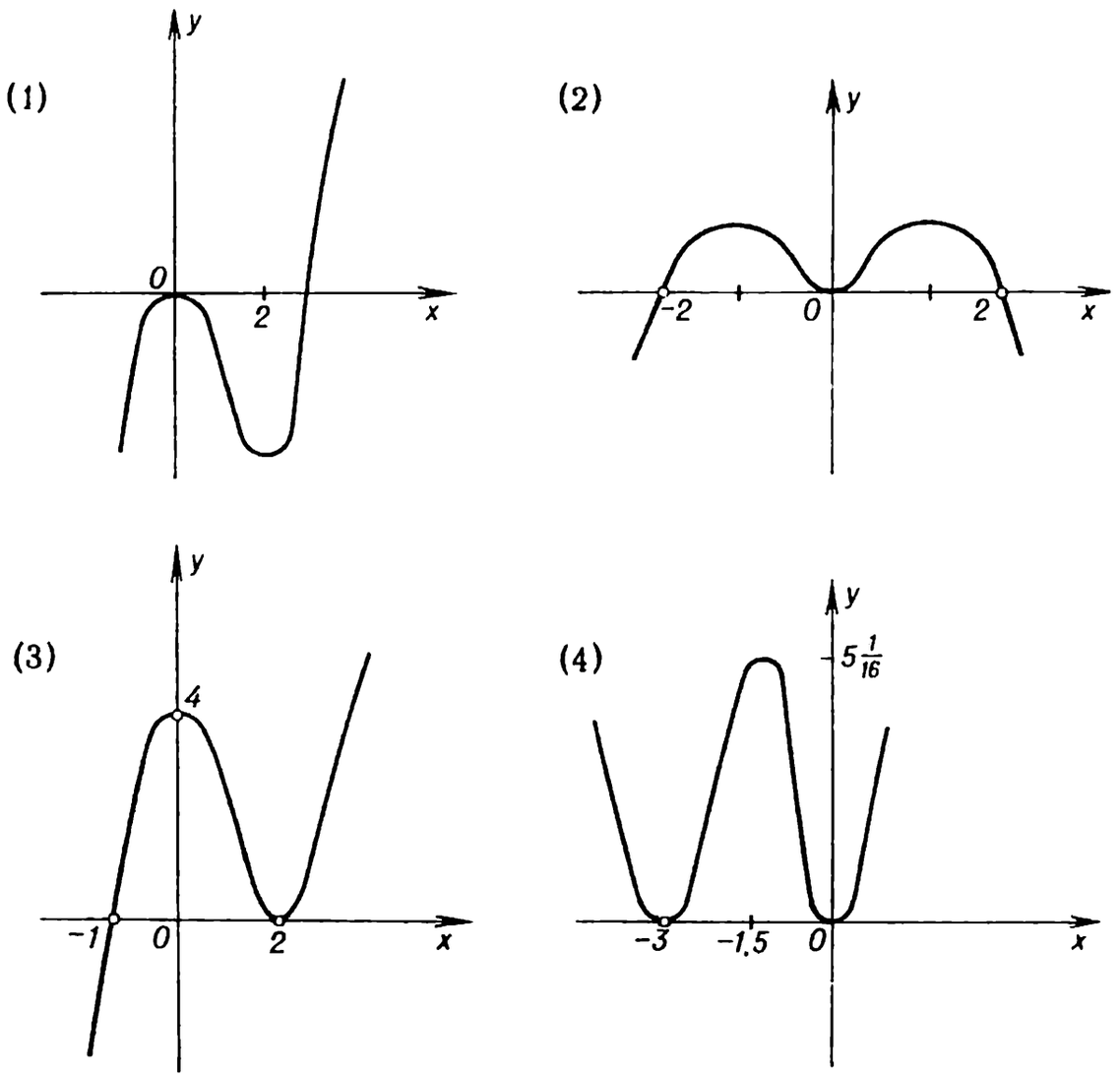
1905. 1. -2. **2.** 1. **1906.** 16 m/s^2. **1907.** (1) 11 m, 10 m/s, 13 m/s^2; (2) 0, 6 m/s, 0; (3) 10 m, 0, $-\pi^2$ cm/s^2. **1909.** $A\omega$, 0.

1910. (1) $y_{\min} = 1\frac{7}{8}$; (2) $y_{\max} = -\frac{11}{12}$; (3) $y_{\max} = 9$; $y_{\min} = -\frac{13}{27}$, (4) $y_{\min} = 2$; (5) investigate with the aid of the first derivative, $y_{\min} = 3$; (6) $y_{\max} = 3-0.8\sqrt{2}$, $y_{\min} = 1.8$, $y_{\max} = 4.2$, $y_{\min} = 3+0.8\sqrt{2}$; (7) $y_{\min} = 1$ for $x = 0$, $y_{\min} = \frac{\pi}{2}(4n+1)$ for $n = -1, -2, -3, \ldots$ at points $x = \frac{\pi}{2}(4n+1)$; $y_{\min} = -\frac{\pi}{2}(4n+3)$ for $n = 0, 1, 2, \ldots$ at points $x = -\frac{\pi}{2}(4n+1)$: $y_{\max} = \frac{\pi}{2}(4n+1)$ for $n = 0, 1, 2, \ldots$ at points $x = \frac{\pi}{2}(4n+1)$; $y_{\max} = -\frac{\pi}{2}(4n+3)$ for $n = -1, -2, \ldots$ at points $x = -\frac{\pi}{2}(4n+3)$; (8) analyse with

the aid of the first derivative; no extrema; (9) $y_{\max} = \frac{\sqrt{2}}{2} e^{\frac{3\pi}{4}+2\pi k}$, $y_{\min} = -\frac{\sqrt{2}}{2} e^{\frac{7\pi}{4}+2\pi k}$; (10) $y_{\max} = -2$, $y_{\min} = 2$. **1911.** (1) $M = 2$ at points $x = 1$ and $x = -1$; $m = -25$ at points $x = -2$ and $x = 2$; (2) $M = 2$, $m = -10$; (3) $M = 2$, $m = -12$, (4) $M = 10$, $m = 6$; (5) $M = \frac{\pi}{2}$, $m = -\frac{\pi}{2}$; (6) $M = 1.5\sqrt{3}$, $m = -2$; (7) $M = 2$, $m = 1$. **1912.** 1. **1913.** (1) 6 and 6; (2) 11 and 1; (3) 6 and 6. **1914.** 3 cm. **1915.** 6 cm. **1917.** 6 m and 6 m.

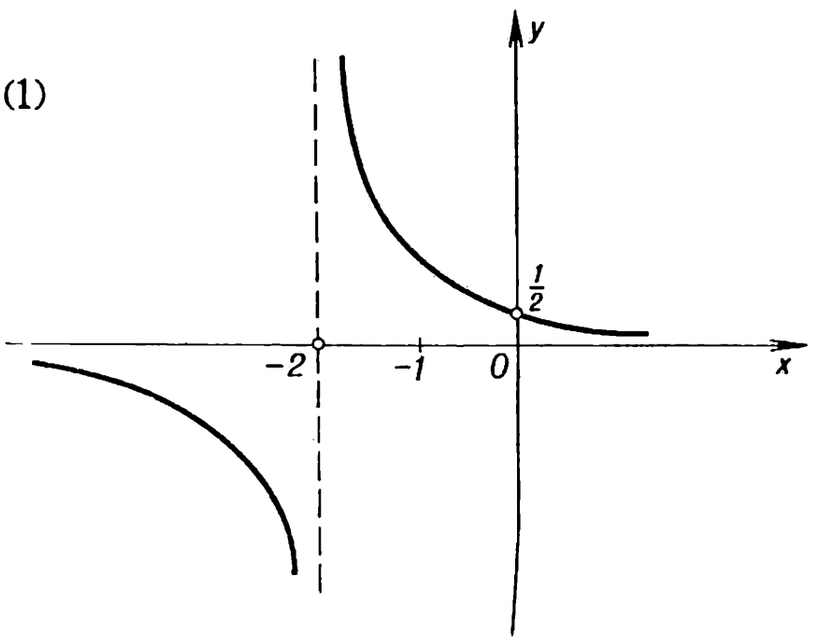
1919. $0.25ah$. **1920.** $\frac{20\sqrt{3}}{3}$ *dm*. **1921.** 1 : 2 : 1. **1922.** The rectangular portion of the window must be a square. **1923.** $h = 0.5a$, where a is the side of the base. **1924.** $D = H$—equilateral cylindrical tank. **1925.** $\approx$ 20 s. **1926.** $\approx$ 9.2 cm.

1928.



1929.

(1)



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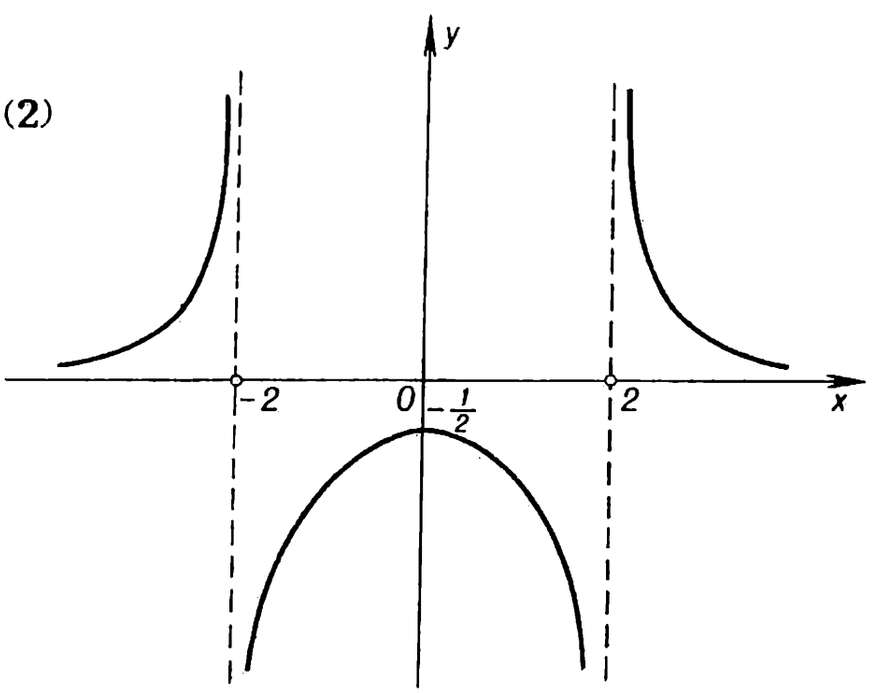

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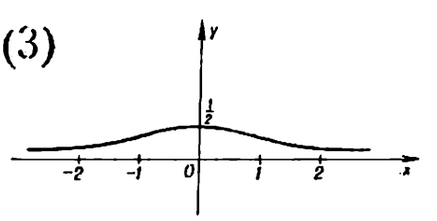
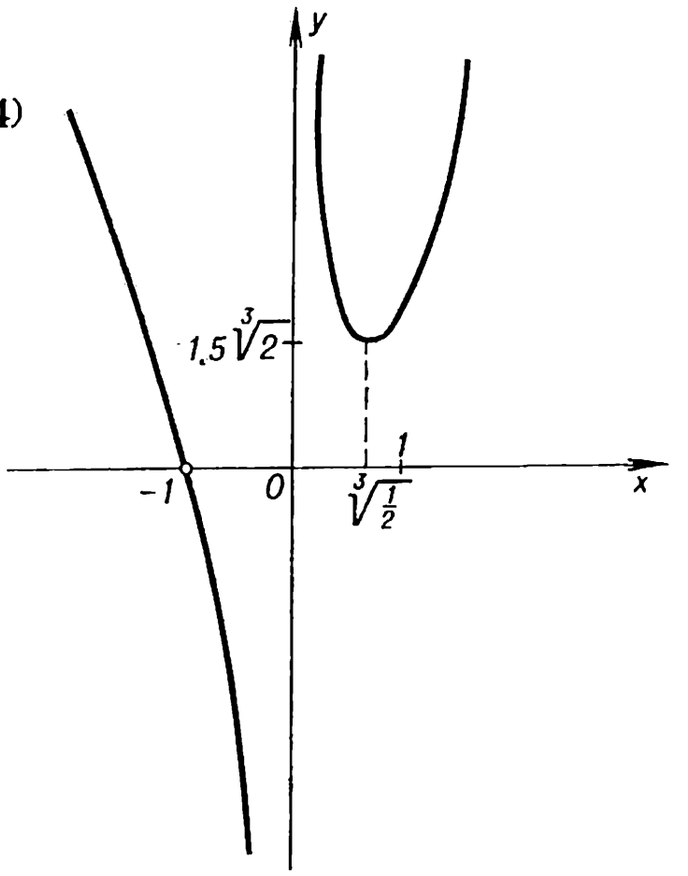
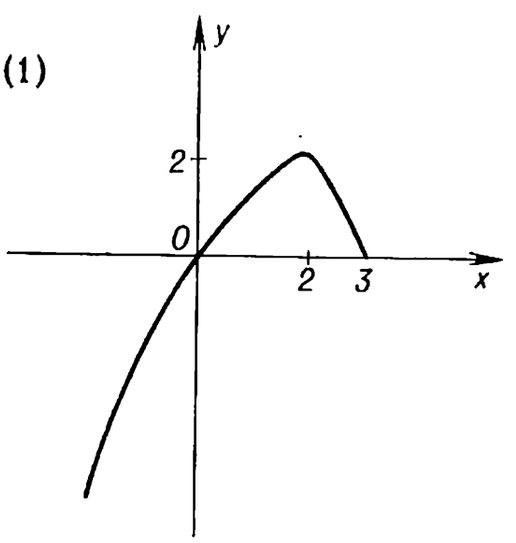


(4)

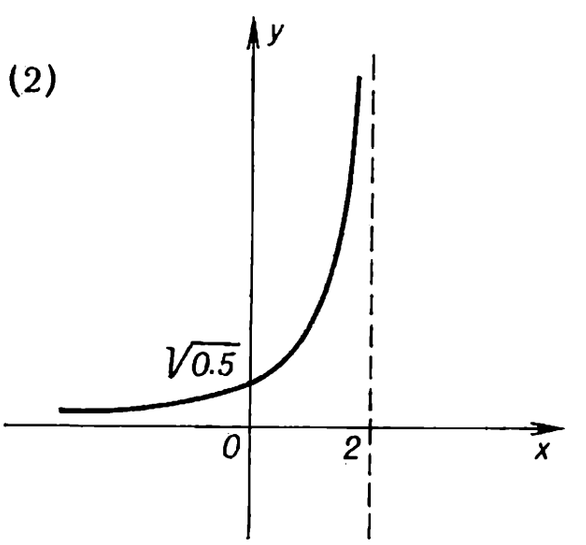

y
$1.5\sqrt[3]{2}$
1
-1
0
$\sqrt[3]{\frac{1}{2}}$
x

1930.

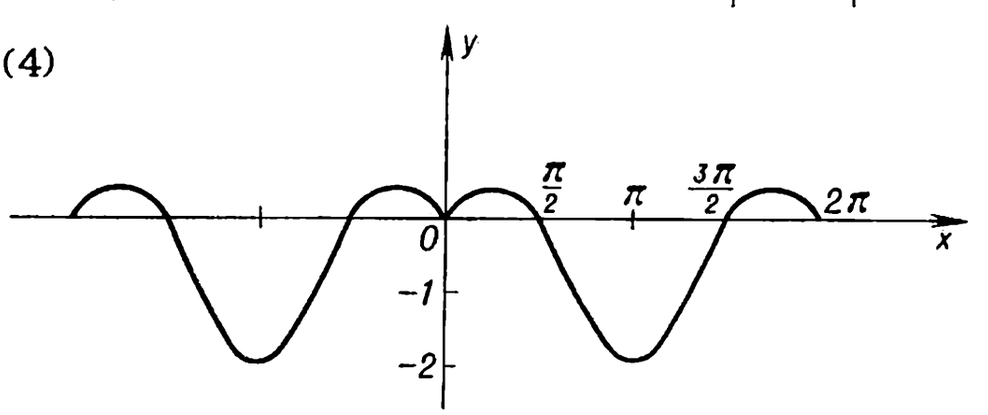
(1)


y
2
0
2
3
x

(2)


y
$\sqrt{0.5}$
0
2
x

(4)


y
$\frac{\pi}{2}$
π
$\frac{3\pi}{2}$
2π
0
-1
-2
x

1931. (1) -1.41 and 1.41; (2) 1.26; (3) 1.20; (4) 2.09; (5) -0.32; (6) 1.93; (7) 0.68; (8) -2.60, 0.34 and 2.26. **1932.** (1) 0.122 and 0.12; (2) 0 and 0; (3) 0.278 and 0.28. **1933.** (1) 0.15, 0.15, 0, 0. (2) 0.10, 0.102, 0.002, 2%; (3) 0.001, 0.001, 0, 0. **1934.** 6 cm^3.

1935. (3) $6x(x^2+a^2)^2\,dx$; (4) $\dfrac{2\,dx}{3\sqrt[3]{x}}$; (5) $\left(\dfrac{1}{3\sqrt[3]{x^2}}+\dfrac{1}{4\sqrt[4]{x^3}}\right)dx$.

1936. (1) $\dfrac{x\,dx}{\sqrt{x^2-a^2}}$; (2) $\dfrac{(1-3x)\,dx}{2\sqrt{1-x}}$; (3) $-\dfrac{4\,dx}{(x-2)^2}$; (4) $-\dfrac{dx}{x^2\sqrt{1+x^2}}$.

1937. (1) $\dfrac{2x\,dx}{x^2+2}$; (2) $(2\ln x+1)x\,dx$; (3) $6x\cos 3x^2\,dx$; (4) $-3\sin 6x\,dx$; (5) $2\sin^2 x\,dx$. **1938.** (1) $-\dfrac{2x\,dx}{\cos^2(1-x^2)}$; (2) $\cot\left(\dfrac{\pi}{4}-x\right)dx$; (3) $-\dfrac{dx}{\sin^2 x}$; (4) $\dfrac{x\,dx}{x^2-4}$. **1939.** 1.6.

1940. 0.32 cm. **1941.** 12.56 cm. **1942.** 0.15 m^3. **1943.** 0.01. **1944.** 0.08 cm. **1945.** 2%. **1946.** 2%. **1947.** (1) 2.03; (2) 24.69; (3) -131.4; (4) 0.807. **1948. 1.** 15 km/h, 10 km/h, 30 km, 20 km; **3.** $2(a^2-a+1)$; **4.** (a) $x\geqslant 0$; (b) $x\leqslant 0$; (c) $-1\frac{2}{3}a$ and a. **5.** (a) 32, 0; (b) $\frac{1}{2}$, $\frac{\pi}{2}$. **1949. 1.** 27, -18, 12, 42 and 3, 6, 12, 18, **2.** (a) $x<-0.75$, $x>0.5$; (b) $-2<x<-0.75$; (c) $-7<x<-1$; (d) $-1<x<0$ and $0<x<1$; (e) $\pi k+\frac{\pi}{3}<x<\frac{\pi}{2}+\pi k$ and $\pi k+\frac{\pi}{2}<x<\frac{2\pi}{3}+\pi k$; (f) $\frac{\pi}{2}k-\frac{\pi}{8}<x<\frac{\pi}{8}+\frac{\pi}{2}k$; (g) $0<x<0.5$; (h) $-3<x<-\frac{2}{3}$. **3.** (a) 1.25; (b) no solution; (c) 3; (d) no solution; (e) 0.5 and 1; (f) -2; 3.25; (g) -5.2; (h) -11; 24 **4.** (a) no solution; (b) $\frac{\pi}{5}+\frac{2\pi}{5}k$, $\frac{2\pi k}{3}$; (c) $\frac{\pi k}{4}$; $\pm\frac{2\pi}{3}+2\pi k$; (d) $\frac{\pi}{4}(4k+1)$; $2\pi k$; $\frac{\pi}{2}(4k+1)$; (5) (a) 2; $\frac{2\pi}{3}$; (b) 2; $\frac{\pi}{3}$; (c) $\frac{1}{2}$; $\frac{\pi}{3}$. **1950. 1.** $20\sqrt{2}$; $10\sqrt{2}$; $5\sqrt{2}$; $2.5\sqrt{2}$ or $2.5\sqrt{2}$; $5\sqrt{2}$; $10\sqrt{2}$; $20\sqrt{2}$. **2.** (a) 2; (b) no solution. **3.** (a) $-1\leqslant x\leqslant 5$; (b) $-3<x\leqslant 3$; (c) $x<-2$ and $x>2$. **5.** (a) $(2, 3)$, $\left(-2, \frac{1}{3}\right)$; (b) $(9, 25)$, $(25, 9)$.

1951. 1. $\dfrac{\sqrt{a^2t^2+2ast}+at}{2t}$ and $\dfrac{\sqrt{a^2t^2+2ast}-at}{2t}$. **2.** (a) $(-2, 3)$, $\left(2, \frac{2}{3}, -\frac{1}{9}\right)$; (b) $(1, -2)$, $\left(\frac{8}{3}, -\frac{16}{3}\right)$, $\left(\dfrac{5+i\sqrt{23}}{3}, \dfrac{-5-i\sqrt{23}}{6}\right)$,

$\left(\frac{5-i\sqrt{23}}{3}, \frac{-5+i\sqrt{23}}{6}\right)$. **3.** (a) $x < -3$ and $4 < x < 7\frac{15}{17}$;
(b) $4 < x < 7$. **4.** (a) $\frac{7\pi}{6}$; (b) $\frac{297}{425}$. **1952. 1.** 3, 7, 11, 15, 19.
2. (a) $x \neq -2$, $x \neq 2$; (b) $x \geqslant 2$; (c) $x > 4$; (d) $x \geqslant -1$;
(e) $-2 \leqslant x \leqslant 5$; (f) $x < -1$ and $x > 4$; (g) $\pi k \leqslant x \leqslant \frac{\pi}{2}+\pi k$;
(h) $2\pi k+\frac{\pi}{2} \leqslant x \leqslant \pi+2\pi k$; (i) $1 < x \leqslant 3$. **3.** (c) $\frac{18\sqrt{6}-21}{19}$;
(d) $\frac{3a^2+b-3a\sqrt{b}}{a^2-b}$; (e) $\frac{\sqrt{a^2-b^2-9}}{b}$; (f) $\sqrt[3]{(a^2-9)^2}$;
(g) $2(\sqrt{10}-\sqrt{5}+2\sqrt{2}-1)$; (h) $0.1(2\sqrt{3}-\sqrt{2})\sqrt{10}$; (i) $\sqrt[3]{16}+\sqrt[3]{4}+1$.
4. $\frac{3n-2}{3n+2}$. **5.** (a) $2(\cos 120°+i \sin 120°)$, $-1+i\sqrt{3}$;
$2(\cos 240°+i \sin 240°)$, $-1-i\sqrt{3}$;
(b) $2[\cos(-60°)+i \sin(-60°)]$; $1-i\sqrt{3}$; (c) $2(\cos 90°+i \sin 90°)$, $2i$, $2(\cos 270°+i \sin 270°)$, $-2i$; (d) $2(\cos 0°+i \sin 0°)$, 2;
$2(\cos 180°+i \sin 180°)$, -2. **1953. 1.** 10 days, 15 days or 16 days and 9.6 days. **2.** (a) 4; (b) 4. **3.** (a) $\frac{\pi}{2}(2k+1)$ and $\frac{\pi}{4}(4k+1)$;
(b) $\frac{\pi}{8}(4k-1)$; πk and $\frac{\pi}{4}(4k+1)$; (c) πk; $\frac{\pi}{4}(2k+1)$;
(d) $(-1)^k \arcsin\frac{\sqrt{2}}{4}+\pi k$; **4.** 1. **5.** $6\sqrt{3}+4$, $6\sqrt{3}-4$.

1954. 1. 24 h and 27 h. **2.** πd. **3.** (a) $2\frac{1}{3}$; (b) 2; (c) $-1, 2, 4$,
(d) 1, (c) 2; (f) ≈ -1.53; (g) 2; (h) 1000; (i) 3 and $-\frac{1}{6}$.
4. $(-2, -1)$ and $(2, -1)$. **5.** (1) $\alpha+\beta = \pi+2\pi k$,
$\alpha = \frac{\pi}{2}+2\pi k$, β—any angle and $\beta = \frac{\pi}{2}+2\pi k$, and α—any angle;
(2) $\frac{\pi}{2}k-\frac{\pi}{8} < x < \frac{\pi}{8}+\frac{\pi}{2}k$; (3) $2\pi k-\frac{2\pi}{3} < x < \frac{\pi}{3}+2\pi k$;
(4) $\frac{2-\sqrt{2}}{4}$ and $-\frac{2+\sqrt{2}}{4}$. **1955. 1.** $\therefore$ 3, 2, $\frac{4}{3}$, ...
2. (a) $-\sin 35°$; (b) 1. **3.** -0.4. **4.** (a) The interior of a circle of radius $R = 2$ with centre at (0, 0); (b) the inside points of a ring between the circles $r = 2$ and $R = 4$ with centre at (0, 0); (c) the upper half of the coordinate plane; (d) the portion of the plane between the rays $\arg z = \frac{\pi}{6}$ and $\arg z = \frac{2\pi}{3}$; (e) a circle of radius $R = 2$ with centre at (0, 0); (f) the portion of the coordinate plane

situated outside the circle $R = 5$ with centre at $(0, 0)$, including the points of the circumference; (g) the points on the circumference with centre $(-3, 0)$ and radius $R = 4$; (h) the points inside the circle with centre at $(2, -1)$ and $R = 2$. **6.** (a) $m > n$; (b) $m < n$; (c) $m < n$; (d) $m < n$; (e) $m < n$; (f) $m > n$. **1956. 1.** (a) $x^2 = 0$; (b) the y-axis, the x-axis, the bisector of the second and fourth quadrants. **2.** (a) $x \neq 0$; (b) $x \geqslant 0$; (c) $x \neq 2$, (d) x any real number. **3.** (a) $2(a^2+b^2)$; (b) $\frac{\sqrt{ax}}{a-2x}$. **4.** (a) No; (b) yes.

1957. 1. 2.4 m and 3.9 m. **2.** a^2. **4.** -2, -1. **1958. 1.** 10t. **2.** $4(x-1)$ if $x > 2$; 4 if $1 \leqslant x < 2$. (4) $x < -3$, $-2 < x < 0$ and $x > 1$. **1959. 1.** 8 days. **2.** $\frac{2b\sqrt{a^2-b^2}}{a^2-b^2}$. **4.** For $m > 2\frac{2}{3}$.

1960. 2. -2, 6. **3.** $\left[\left(\frac{a+\sqrt{a^2-4b}}{2}\right)^4, \left(\frac{a-\sqrt{a^2-4b}}{2}\right)^4\right]$ and $\left[\left(\frac{a-\sqrt{a^2-4b}}{2}\right), \left(\frac{a+\sqrt{a^2-4b}}{2}\right)^4\right]$. **4.** -4.5 for $x = 1$, for $x < -2$ and $x > 4$. **1961. 2.** (a) Yes; (b) no; (c) yes; (d) yes; (e) yes.

3. For $n \geqslant 0$ and $p > 0$. **4.** -1 and 6. **5.** πk. **1962. 1.** $0.5\pi l$

4. (a) ± 0.8; (b) $\pm\frac{\pi}{3}+\pi k$; (5) 0. **1963. 1.** 14 s. **3.** (a) $\pm 1, \pm 3$; (b) $\frac{\pi}{4}-\frac{\alpha}{2}+\frac{\pi}{2}k$; (c) $\arctan 2+\pi k$. **1964.** (1) $t = 6$ s; 64 m.

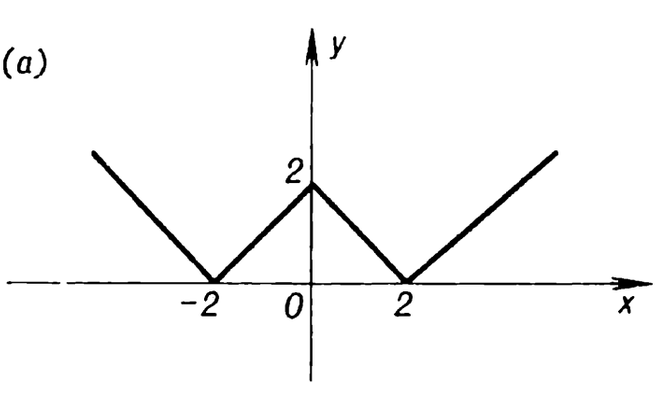
2. (a) Yes; (b) no; (c) yes. **4.** (a) 1000; (b) 10.

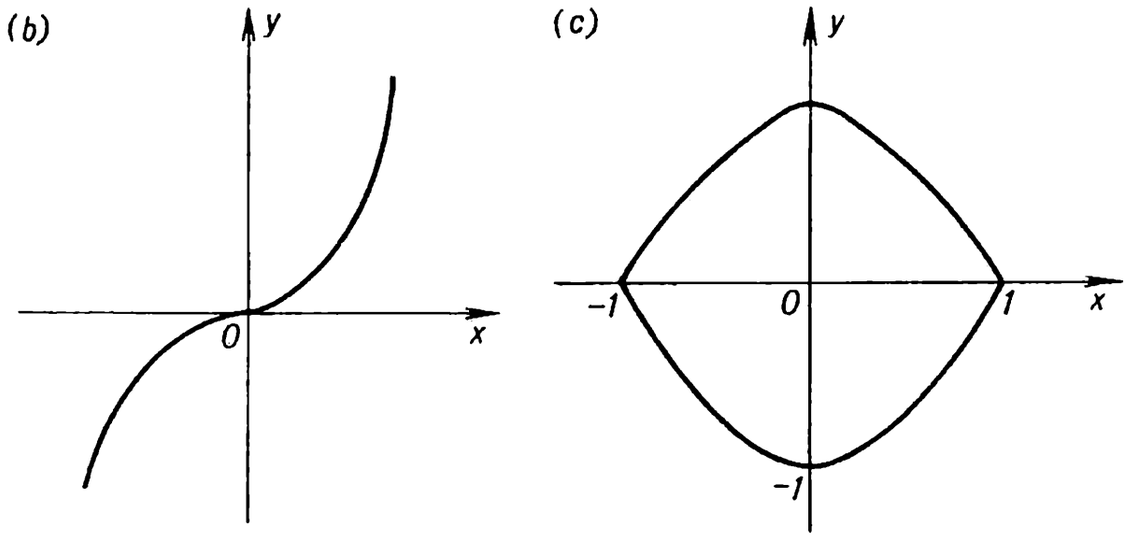
1965. 1. ≈ 13.3 l. **2.** (a) 0.5. **4.** (a) 7; (b) 0.5. **5.** $-i$, -3, 1.

1966. 1. ≈ 17.6. **2.** $a < -7.5$. **3.** $A = 2$, $B = 2$. **4.** (a) 5; (b) $1.5-2i$; (c) $(-1)^k\frac{\pi}{12}+\frac{\pi}{2}k$; (2) πk, $\frac{\pi}{4}(4k+1)$; (e) $\frac{\sqrt{3}}{2}$; (5) $1-\frac{m^2+n^2}{2}$. **1967. 1.** ≈ 265 mm. **2.** (a) $a = -5$, $b \neq 4$; (b) $a = -1.5$, $b = -1.25$. **3.** $20\sqrt{2}(\sqrt{3}-1)$. **4.** $1+2i$, $\pm\sqrt{2}$, $\pm\frac{\sqrt{3}}{3}$.

5. 1. **6.** 100. 0.

7. (a)





1968. 1. The first instantaneous stop of the body occurs at the time $t = 4s$ at a distance of 64 m from the point A. The second stop happens at the point A (the body returned to the initial position) at the time $t = 8$s. After this time the body will have no stops and will move in one direction. **3.** (a) one; (b) one; (c) a set; (d) none; (e) a set; (f) for $a \neq -6$ one solution, for $a = -6$ no solution. **5.** (a) $\frac{10^n(9n-1)+1}{9\times 10^n}$; (b) $\frac{9\sqrt{2}}{4}$. **6.** $\frac{m}{2}, \frac{m}{2}, 90°$.

7. 25°34′.

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